X-Ray Deficiency in Mesonic Atoms

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We present an analysis of π-mesonic atoms, based upon cascade calculations taking into account the known processes of radiation, Auger transitions, and nuclear absorption. This analysis, together with the previous one on μ-mesonic atoms, is intended to provide a deeper insight into the unsolved problem of the deficiency of x rays in mesonic atoms. It is shown that the π-mesonic L x-ray yields (for $Z\leq 20$) are quite insensitive to the strength of nuclear absorption and depend only upon the chosen initial meson population of the higher levels. Similarly, the ratios of basic (K_α, L_α, etc.) to higher x-ray yields, both for μ and π mesons, depend strongly on the initial distribution. The best agreement between the calculations and experiment was obtained for a "modified statistical" initial population of the form $(2l+1)e^{al}$, with $a=0.2$, in the $n=14$ level. From the existing experimental data on π-mesonic K x rays, the mean life of the π meson in nuclear matter was deduced: $\tau_c=2.75\times10^{-23}$ sec. Within the framework of the present theory we are still unable to account for the x ray deficiency in the light atoms. However, it is shown that the quantum loss as a function of energy is different for π- and μ-mesonic atoms, and therefore it is very probably due to a real physical effect. Furthermore, by comparing our predicted Auger electron yields with the experimental data, we can rule out any hypothetical simple Auger process in which the full energy of the "missing" quantum is given to a single electron.

1. INTRODUCTION

THE question of the deficiency of x rays in mesonic atoms[1] has been the subject of many investigations[2–7] during the past few years. No satisfactory solution to this problem has been offered so far.

In a series of experiments measuring the characteristic K and L x-ray yields of mesonic atoms, Stearns and Stearns[8–10] discovered a very large discrepancy between the observed and calculated values. The experimental yields for the light elements were smaller (by a large factor) than the predicted yields obtained from a simple theory of this process.[11] Attempts to explain the above discrepancy by some kind of an "external" Auger effect[2] turned out to be unsuccessful.[3,4] The possibility[5] that a large number of mesons reach the metastable $2s$ state—from which no radiation and only Auger $2s\rightarrow 1s$ transitions are possible—was examined critically by Ruderman[7] who arrived at the conclusion that this, very probably, cannot explain the x-ray deficiency. In fact, one can state more generally that the experimental evidence on Auger electrons associated with the capture of μ-mesons in the light elements (C, N, O) of nuclear emulsions excludes the possibility of explaining the "missing" K x rays of carbon in terms of any effect leading to the emission of single Auger electrons (see reference 12 and Sec. 6 of the present work).

In a previous paper, I, we discussed μ-mesonic atoms on the basis of straightforward electromagnetic cascade calculations and compared the results with the experiments on mesonic x rays and Auger electrons. The only free parameter in the theory was the initial meson distribution.

In the present paper we shall present our results for π-mesonic atoms as well as a detailed discussion of the entire subject, in the hope that our more precise predictions (on the basis of ordinary electromagnetic theory) will bring about a better understanding of the problem.

The analysis of π-mesonic atoms differs from the previous calculations of μ-mesonic atoms in three respects:

(1) Because of the more intense π-meson beams available, the experimental x-ray yields of Stearns and Stearns[9,10] were much more precise and given in absolute terms (number of quanta per stopped meson) and not, as for μ-mesons, only with respect to oxygen (K x rays) and silicon (L x rays). Therefore, we will be able to compare the calculated and observed yields in an absolute way and to study the influence of the initial population upon absolute K and L x-ray yields, as well as upon the ratio of the basic (K_α and L_α) to all K and L x rays.

(2) π mesons are absorbed by the nucleus from states higher than the ground state and this affects the cascade calculations in a very sensitive way. Absorption is described in the present work by means of a single parameter, the lifetime of the π meson in nuclear matter.

