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# Free-Carrier Voigt Effect in Semiconductors

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Measurements of the Voigt and the Faraday effects have been made on samples of $n$-type InAs and InSb. The Voigt and Faraday data are used together to obtain values of both the effective mass and concentration of free carriers which are consistent with values given by other methods. An experimental technique for measurement of small phase shifts in the Voigt effect is described.

## I. INTRODUCTION AND THEORY

RECENTLY, Teitler and Palik[1] observed the free-carrier Voigt effect in $n$-type InAs and InSb. In the present paper we discuss an extension of the techniques used in the earlier work and show how the Voigt and Faraday effects may be used to yield complementary information about a given semiconductor.

In the Voigt-effect experiment, radiation propagates through an optically isotropic substance in a direction normal to a static magnetic field. We consider free carriers in the Drude-Zener approximation with effective mass $m^*$ and relaxation time $\tau$ for samples whose geometry is such that depolarization effects are negligible. If $\omega$, the angular frequency of the incident radiation, is away from the cyclotron frequency and is such that $\omega\tau \gg 1$, the indexes of refraction for radiation with polarization in the directions parallel and perpendicular to the static magnetic field are respectively

$$\begin{aligned} n_{\parallel} &= \epsilon^{\frac{1}{2}}(1-\omega_p^2/\omega^2)^{\frac{1}{2}}, \\ n_{\perp} &= n_{\parallel}\{1-[\omega_p^2/(\omega^2-\omega_p^2)][\omega_c^2/(\omega^2-\omega_p^2-\omega_c^2)]\}^{\frac{1}{2}}. \end{aligned} \tag{1.1}$$

Here $\epsilon$ is the static dielectric constant of the host lattice; $\omega_c$ is the cyclotron frequency, $eH/m^*c$; $\omega_p^2$, the plasma frequency squared, equals $4\pi Ne^2/\epsilon m^*$; and we note that $n_{\parallel}$ is just the zero-magnetic-field index of refraction. We have used unrationalized cgs units.

The difference in the indexes of refraction in Eqs. (1.1) is the basis of the occurrence of magnetic double refraction, i.e., the Voigt effect. When radiation propagates through a sample in a direction normal to a static magnetic field, we may define a phase shift

$$\delta = (\omega d/c)(n_{\parallel}-n_{\perp}), \tag{1.2}$$

where $d$ is the thickness of the sample. Expanding $n_{\perp}$ around $n_{\parallel}$ in Eq. (1.1), we may write

$$\delta = \frac{\epsilon d}{2cn_{\parallel}} \frac{\omega_p^2}{\omega} \frac{\omega_c^2}{\omega^2-\omega_p^2-\omega_c^2} \times\left[1+\frac{1}{4}\frac{\epsilon}{n_{\parallel}^2}\frac{\omega_p^2}{\omega^2}\frac{\omega_c^2}{\omega^2-\omega_c^2-\omega_p^2}+\cdots\right]. \tag{1.3}$$

For impure samples at low magnetic fields, i.e., when $\omega \gg \omega_p \gg \omega_c$, we have

$$\delta \simeq \frac{\epsilon d}{2cn_{\parallel}} \frac{\omega_p^2}{\omega} \frac{\omega_c^2}{\omega^2-\omega_p^2}. \tag{1.4}$$

In the complementary case of the Faraday effect, the radiation propagates through the sample in a direction parallel to the static magnetic field. For $(\omega-\omega_c)\tau \gg 1$, the indexes of refraction for plus and minus circularly polarized light are[2]

$$n_{\pm} = \epsilon^{\frac{1}{2}}\{1-[\omega_p^2/\omega(\omega\mp\omega_c)]\}^{\frac{1}{2}}. \tag{1.5}$$

