Test of the ξ-Approximation in Some First-Forbidden $2^- \to 2^+$ β Transitions*

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The β-γ directional correlations of the first-forbidden nonunique $2^- \to 2^+$ β transitions of K^{42}, Sb^{122}, and Au^{198} were investigated and compared with the predictions of the ξ approximation, whose range of applicability is discussed. Upper limits for the contribution of the tensor-type matrix element $\int B_{ij}$ to the β transitions were estimated on the basis of the modified B_{ij} approximation.

The anisotropy coefficient $A_2(W)$ in the β-γ directional correlation involving the 1.98-Mev β transition of K^{42} varies from $A_2(1.66) = -0.009 \pm 0.002$ to $A_2(4.60) = -0.049 \pm 0.002$, where W is in units of mc^2. The energy dependence of $A_2(W)$ deviates from the predictions of the ξ approximation by about 40% over the measured energy range. A rough estimate of the upper limit for the contribution from the B_{ij} component is: $|C_A \int B_{ij}| < 0.3(|V_0|+|Y_1|)$ (in the notation of Kotani).

The anisotropy factor $A_2(W)$ of the β-γ directional correlation involving the 1.40-Mev β transition of Sb^{122} varies from $A_2(1.96) = +0.035 \pm 0.003$ to $A_2(3.5) = +0.081 \pm 0.004$. The energy dependence of $A_2(W)$ is well represented by the factor $\lambda_2(Z,W)\ (W^2-1)/W$ as predicted by the ξ approximation. The upper limit of the $\int B_{ij}$ contribution to this β transition is estimated as: $|C_A \int B_{ij}| < 0.15|Y_1|$ or $|C_A \int B_{ij}| < 0.2|V_0|$.

The anisotropy factor $A_2(W)$ of the Au^{198} β-γ directional correlation involving the 0.96-Mev β transition varies between $A_2(1.39) = +0.0076 \pm 0.0010$ and $A_2(2.78) = +0.0286 \pm 0.0010$, and its energy dependence agrees very well with the predictions of the ξ approximation. The upper limit for the $\int B_{ij}$ matrix element is estimated as: $|C_A \int B_{ij}| < 0.1|Y_1|$.

1. INTRODUCTION

THE recent theoretical and experimental developments in beta decay have led to some clarification of the laws of the beta interaction. The results of β-ν directional correlation experiments showed conclusively that the vector (V) and axial vector (A) couplings give the main contributions (>90%) to β decay.[1,2] Experiments on the β decay of the neutron[3] lead to an accurate determination of the ratio of the axial vector coupling constant C_A to the vector coupling constant C_V: $C_A/C_V = \rho e^{i\phi}$, with $\rho^2 = 1.46 \pm 0.06$ and $\phi = \pi \pm 0.14$, so that $C_A = -(1.21 \pm 0.03) C_V$. From measurements of the ft values of $0 \to 0$ transitions[4] one obtains for the absolute value of C_V: $C_V = g = (1.41 \pm 0.01) \times 10^{-49}$ erg cm^3 or $C_V = g = 2.97 \times 10^{-12}$ in units $\hbar = m = c = 1$. All available experimental facts point to the validity of the two-component neutrino theory, which implies that the odd coupling constants C_V' and C_A' are equal to the even coupling constants C_V and C_A, respectively. The actual results extracted from parity experiments[5] are $C_A' = (1.0 \pm 0.2) C_A$ and $C_V' = (1.0_{-0.6}^{+1.5}) C_V$.

We must recognize, however, that the values of the coupling constants given above are obtained on the basis of experiments designed to determine one or two of a total of 35 independent coupling constants. In evaluating those experiments many of the remaining coupling constants are set equal to zero although experimental evidence only indicated that they are considerably smaller than the "main" coupling constants. An attempt to deduce the values of the 35 independent coupling constants from a simultaneous least-squares fit of all the experimental results (similar to the methods employed by Dumond and co-workers[6] to determine the atomic constants) gives a determination of the β-coupling constants which is far from satisfactory. Thus the errors quoted above from the literature seem to be rather unrealistic. In the following, however, we follow the accepted custom and assume $C_A' = C_A$, $C_V' = C_V$, $C_T = C_S = C_P = C_T' = C_S' = 0$.