(3) On the other hand, the theoretical uncertainty as to the de-excitation mode of the metastable $2s$ state which we had to consider in μ-mesonic atoms, will not

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[1] For a comprehensive review, see D. West, *Reports on Progress in Physics* (The Physical Society, London, 1958), Vol. 21, p. 271.
[2] T. B. Day and P. Morrison, Phys. Rev. **107**, 912 (1957).
[3] J. Bernstein and T. Y. Wu, Phys. Rev. Letters **2**, 404 (1959).
[4] T. B. Day and J. Sucher, Air Force Office of Scientific Research Report TN-59-771, 1959 (unpublished).
[5] N. A. Krall and E. Gerjuoy, Phys. Rev. Letters **3**, 142 (1959).
[6] R. A. Ferrell, Phys. Rev. Letters **4**, 425 (1960).
[7] M. A. Ruderman, Phys. Rev. **118**, 1632 (1960).
[8] M. B. Stearns and M. Stearns, Phys. Rev. **105**, 1573 (1957).
[9] M. Stearns and M. B. Stearns, Phys. Rev. **107**, 1709 (1957).
[10] M. B. Stearns, M. Stearns, and L. Leipuner, Phys. Rev. **108**, 445 (1957).
[11] G. R. Burbidge and A. H. de Borde, Phys. Rev. **89**, 189 (1953), and A. H. de Borde, Proc. Phys. Soc. (London) **A67**, 57 (1954).
[12] Y. Eisenberg and D. Kessler, Nuovo cimento **19**, 1195 (1961), hereafter referred to as I.

appear again in the present context, because in all s states the π meson is predominantly absorbed.

In Sec. 2, our results on μ-mesonic atoms are summarized. In Sec. 3, it will be shown that the π-mesonic L x rays do not depend on the strength of the absorption of the meson by the nucleus, but depend quite sensitively on the chosen initial population. The latter can thus be determined uniquely.

In Sec. 4, we consider the K x-ray yields which depend strongly on the absorption parameter, once a definite initial population is adopted. The lifetime of the π meson in nuclear matter is thus determined. The results for M x rays are also included in this section, although no experimental data are yet available.

The expected Auger electron yields and spectra from π mesons stopped in nuclear emulsions are given in Sec. 5 and compared with the rather scarce experimental data.

The results are discussed in Sec. 6 and summarized in Sec. 7.

2. SUMMARY OF RESULTS ON μ-MESONIC ATOMS

Cascade calculations for μ mesons captured in a number of elements ranging from Li to Ag were performed[12] by using the calculated radiative and Auger transition probabilities. The calculations were started from the $n=14$ level and hydrogenic wave functions were assumed, since from this level downwards the μ meson is already below the electronic K shell.

Various initial populations in the $n=14$ level were tried because we have no definite *a priori* knowledge of the distribution of the mesons among the substates of a given higher level, except for an intuitive preference of the statistical $(2l+1)$ distribution. Comparison of the calculated x-ray yields with the experimental values, however, did not allow us to draw conclusions concerning the initial population. Indeed, the absolute K and L x-ray yields were not determined precisely enough by experiment, and only the relative values with respect to oxygen and silicon, respectively, were sufficiently accurate for comparison. It turned out, however, that the calculated values, if similarly normalized, were essentially independent of the assumed initial population.

The results showed that the experimental K x-ray yields for elements with $Z<6$ and the L x-ray yields for $Z<14$ are much lower than the predicted values. If plotted as a function of energy, the ratio of observed to expected yields of both K and L x rays increases roughly linearly from about 0.20 at 20 kev to 1.00 at about 90 kev. Above this energy the agreement was satisfactory.

On the other hand, we were able to show that, in contrast to the (normalized) absolute yields, the relative yields of the basic K_α line to all K lines and of L_α to all L are indeed sensitive to the assumed initial population. As the discrepancy between experiment and calculation seems to depend upon the quantum energy and not on the specific transition, it is expected that the discrepancies in the above relative yields should cancel and that the observed values should agree with the calculated ones. In order to achieve this agreement, the initial population must be chosen more peaked towards the high l values than the statistical $(2l+1)$ distribution. We have tried distribution of the form $(2l+1)\exp(al)$ and the best fit was obtained for the choice $a=0.2$. Such an initial distribution was shown to be not unreasonable because a statistical distribution in a level with very high principal quantum number will gradually become more peaked towards high orbital momentum states while the meson cascades down.[13]

The above considerations do not depend strongly on whether the transition from the metastable $2s$ state to the ground state takes place by an Auger S transition or by a "mixed" transition as proposed by Ruderman.[7] However, the expected number of Auger electrons in the light elements of nuclear emulsion depends very strongly on this transition, and comparison of our predictions with the experimental data of Pevsner *et al.*[14] showed that the "mixed" transition must be preponderant. More generally, the small number of Auger electrons experimentally observed does not leave much room for explaining the x-ray deficiencies in the light elements by any competing pure Auger process.