The Faraday rotation is

$$\begin{aligned} \theta &= (\omega d/2c)(n_{-}-n_{+}) \\ &= \frac{\epsilon d}{2n_0 c}\frac{\omega_p^2\omega_c}{\omega^2-\omega_c^2}\left[1+\frac{1}{2}\frac{\epsilon}{n_0^2}\frac{\omega_p^2}{\omega^2}\frac{\omega_c^2}{\omega^2-\omega_c^2}+\cdots\right], \end{aligned} \tag{1.6}$$

and we note that $n_0 = n_{\parallel}$. For impure samples at low fields, i.e., when $\omega \gg \omega_p \gg \omega_c$, we have

$$\theta \simeq (\epsilon d/2n_0 c)(\omega_p^2\omega_c/\omega^2). \tag{1.7}$$

For the same frequency and magnetic field, we may combine Eqs. (1.4) and (1.7) to obtain

$$\delta/\theta = \omega\omega_c/(\omega^2-\omega_p^2). \tag{1.8}$$

We may solve Eq. (1.8) for the effective-mass ratio as follows:

$$m^*/m = (\omega_c^0\theta/\omega\delta)[\omega^2/(\omega^2-\omega_p^2)], \tag{1.9}$$

where $\omega_c^0 = eH/mc$ and $m$ is the free-electron mass. Unfortunately, Eq. (1.9) is complicated by the fact that $\omega_p^2$ depends on $m^*$ and the carrier concentration. However, Eqs. (1.9) and (1.7) or (1.4) may be used as a basis for an iterative procedure. First we determine $\theta$ and $\delta$ at a given magnetic field and frequency. We then neglect the quantity in square brackets in Eq. (1.9) and estimate $m^*/m$. We use this value of $m^*/m$ to determine $\omega_p^2$ from Eq. (1.7) or Eq. (1.4), also by iteration. This value of $\omega_p^2$ is used in the complete Eq. (1.9) to determine a more accurate value of $m^*/m$. The procedure is repeated until consistent values of $m^*/m$ and $\omega_p^2$ (or carrier concentration) are obtained. Thus by measuring both Faraday rotation and the

[1] S. Teitler and E. D. Palik, Phys. Rev. Letters **5**, 546 (1960).

[2] See, e.g., M. J. Stephen and A. B. Lidiard, J. Phys. Chem. Solids **9**, 43 (1958).

Voigt effect, we determine $m^*/m$ and carrier concentration by optical methods.

If fields of the order of 250 kgauss are used for experiments at wavelengths around $20\mu$, the above procedure could be used to determine an effective mass ratio up to about 0.6 with a carrier concentration of about $10^{18}/\mathrm{cm}^3$. However, in considering the feasibility for studying any given material, one must make sure the mobility is sufficiently high that the condition $\omega\tau \gg 1$ is valid and that the free carrier absorption is sufficiently low.

## II. EXPERIMENTAL TECHNIQUES AND RESULTS

The Voigt effect and Faraday rotation were measured at 80°K for $n$-type samples of InSb and InAs in an Arthur D. Little iron core magnet which was used to obtain fields as high as 25 kgauss. Measurements were made in the spectral region from 15 to $24\mu$ using a KBr prism monochromator. A typical optical arrangement is shown schematically in Fig. 1. This is the case for Faraday rotation. The sample, mounted in a low-temperature Dewar equipped with CsBr windows, is placed so that the radiation passes through it at normal incidence in a direction parallel to the magnetic field. The polyethylene pile-of-plates linear polarizer before the monochromator is fixed to pass the component of polarization perpendicular to the slit and the other polyethylene polarizer is rotated to measure the angle of Faraday rotation.

For the Voigt effect, the two mirrors between the pole pieces are removed and the sample is placed so that the radiation passes through it at normal incidence in a direction perpendicular to the static magnetic field. Again the experimental setup used has polarization optics in reverse. In our discussion we reverse the radiation beam direction and consider normal polarization optics. The polyethylene polarizer before the monochromator is fixed at 45° so that there are equal components in the directions parallel and perpendicular to the magnetic field for the radiation incident on the sample. If the phase shift due to the Voigt effect is less that $\pi/2$ and there is no differential attenuation of the equal components, the resultant radiation emerging from the sample is elliptically polarized with major axis along the direction of the incident linearly polarized

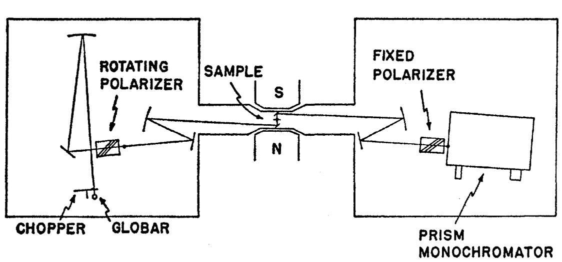


FIG. 1. Schematic optical arrangement for Faraday rotation.