After the "clarification" of the interaction laws of β decay, it seems desirable now to study the matrix elements involved in beta transitions. Experimental data of the β-transition matrix elements may then be compared with calculations based on some specific nuclear models. Calculations of this kind require exact knowledge of the wave functions of the nuclear states involved, and only in a few selected cases is it possible, at present, to perform such β matrix element calculations with some degree of confidence. Nevertheless, an accumulation of experimental data concerning β matrix elements may provide a useful table of information for nuclear theorists.

In allowed β transitions the matrix elements are solely determined by their ft values, if the β transitions are either pure Fermi transitions ($0 \to 0$, matrix element $\int 1$), or pure Gamow-Teller transitions ($\Delta I = \pm 1$, matrix element $\int \boldsymbol{\sigma}$). Mixed transitions ($\Delta I = 0$, not $0 \to 0$) require, in addition, the determination of the mixing ratio $y = (C_V \int 1)/(C_A \int \boldsymbol{\sigma})$ of Fermi to Gamow-Teller component. Measurements of the β-γ circular polarization correlation or of the angular distribution

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[4] R. K. Bardin, C. A. Barnes, W. A. Fowler, and P. A. Seeger, Phys. Rev. Letters 5, 323 (1960).
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of the radiation emitted from polarized nuclei allow us to determine this ratio.[5,7,8]

The situation is more complex for first-forbidden transitions. Six matrix elements ($\int i\gamma_5$, $\int \boldsymbol{\sigma}\cdot\mathbf{r}$, $\int \mathbf{r}$, $\int i\boldsymbol{\alpha}$, $\int i\boldsymbol{\sigma}\times\mathbf{r}$, $\int B_{ij}$) may, in general, contribute to a β transition without spin change ($\Delta I=0$), whereas four ($\int \mathbf{r}$, $\int i\boldsymbol{\alpha}$, $\int i\boldsymbol{\sigma}\times\mathbf{r}$, $\int B_{ij}$) may contribute to a $\Delta I=\pm1$ transition. Only the $\Delta I=\pm2$ transition is unique, i.e., one matrix element ($\int B_{ij}$) is involved, which may be determined from the ft value of the transition. The determination of matrix elements for first-forbidden transitions of the types $\Delta I=0$ and $\Delta I=\pm1$ requires the combined evaluation of experimental data of different kinds, such as shape measurements, β-γ angular correlation studies, etc., and is practically only possible in cases where the so-called ξ approximation fails. A few cases of this kind have been investigated, e.g., RaE,[9] Sb^{124},[10,11] and Eu^{152}.[12] The great majority of first forbidden transitions is well represented by the ξ approximation and only certain relationships between the different matrix elements can be established. A determination of the individual matrix elements in these cases would require measurements of very high precision in order to extract the information from higher order effects.

The purpose of this paper is to test the validity of the ξ approximation in some $2^-\rightarrow 2^+$ β transitions of widely different Z (K^{42}, Sb^{122}, Au^{198}), by means of careful measurements of the β energy dependence of the β-γ directional correlation. The results of the measurements indicate excellent agreement with the ξ approximation for Sb^{122} and Au^{198}; the agreement in the case of K^{42} is less satisfactory. From the degree of agreement with the ξ approximation upper limits of possible contributions from the $\int B_{ij}$ matrix element component may be determined.