3. THE π-MESONIC L X RAYS AND INITIAL DISTRIBUTION

The calculations are similar to those in I, except that now absorption from the low angular momentum states takes place in addition to radiation and Auger effect. The absorption probability was taken equal to the overlap of the meson wave function with the nucleus divided by the lifetime (τ_c) of the π meson in nuclear matter (see Appendix). This latter parameter must be determined from the experiment. The overlap integral varies only slowly with the principal quantum number n, but depends strongly upon the orbital quantum number l. This fact arises, of course, from the behavior of the hydrogenic wave functions near the origin. Thus, the absorption probability decreases by about three orders of magnitude between any l and $l+1$. Radiation and Auger effect, on the contrary, vary only relatively slowly with n and l. Competition between absorption, on the one hand, and radiation and Auger effect, on the other hand, therefore usually takes place at a certain orbital momentum l. Below this l, capture is practically 100% effective, whereas for higher l's the electromagnetic processes dominate. Thus the gross behavior of the cascade is determined mainly by the intrinsic strong

[13] Recently a different initial population was proposed by R. A. Mann and M. E. Rose, Phys. Rev. **121**, 293 (1961). We have performed a cascade calculation (for μ^- mesons in carbon) by using this distribution. The resulting K and L x-ray yields were essentially the same as ours from C^{III} and C^{IV} (see Table 3 of I). However, the K_α/all K ratio was too low, 0.46, as compared with our value from C^{IV}, 0.66, and the experimental value, 0.80.

[14] A. Pevsner, R. Strand, L. Madansky, and T. Toohig, Nuovo cimento **19**, 409 (1961).

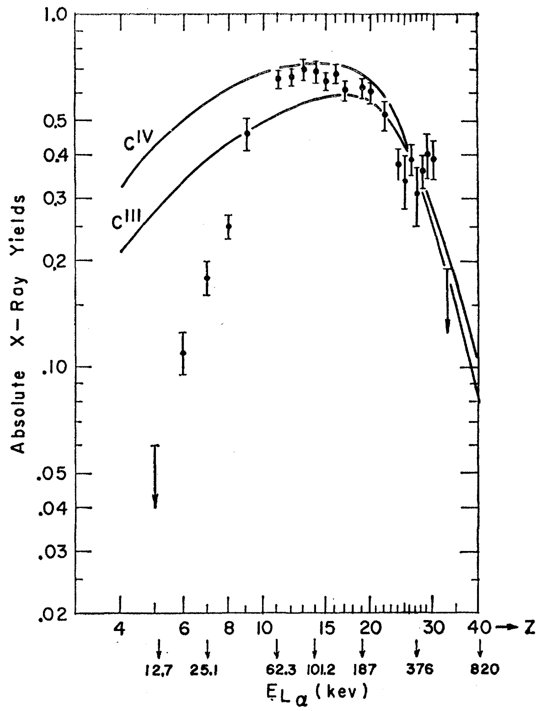

FIG. 1. Calculated total L x-ray yields for the "statistical" (C^{III}) and "modified statistical" (C^{IV}) initial meson distributions. The experimental points are those of Stearns *et al.*

l dependence of the overlap integral. Since absorption from the d states begins to compete significantly with electromagnetic processes only at $Z \gtrsim 20$, the L x-ray yields of the light elements are not affected at all by the choice of τ_c.

