Fermi Surface and Positron Annihilation in Sodium

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The angular correlation of photons from positrons annihilating in polycrystalline sodium has been measured. The results show two things: (a) In comparison with a free-electron theory, the Fermi surface in Na is probably anisotropic by an amount of the order of 5% of p_F. (b) The probability of annihilation is not very velocity dependent over the range of conduction-electron velocities in Na. This is not inconsistent with the calculations of either Daniel and Friedel or of Kahana.

RECENT work by Migdal,[1] Luttinger,[2] (also with Kohn and with Ward), and Daniel and Vosko[3] has caused reappraisal and discussion[4] of the existence of a Fermi surface in metals when electron-electron interactions are considered. These several writers point out that since electron-electron interactions energies are of the order of the Fermi energy, one would expect very considerable smearing of the Fermi surface even at $T=0$. Although the analyses given all show a discontinuity in occupation of states at the $T=0$ Fermi surface of the free electron theory, it has not yet proved possible to calculate the extent of this discontinuity. Daniel and Vosko[3] show that at the Fermi surface of sodium the fractional discontinuity in occupation number, $\Delta n/n$, $\approx\frac{1}{2}$ when only electron hole pair correlation is considered and that it will probably increase if exchange interactions are considered.

This short paper presents some experimental data concerning the "sharpness" of the Fermi surface in sodium. For some time now experimental measurements of the angular correlation of photons from positron annihilation in metals have yielded information about the momentum distribution of the conduction electrons in the metal. (For a recent review see Wallace.[5]) We have observed the angular correlation of photons from positrons annihilating in polycrystalline sodium with better resolution and statistical accuracy than has been heretofore reported. The apparatus was similar to that used before[6,7] and will not be described here. Also as before, we will present the data as the slope of chords of the angular correlation curve. This presentation shows more significantly the experimental statistical errors and is more closely related to the momentum distribution. The slopes are in fact, proportional to the momentum times occupation number in momentum space. In this form the results are shown in Fig. 1, on which we have also indicated an estimate of the instrumental resolution. The resolution is composed of two terms; first, the slit angle subtended at the source which was 0.0003 rad and second, the thickness of the Na source. The latter was taken to be an average of 0.01 in., allowed for surface irregularities, and the range of positron penetration into the Na metal. For the penetration we used the range-energy data for β spectra. This is certainly too large because of back scattering and annihilation. The resolution function shown was obtained by combining these two effects, which are comparable in size, slit width, and source thickness.

These comments regarding the Fermi surface follow from an examination of the results: If one supposed a free electron theory cutoff in the momentum distribution and averaged it over the estimated instrument resolution, the slope expected would be given by the dotted line in Fig. 1. The results appear to favor a slightly more gradual slope. While a wider instrumental resolution function could account for this, the increase would be about 40% and it is thought that the estimate of resolution is more accurate than that. If, on the other hand, the width of the resolution function has been estimated correctly, this implies either a slightly sloped cutoff in the momentum distribution or a nonspherical Fermi surface in Na, the nonsphericity being as much as 5% of p_F. In any case, the cutoff is much greater than, for example, might be expected from Luttinger's

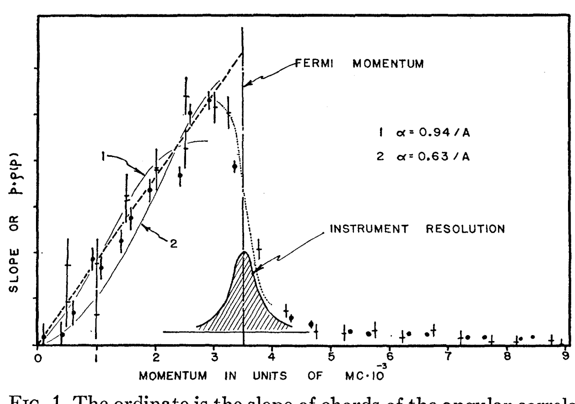

FIG. 1. The ordinate is the slope of chords of the angular correlation data. This slope is proportional to momentum times momentum space density, $p\cdot\rho(p)$. The small shaded figure is the instrumental resolution.

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[3] E. Daniel and S. H. Vosko, Phys. Rev. **120**, 2041 (1960).

[4] See Luttinger's article and the discussion in *The Fermi Surface* (John Wiley & Sons, Inc., New York, 1960).

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schematic drawing[8] and possibly sharper than the cutoff given by Daniel and Vosko[3] for sodium. Finally, it should be stated that this experiment observes the cutoff in the momentum distribution which generally is not the same as the Fermi surface. However, in the special case of sodium one expects that the two are almost identical since the effective mass is known to be very close to the real mass.

A further conclusion, this one regarding annihilation probability as a function of velocity, can be drawn from the results. The experiment measures the distribution in momentum of photons from annihilation events, $N(p)$. One may consider this to be the momentum distribution of electrons in the metal, $n(p)$, times the probability of annihilation, $P(p)$. That is, $N(p) = n(p)P(p)$. If one takes $n(p) \propto p^2$ following the free-electron theory, then the results can be used to determine $P(p)$. To the extent that the results fit the dashed straight line, $P(p)=$constant over this range of p.

Various theories of $P(p)$ have been discussed. Without Coulomb interactions $P(p)$ is a constant. With a Coulomb interaction between an otherwise free electron and positron pair, $P(p) \propto 1/p$ in clear disagreement with this experiment. Daniel and Friedel[9,10] discussed a theory using a screened Coulomb interaction which enhances low-velocity annihilation too much for agreement with experiment but less than the $1/p$ dependence of a pure Coulomb field. They combined this interaction with the effect discussed by Landsberg[11] to account for the low-energy "tails" of soft x-ray spectra. Landsberg suggested that a vacancy in the conduction band has a very short lifetime because it is quickly filled by nonradiative transitions among electrons of the conduction band. This short lifetime broadens the energy of states at the bottom of the band sufficiently to give the observed x-ray "tails." In terms of this experiment, Daniel and Friedel suggested that the broadening of energy levels reduces the density of states in the low-energy region and hence diminishes the fraction of low-momentum annihilation events. Following these authors we have calculated the relative annihilation probability for two values of the screening constant, $\alpha=0.94/\text{A}$ and $\alpha=0.63/\text{A}$. From the figure it is seen that a value of α between these two will probably give a fair fit to the results. Other values of α for Na are 0.65/A[12] and 0.85/A.[13] A more fundamental theory of the annihilation probability has been given by Kahana[14] who obtained approximate solutions of a Bethe-Goldstone equation for an electron positron pair within a sea of interacting electrons. He found that both the screening of a pure Coulomb interaction and the exclusion principle enhance the annihilation rate for electrons at the Fermi surface and diminish rate for low-velocity electrons sufficiently that $P(p)$ is again almost constant over the range of velocities of conduction electrons. For Na he estimated that $P(0)$ is within about 2% of $P(p_F)$, which is certainly compatible with these experimental results.

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[8] See Fig. 3 of Luttinger's article, reference 4.
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