II. THE ξ APPROXIMATION FOR NONUNIQUE FIRST-FORBIDDEN β TRANSITIONS

It is an experimental fact that most nonunique first-forbidden β transitions exhibit, within experimental error (a few percent), a statistical shape just as do the allowed β transitions. This was first explained by Konopinski and Uhlenbeck,[13] who expanded the beta transition probability in powers of $\mathbf{p}\cdot\mathbf{r}$ and $\mathbf{q}\cdot\mathbf{r}$, where $\mathbf{p}$ and $\mathbf{q}$ are the electron and neutrino momentum, respectively, and $\mathbf{r}$ is the radius vector. Actually this (multipole) expansion involves an expansion of the relativistic lepton wave functions in the Coulomb field of a nucleus of charge Ze. The various terms obtained in this expansion can be grouped such that their orders of magnitude form a series of descending powers of the parameter $\xi=\alpha Z/2R$ ($\alpha=e^2/\hbar c=1/137$, $R=$nuclear radius). If $Z>20$, and if the maximum energy W_0 of the β transition is reasonably small: $\xi\gg W_0-1$, the term of highest power in ξ predominates over all other terms. Retaining only this term leads to the so-called *ξ approximation*, which corresponds to the physical situation where the Coulomb energy $2\xi=\alpha Z/R$ of the electron at the nuclear surface is much larger than the kinetic energy W_0-1 of the electron. In other words, the amount of distortion of the electron wave function by the Coulomb field of the nucleus is more significant than the next higher terms in the pr expansion. The error introduced by the ξ approximation is obviously of the order $(W_0-1)/\xi$.

The terms in the expansion which contain the parameter ξ come from the $j=\frac{1}{2}$ electron wave functions whose radial parts are of the form $g_{\frac{1}{2}}^{\mathrm{el}}\approx pr[1+3\xi(W-1)/p^2]$. The amplitudes of the $j=\frac{3}{2}$ waves are of the form $g_{\frac{3}{2}}^{\mathrm{el}}\approx pr$ and do not contain ξ. The radial parts of the neutrino wave functions are of the form $f_{\frac{1}{2}}^{\nu}=1/\sqrt{2}$ or $f_{\frac{1}{2}}^{\nu}=[1/(18)^{\frac{1}{2}}]qr$ for $j=\frac{1}{2}$, and $f_{\frac{3}{2}}^{\nu}=[1/(18)^{\frac{1}{2}}]qr$ for $j=\frac{3}{2}$. Thus the component of the lepton field, which carries away two units of angular momentum, and whose amplitude is proportional to the tensor matrix element $\int B_{ij}$, is described by combinations of electron and neutrino waves (e.g., $g_{\frac{3}{2}}^{\mathrm{el}}f_{\frac{1}{2}}^{\nu}$; $g_{\frac{1}{2}}^{\mathrm{el}}f_{\frac{3}{2}}^{\nu}\rightarrow 0$) which do not contain ξ. Consequently, the contribution of the $\int B_{ij}$ matrix element is neglected in the ξ approximation. This implies that the validity of the ξ approximation also requires that the matrix element $\int B_{ij}$ of tensor rank $\lambda=2$ must be much smaller than ξ times the "normal" first-forbidden matrix elements of tensor rank $\lambda=1$ and $\lambda=0$.

On the basis of the ξ approximation the spectrum shape correction factor $C(W)$ becomes independent of the energy of the β particles and may be expressed as

$$C^{(0)}=|V_0|^2+|Y_1|^2, \qquad (1)$$

where the parameter V_0 is a linear combination of the first-forbidden nuclear matrix elements of tensor rank $\lambda=0$ (selection rules: $\Delta I=0$, $\Delta\pi=$yes):

$$V_0=\xi\left(\Lambda C_A\int i\gamma_5+C_A\int \boldsymbol{\sigma}\cdot\mathbf{r}\right). \qquad (2)$$

The parameter Y_1 is a linear combination of the nuclear matrix elements of tensor rank $\lambda=1$ (selection rules: $\Delta I=0, \pm1$, $\Delta\pi=$yes):

$$Y_1=\xi\left(-\Lambda' C_V\int i\boldsymbol{\alpha}+C_V\int \mathbf{r}-C_A\int i\boldsymbol{\sigma}\times\mathbf{r}\right). \qquad (3)$$