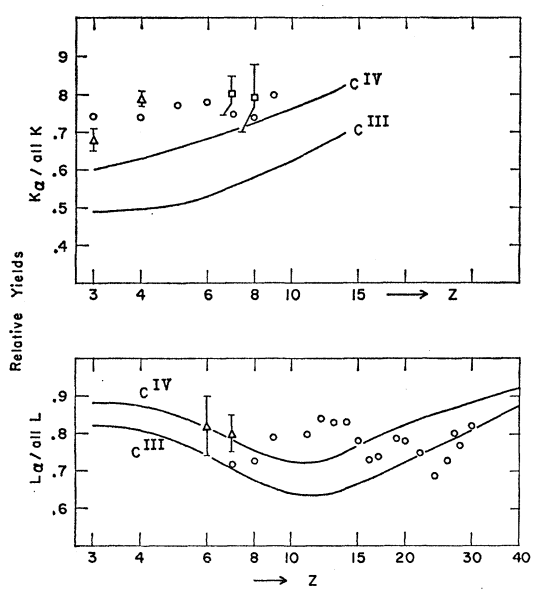

FIG. 2. Calculated ratios of basic to all K and L x-ray yields for cascades C^{III} and C^{IV}. The experimental points are: ○ Stearns *et al.*, □ Camac *et al.*, and △ West *et al.*

In Fig. 1 we plot the calculated values of the π-mesonic L x-ray yields for two initial distributions, the statistical $[C^{III} \propto (2l+1)]$ and the "modified statistical" $[C^{IV} \propto (2l+1)\exp(0.2l)]$. The ratios L_α/all L are shown in Fig. 2. The results of Fig. 1 and 2 were obtained by using the value $\tau_c = 2.75 \times 10^{-23}$ sec, but as discussed above, the calculated L x-ray yields are quite insensitive to the choice of τ_c. It can be seen from Fig. 1 that agreement with experiment (for $Z \geq 11$) is much better for the "modified statistical," than for the statistical initial population. The same is true for the ratios L_α/all L (Fig. 2). For these, the experimental values do not reflect the detailed behavior predicted by theory, but in the average the agreement can be considered as

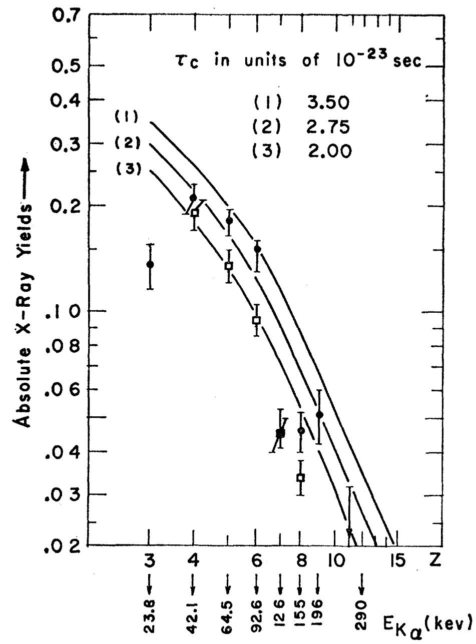

FIG. 3. Calculated (C^{IV}) total K x-ray yields for 3 different values of τ_c. The experimental points are ● Sterns *et al.*, □ Camac *et al.*

satisfactory. The K_α/all K ratios will be discussed in Sec. 3.

It is very gratifying to note that the same initial population, namely the "modified statistical" distribution, produces reasonable agreement between observed and calculated values of all those quantities which depend on the initial population: the π-mesonic L x-ray yields and the ratios of basic to higher transitions in μ- and π-mesonic atoms. From now on this distribution will be used exclusively.

It is seen from Fig. 1 that for the light elements from $Z=9$ down there is an increasing discrepancy between the observed and calculated yields. We shall discuss this point in Sec. 6.

4. THE π-MESONIC K X-RAY FIELDS AND THE STRENGTH OF NUCLEAR ABSORPTION

In contrast to the L x rays, the K x rays depend on τ_c as a consequence of the competition between absorption and electromagnetic transitions which takes place in the p states of the lighter elements ($Z \leqq 15$). K x-ray yields were calculated with the "modified statistical" initial population and with τ_c as a free parameter. Curves for three values of τ_c (2.00, 2.75, and 3.50×10^{-23} sec) are displayed in Fig. 3 together with the experimental data of Camac *et al.*[15] and Stearns and Stearns.[9] The results of Camac *et al.*, while showing the same general trend, are consistently lower than those of Stearns and Stearns. Until more experimental results are available we accept the more recent data of Stearns and Stearns. These agree with curve 2 ($\tau_c = 2.75 \times 10^{-23}$ sec), except for a closed shell effect at $Z = 7$, 8 and the x-ray deficiency at $Z = 3$. It is clear that an accurate determination of τ_c will be possible with the help of better experimental data for the K x-ray yields of light elements.