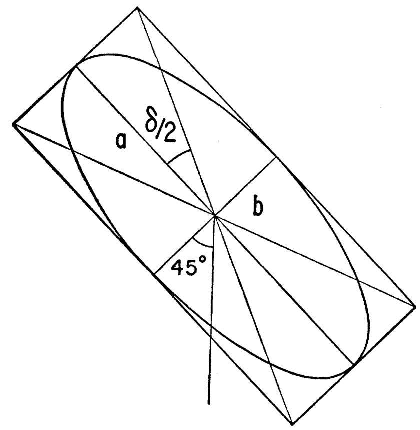


FIG. 2. Schematic diagram of elliptically polarized radiation with pertinent parameters.

light. A proper combination[3] of elliptically polarized radiation and a quarter-wave plate yields linearly polarized radiation in a direction making an angle $\tan^{-1}(b/a)=\delta/2$ with the major axis. Here $b$ is the minor and $a$ is the major axis of the elliptically polarized radiation as shown in Fig. 2. A CsI Fresnel rhomb, placed between the sample and rotating polarizer and oriented along either the major or minor axis, was used as a quarter-wave plate. The combination of magnetic double refraction and properly oriented quarter-wave plate provides an effective rotation of linearly polarized radiation by an angle of $\delta/2$. Thus the measurement of $\theta$ and $\delta/2$ are of equivalent difficulty. Both $\theta$ and $\delta/2$ may be measured to an accuracy of 1° or less. However, for a given magnetic field, the magnitude of $\theta$ is considerably greater than $\delta/2$ as is indicated by Eq. (1.9).

It should be mentioned that care must be taken to properly place the sample in the Voigt experiment so that the radiation is normal to both sample and magnetic field. If the sample is slightly skew, there is a Faraday rotation of the ellipse that emerges from the sample which causes a slight complication in the measurement of $\delta/2$ in the above scheme. A simple way to check that there is no such Faraday rotation of the ellipse is to utilize a method mentioned in our previous report[1] which we used there to measure $\delta$. This is the measurement of the ellipse (with CsI rhomb removed) by using a polarizer fixed at 45° before the monochromator and rotating the first polarizer. For no attenuation, the ratio of the transmission along the minor axis to that along the major axis is $\tan^2(\frac{1}{2}\delta)$. This method has the difficulty that the polarizers are not perfect and it is difficult to assign a zero for the determination of transmission. However, if there is some skewness of the sample position, the extrema in transmission corresponding to the axes of the elliptically polarized light are shifted with respect to the extrema in the zero-field transmission. This difficulty can be

[3] J. Valasek, *Introduction to Theoretical and Experimental Optics* (John Wiley & Sons, Inc., New York, 1949), p. 406.

removed by properly orienting the sample. Once this is done, one can put the properly oriented CsI rhomb in place and proceed with the measurement of $\delta/2$.

A rotation of the ellipse also occurs if there is some cyclotron resonance absorption which amounts to a differential attenuation of the equal components of the incident radiation. However, for the fields and samples used in our present experiments no such differential absorption occurred. Ellipticity produced by reflections from the mirrors in the optical systems was negligible. Polarization of the radiation by the prism also has little or no effect on the measurements since the polarization of the radiation entering the monochromator was always fixed.

Typical results are shown in Figs. 3 and 4. In Fig. 3 we have plotted $\delta$ vs $H^2$ and in Fig. 4 we have plotted $\theta$ vs $H$ for two $n$-type InSb samples at fixed wavelengths. The points fit a straight line within experimental error. This is consistent with Eqs. (1.4) and (1.7) if $m^*/m$ is independent of magnetic field. It is known from cyclotron resonance data[4] on relatively pure samples that the effective-mass ratios for InSb and InAs increase roughly linearly with magnetic field. However, in unpublished results for higher magnetic fields, we have found that the slope of the effective-mass ratio versus magnetic field curve is considerably lower for Voigt- and Faraday-effect measurements on more impure samples. Also in Eqs. (1.4) and (1.7) we have neglected higher order terms in $\omega_c$ which tend to cancel