The numbers Λ and Λ' which indicate the relative contri-

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[8] E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, Phys. Rev. **108**, 503 (1957).
[9] A. I. Alikhanow, G. P. Eliseyev, and V. A. Luibimov, Nuclear Phys. **13**, 541 (1959).
[10] R. M. Steffen, Phys. Rev. Letters **4**, 290, (1960).
[11] G. Hartwig and H. Schopper, Phys. Rev. Letters **4**, 293 (1960).
[12] J. W. Sunier, P. Debrunner, and P. Scherrer, Nuclear Phys. **19**, 62 (1960).
[13] E. J. Konopinski and G. F. Uhlenbeck, Phys. Rev. **60**, 308 (1941).

butions of the relativistic matrix elements as compared to the moment-type matrix elements, are of order unity. Theoretical predictions of $\Lambda=1$ and $\Lambda=2$ have been made.[14,15] After the introduction of the parameters V_0 and Y_1 the requirements for the validity of the ξ approximation may be summarized as: $\xi \gg W_0-1$, $|C_A \int B_{ij}| \ll (|V_0|+|Y_1|)$.[16] Generally, it can be shown that the expressions for all the observables (e.g., β-γ circular polarization correlation, longitudinal polarization of the β particles, etc.) known from allowed β transitions are, within the validity of the ξ approximation, the same for first-forbidden transitions, if one replaces in the formulas for allowed transitions the Fermi to Gamow-Teller ratios $y=C_V \int 1/C_A \int \boldsymbol{\sigma}$ by[17]

$$y'=V_0/Y_1. \tag{4}$$

The β-γ directional correlation is isotropic for allowed transitions, if higher order terms (e.g., cross terms of allowed matrix elements with second-forbidden matrix elements, Gell-Mann terms, etc.) are neglected. Thus the above-mentioned substitution is not applicable for β-γ directional correlations. In fact, the computation of the β-γ directional correlation of first-forbidden transitions requires consideration of terms in the pr expansion which are of higher order (in pr), than the leading terms used in the ξ approximation of the shape and of the β-γ circular polarization correlation. As a result, the first-forbidden β-γ directional correlation depends not only on $y'=V_0/Y_1$, but also on the matrix element ratios[18–20]:

$$r=C_V \int \mathbf{r}/Y_1,$$
$$s=C_A \int i\boldsymbol{\sigma}\times\mathbf{r}/Y_1, \tag{5}$$
$$t=C_A \int B_{ij}/Y_1,$$

which are of the order $1/\xi$.

The directional correlation between a β particle emitted in a first-forbidden β transition and the following γ ray according to the decay scheme $I_0(\beta)I_1(\gamma)I_2$, is given by

$$W_{\beta\gamma}(\theta)=1+A_2(W)P_2(\cos\theta), \tag{6}$$

where

$$A_2(W)=A_2{}^\beta(W)A_2{}^\gamma. \tag{7}$$

The factor $A_2{}^\gamma$ characterizes the γ transition. For a γ transition of pure multipolarity L, the factor $A_2{}^\gamma$ is identical with the F coefficient defined by Rose and Biedenharn[21]

$$A_2{}^\gamma=F_2(LLI_2I_1). \tag{8}$$

The F coefficients $F_2(LL'I_2I_1)$ are tabulated in reference 20. The factor $A_2{}^\beta(W)$ characterizes the β transition of the β-γ cascade. It may be expressed in the following form[17]:

$$A_2{}^\beta(W)=\lambda_2(Z,W)(p^2/W)M(I_0,I_1). \tag{9}$$

The factor $\lambda_2(Z,W)$ which takes into account effects of the Coulomb field of the order $\alpha ZW/p$ is practically energy independent and close to unity. Values of $\lambda_2(Z,W)$ are tabulated elsewhere.[17] The factor $M(I_0,I_1)$ depends on the nuclear matrix elements involved in the β transition, and, in general, on the energy of the β particles. If the ξ approximation is applicable, $M(I_0,I_1)$ is independent of the β energy and contains the matrix element parameters y', r, s, and t:

$$M(I_0,I_1) = \frac{(\tfrac{2}{3})^{\frac{1}{2}}g_{02}(2)y't-(\tfrac{2}{3})^{\frac{1}{2}}g_{11}(2)(s-2r)-g_{12}(2)t}{1+y'^2}. \tag{10}$$