The relative yields K_α/all K depend again on the initial distribution. Figure 2 shows that the statistical distribution (C^{III}) is completely excluded. Agreement with C^{IV} can be considered as satisfactory pending more precise experimental results.

Although M x rays were seen in the experiments of Stearns and Stearns, no yields were quoted. However, we plot the calculated yield curves (C^{IV}) in Fig. 4, for future comparison with experiment.

The extent to which various angular momentum states contribute to the absorption of the π mesons in the different elements is displayed in Fig. 5. (These results were obtained from the best description that we now have—namely cascade C^{IV} with $\tau_c = 2.75 \times 10^{-23}$ sec.)

The largest contribution to the absorption from a given l state comes from the lowest possible levels: $1s$, $2p$, $3d$, etc. For example, 52% of all p-state absorption in Li is due to the $2p$ state; this ratio increases regularly, it is 73% at the peak of the total p absorption ($Z = 12$) and reaches 91% at $Z = 27$, where p absorption ceases to be important. Similar behavior is observed from the other angular momentum states.

5. AUGER-ELECTRON YIELDS FROM π-MESONIC ATOMS

Hitherto, Auger electrons from π-mesonic atoms have been measured only very roughly in nuclear emulsions.[16] In Fig. 6 we have displayed the predicted Auger-electron spectra (above 15 kev) for the light and heavy emulsion elements, separately. The total predicted yield in emulsions is 37.5%, whereas 22% were found experimentally.[16] This is not too surprising, considering the experimental difficulties.

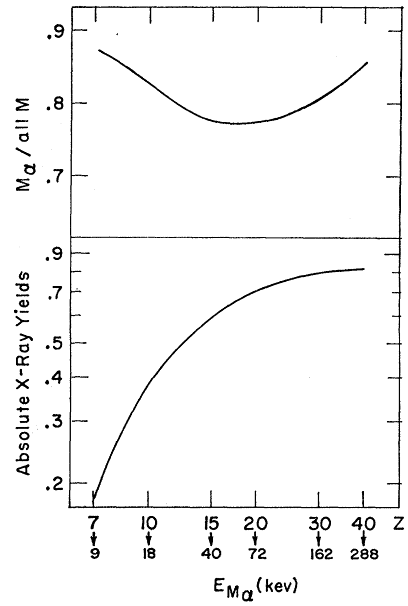

FIG. 4. Calculated (C^{IV}) total M x-ray yields and relative M_α/all M ratios.

In view of the x-ray deficiency in the light elements, it would have been interesting to measure the Auger-electron yields for π-mesonic atoms in the light and heavy emulsion elements separately. Such an experiment seems, however, very difficult to perform.

On the other hand, the present calculations show that only 7.5% of the Auger electrons of more than 15-kev energy in nuclear emulsion are expected to arise from capture in the light elements C, N, O. If the energy cutoff is 30 kev, the contribution from the light elements will reduce to 4%.

It should be pointed out that, owing to the big difference of the Auger-electron yields from light and heavy

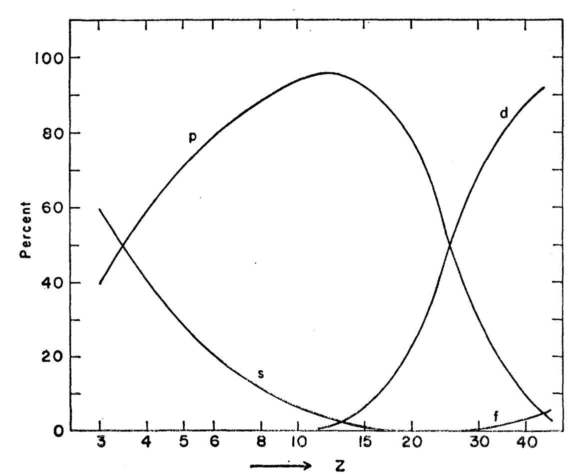

FIG. 5. Percentage contribution of different angular momentum states to π-meson absorption, as a function of Z, in light and medium nuclei.

[15] M. Camac, A. D. McGuire, J. B. Platt, and H. J. Schulte, Phys. Rev. 99, 897 (1955).

[16] E. B. Chesick and J. Schneps, Phys. Rev. 112, 1810 (1958).