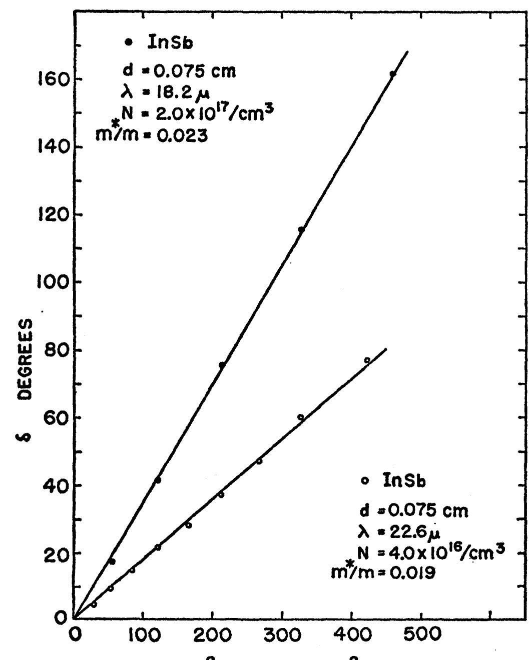


FIG. 3. Voigt phase shift $\delta$ vs $H^2$ for two samples of $n$-type InSb at liquid nitrogen temperature.

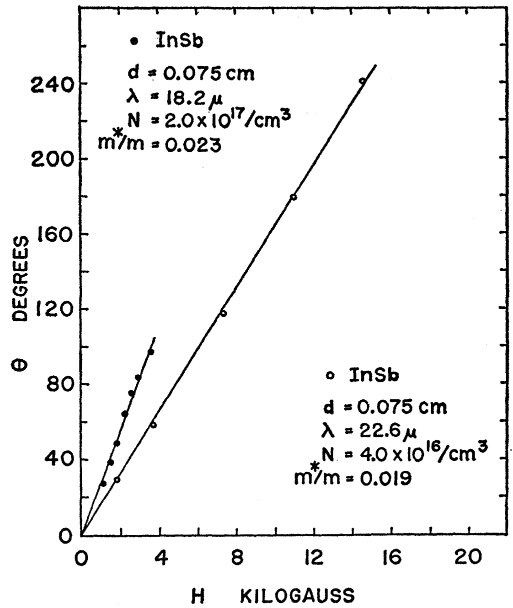


FIG. 4. Angle of Faraday rotation $\theta$ vs $H$ for two samples of $n$-type InSb at liquid nitrogen temperature.

the effect of the increase in effective mass as magnetic field is increased. We shall assume that in this linear region the cancellation has indeed occurred within our experimental error. Then the experimental points at the higher fields may be used to fix the line giving low field values of $\delta$ and $\theta$ for which Eqs. (1.4), (1.7), and (1.9) surely apply. Thus, we may determine the zero-field effective-mass ratio.

We have used the iterative procedure indicated in the first section on two samples of InSb and one of InAs for values of $\delta$ and $\theta$ at 1 kgauss. The results for the InSb samples at 80°K are an effective mass ratio of 0.023 with a carrier concentration of $2.0\times10^{17}/\mathrm{cm}^3$ and an effective mass ratio of 0.019 and carrier concentration of $4.0\times10^{16}/\mathrm{cm}^3$. These values fit, within experimental error, the curve of mass vs carrier concentration obtained by Smith *et al.*[5] using low-field Faraday-rotation and Hall-effect measurements. The calculated plasma frequencies obtained for the above samples are consistent with those measured by Spitzer and Fan.[6]

For the InAs sample at 80°K we obtained an effective-mass ratio of 0.030 with a carrier concentration of $9.6\times10^{16}/\mathrm{cm}^3$. These results are consistent with Cardona's recent report[7] of an effective-mass ratio of 0.026 with a carrier concentration of $4.9\times10^{16}/\mathrm{cm}^3$ obtained by nitrogen-temperature low-field Faraday-rotation and Hall-effect measurements. The calculated plasma frequency is also consistent with the values measured by Cardona and those measured by Spitzer and Fan.

[4] See, e.g., E. D. Palik *et al.*, International Conference on Semiconductors, Prague, 1960 Czechoslovak J. Phys. (to be published).

[5] S. D. Smith, T. S. Moss, and K. W. Taylor, J. Phys. Chem. Solids **11**, 131 (1959).

[6] W. G. Spitzer and H. Y. Fan, Phys. Rev. **106**, 882 (1957).

[7] M. Cardona, Phys. Rev. **121**, 752 (1961).