The factors $g_{\lambda\lambda'}(n)$ contain the "geometrical" part of the β transition; they describe the rotational invariant part of the system nucleus plus radiation field:

$$g_{\lambda\lambda'}(n)=(-1)^{I_1-I_0}W(I_1I_1\lambda\lambda';nI_0)(2I_1+1)^{\frac{1}{2}}. \tag{11}$$

The number $W(I_1I_1\lambda\lambda';nI_0)$ is a Racah coefficient. It is noteworthy that, within the framework of the ξ approximation, the anisotropy factor $A_2{}^\beta(W)$ is proportional to $\lambda_2(Z,W)p^2/W$. The coefficients $g_{\lambda\lambda'}(n)$ are roughly of order of magnitude 1 and it is easily seen from the definitions of r, s, and t, that the order of magnitude of $M(I_0,I_1)$ is $1/\xi$.

III. DEVIATIONS FROM THE ξ APPROXIMATION

The ξ approximation as discussed above applies to a first-forbidden nonunique beta transition ($\Delta I=0, \pm1$) in which the nuclear matrix elements are of "normal" relative magnitude such that $|C_A \int B_{ij}| \ll (|V_0|+|Y_1|)$. The ξ approximation cannot be used if the normally dominant terms in the pr expansion are reduced such that the next higher order terms must be taken into account. Three reasons may be responsible for the reduction of the main term: (a) In β transitions of low-Z nuclei and large maximum β energy the condition $\xi \gg W_0-1$ does not hold. (b) The matrix elements of tensor rank $\lambda=0$ and $\lambda=1$ are greatly reduced by virtue of selection rule effects (j selection rule,[22] K forbidden-

[14] T. Ahrens and E. Feenberg, Phys. Rev. **86**, 64 (1952).
[15] D. L. Pursey, Phil. Mag. **42**, 1193 (1951).
[16] The form of this expression does not imply that V_0 and Y_1 appear in the form $|V_0|+|Y_1|$. It merely indicates that $C_A \int B_{ij}$ must be much smaller than the larger of the two parameters $|V_0|$ and $|Y_1|$.
[17] T. Kotani and M. Ross, Phys. Rev. **113**, 622 (1959).
[18] T. Kotani and M. Ross, Progr. Theoret. Phys. (Kyoto) **20**, 643 (1958).
[19] M. Morita and R. S. Morita, Phys. Rev. **109**, 2048 (1958).
[20] K. Alder, B. Stech, and A. Winther, Phys. Rev. **107**, 728 (1957).
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[22] R. W. King and D. C. Peaslee, Phys. Rev. **94**, 1284 (1954) and C. E. Johnson and R. W. King, Bull. Am. Phys. Soc. **4**, 58 (1959).

ness,[23] etc.). (c) Mutual cancellation of the $\lambda=0$ and $\lambda=1$ matrix elements causes the usually dominant terms to vanish, e.g., RaE. The contribution of the $\int B_{ij}$ matrix element ($\lambda=2$), which may be little or not at all affected by these selection rules and cancellations, may then become more important or may even predominate. First-forbidden β transitions of this kind are characterized by their large ft values ($\log ft \approx 9-12$). In these cases higher order terms in the pr expansion must be considered. The shape factor is then of the form[24]

$$C^{(1)}(W)=k+akW+(bk/W)+ckW^2. \quad (12)$$

The coefficients k, ak, bk, and ck contain the nuclear matrix elements but are independent of the β energy. Exact expressions for k, ak, bk, and ck have been given by Kotani.[24] Also, if $C_A\int B_{ij}$ contributes significantly, the energy dependence of $A_2^{\beta}(W)$ is not simply given by $\lambda_2 p^2/W$. It is, in general, a complicated function of the energy W and of the various matrix elements which contribute to the β transition. In a first approximation, transitions of this kind may be analyzed on the basis of the so-called "modified B_{ij} approximation, which was suggested by Matumoto *et al.*[25] Within the framework of this approximation, which presupposes