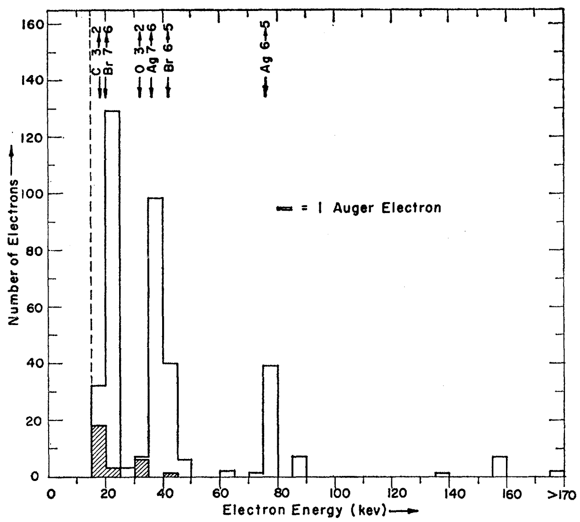

FIG. 6. Calculated energy spectrum of Auger electrons (≥ 15 kev) normalized to 1000 stopping π mesons in nuclear emulsions. Shaded area: capture in C, N, O.

elements, the total yield depends sensitively on the assumed capture distribution between the gelatine and the AgBr. In our calculations, the μ-capture distribution observed by Pevsner *et al.*[14] was used.

6. DISCUSSION

Within the framework of the present theory, the discrepancies between the predicted and observed x-ray yields in the light atoms are still unexplained. The x-ray efficiencies (observed/calculated yields) were plotted as a function of quantum energy in Fig. 7, for μ- and π-mesonic atoms separately. Both show a characteristic

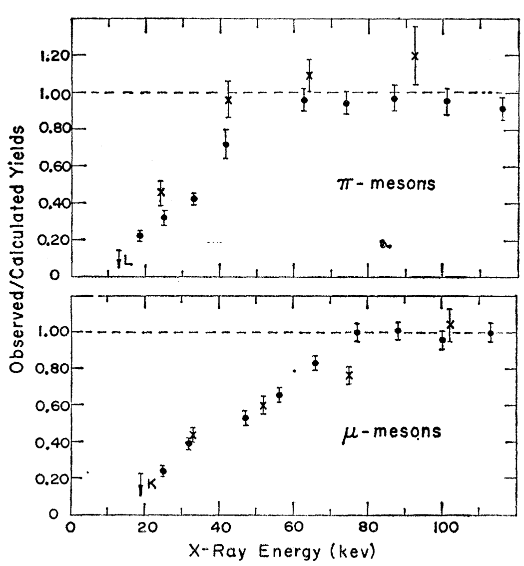

FIG. 7. Ratio of observed-to-calculated x-ray yields as a function of quantum energy. The experimental values were taken from Sterns *et al.*: ×—*K* lines, ●—*L* lines.

increase of efficiency with energy. It is clear from the graphs, however, that the behavior of the two types of mesons is quite different: The efficiency of μ-mesonic atoms approaches 100% only in the region of about 90 kev, whereas this already occurs, for π-mesons, at about 40 kev. Thus, for instance, the 42.1-kev π-mesonic K x ray of Be shows an efficiency of $96\pm10\%$, and the 62.3-kev L line of Na, $96\pm6\%$, whereas the experimental/predicted yield for the 47-kev μ-mesonic L x ray of Na is as low as $53\pm4\%$, and the 52.1-kev μ-mesonic K x ray of B, $60\pm5\%$. This could not have been noticed by Ferrell,[6] who plotted only the observed yields of x rays as a function of the quantum energy. By pure chance, the π-mesonic L x-ray yields around 100 kev (Si-lines) are about 0.80 quanta per stopping meson, which is also the normalization value used by Stearns and Stearns in the μ experiment. Thus, both π and μ observed yields coincide at $\sim$100 kev. This coincidence is really irrelevant; the only relevant quantities in this connection are the observed/calculated yields, such as those plotted in Fig. 7.

Even within the present experimental uncertainties, it seems impossible to describe both the μ- and π-mesonic x-ray yields with the help of a single efficiency curve. It therefore seems difficult to put the blame of the "missing x rays" on the experimenters, especially as both μ- and π-mesonic x rays were measured under the same experimental conditions by the same observers and with the same equipment.

On the other hand, it seems that all attempts to explain the "missing x rays" with the help of some unconventional Auger-process are doomed to failure. We have shown previously, in our work on μ-mesonic atoms, that the experiment of Pevsner *et al.*[14] hardly accounts for the "ordinary" Auger electrons predicted by present theory: Only if we assume that *all* metastable 2*s* states de-excite through the Ruderman "mixed" transition[7] can we predict the correct number of 2 observed Auger electrons for the above experiment; otherwise, up to at least 50 Auger electrons would have been expected. If Auger electrons were to account for the "missing x rays," an additional 120 ± 30 electrons should have been observed (mainly from carbon) in Pevsner's experiment, so that this possibility is completely ruled out.

7. CONCLUSIONS

(a) There still is a very serious discrepancy between the observed and calculated x-ray yields in the light elements, for both π- and μ-mesonic atoms. In other words, even a detailed cascade calculation, in which all the important Auger and radiative transition probabilities of all orders are taken into account, is incapable of explaining the observed data.

(b) Since the discrepancy is *not a unique function* of the quantum energy (being different for K lines and L lines and for μ mesons and π mesons; see Fig. 7) it could not be attributed to a simple experimental loss of

detection efficiency at low quantum energy. It must be due, very probably, to some real physical effect.

(c) The solution to the problem should not be sought after in any mechanism which transfers the entire quantum energy into a single Auger electron, since, at least for μ mesons in carbon, the experimental evidence[14] barely accounts for the numbers of "ordinary" Auger electrons expected from the theory.[12]

(d) The experimental L x-ray yields, for π mesons above the discrepancy region, as well as the ratios K_α/all K and L_α/all L, for μ and π mesons, all seem to indicate that the "modified statistical" population $[(2l+1)e^{0.2l}]$ of the $n=14$ level gives the best description of the actual physical situation. More precise experiments might help in understanding the so far unexplained discrepancies: Absolute determination of the μ-mesonic x rays, determination with better resolution of the relative intensities of K_α, K_β, K_γ, and L_α, L_β, L_γ in both μ- and π-mesonic atoms, measurement of the yields of π-mesonic M x rays in the 20–60 kev region, determination of the Auger electron spectra of μ- and π-mesonic atoms in light and medium elements. It also seems worthwhile to attempt the measurement of K^--mesonic x rays, for which calculations are now in progress. With the intense K^- beams now being developed, such an experiment will soon become feasible.

ACKNOWLEDGMENT

We are very grateful to M. Schatz for his help in the numerical computations.

APPENDIX

Radiative and Auger transition probabilities were given in I. The π-meson capture probability by the nucleus was taken as

$$P_c=J/\tau_c,$$

where J is the overlap integral of the pion wave function with the nucleus, and τ_c is the mean lifetime of the meson in nuclear matter. τ_c was determined by comparison with experiment (Sec. 4).

Using hydrogenic wave functions for describing the π meson, we get

$$J=\int^{R}4\pi r^2|\psi_{nl}|^2dr=\frac{(n-l-1)!(n+l)!}{2n}\times\int_0^{2ZR/na_\pi}\left[\sum_{\lambda=0}^{n-l-1}\frac{(-\rho)^\lambda}{\lambda!(n-l-1-\lambda)!(2l+1+\lambda)!}\right]^2\times e^{-\rho}\rho^{2l+2}d\rho,$$

where R is the radius of the nucleus involved, $R=R_0A^{\frac{1}{3}}$ (R_0 was taken as 1.2×10^{-13} cm), and a_π is the radius of the first Bohr orbit of the π meson.

The integrals were calculated with the help of the formula:

$$\int_0^x e^{-\rho}\rho^k d\rho=k!-e^{-x}[x^k+kx^{k-1}+k(k-1)x^{k-2}+\cdots],$$

or with the help of the expansion:

$$\int_0^x e^{-\rho}\rho^k d\rho=e^{-x}\frac{x^{k+1}}{k+1}\left[1+\frac{x}{k+2}+\frac{x^2}{(k+2)(k+3)}+\frac{x^3}{(k+2)(k+3)(k+4)}+\cdots\right],$$

which converges rapidly for small x.