$$C_A\int B_{ij}\approx(|V_0|+|Y_1|), \quad \text{but} \quad r\ll 1, \quad s\ll 1,$$

the anisotropy factor for a $2^-(\beta^-)2^+(\gamma)0$ β-γ cascade is given by

$$A_2^{\beta\gamma}(W) =\lambda_2\frac{p^2}{W}\,\frac{(1/56)^{\frac{1}{2}}t-(1/21)^{\frac{1}{2}}y't-(\lambda_1/\lambda_2)(1/112)t^2W}{1+y'^2+\frac{1}{12}[(W_0-W)^2+\lambda_1(W^2-1)]t^2}. \quad (13)$$

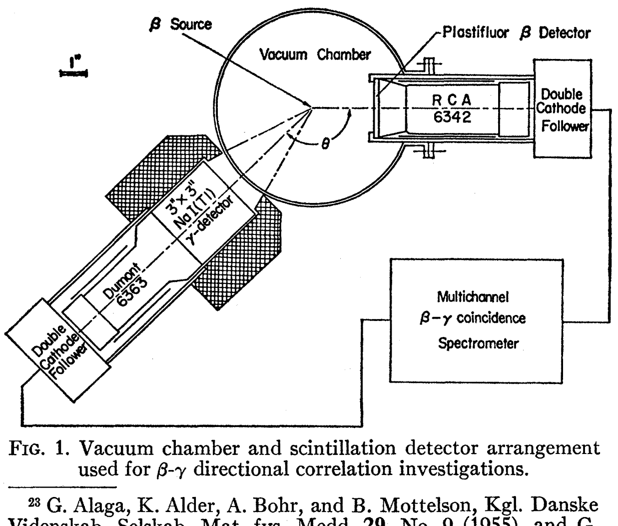

FIG. 1. Vacuum chamber and scintillation detector arrangement used for β-γ directional correlation investigations.

A precise measurement of $A_2(W)$ as a function of the β-energy W may make it possible to estimate the relative magnitude of t in a given nonunique β transition, and the magnitude, or at least an upper limit, of the $\int B_{ij}$ matrix element may be determined. If only the $\int B_{ij}$ component contributes significantly to the β transition, the anisotropy factor is

$$A_2(W)=-\lambda_2(3/28)\frac{p^2}{(W_0-W)^2+\lambda_1 p^2}, \quad (14)$$

which assumes the relatively modest value of $A_2(W_0) = -0.107$ at the maximum β energy W_0. It may be interesting to note that the corresponding values of $A_2(W_0)$ for $1^-(\beta)2^+(\gamma)0^+$ and $3^-(\beta)2^+(\gamma)0^+$ transitions are considerable larger: $+0.251$, and -0.287, respectively. Therefore a possible $\int B_{ij}$ contribution shows up less conspicuously in the directional correlation of a $2^-(\beta)2^+(\gamma)0^+$ cascade than in either a $3^-(\beta)2^+(\gamma)0^+$ or a $1^-(\beta)2^+(\gamma)0^+$ cascade.

IV. APPARATUS AND EXPERIMENTAL PROCEDURE

The β-γ directional correlation measurements described below were performed with the aid of the vacuum chamber and the scintillation counter arrangement shown in Fig. 1. The details of the arrangement have been described before.[26] The thickness of the Pilot B plastic scintillator disk of the β counter was chosen such that it exceeded the range of the most energetic electrons emitted by the source under investigation.

The coincidence electronics (Fig. 2), which was of the usual fast-slow type, had four beta-energy selection channels (β_1, β_2, β_3, and β_4) and two gamma energy selection channels (γ_1 and γ_2) which permitted to measure 8 coincidence events ($\beta_1\gamma_1$, $\beta_2\gamma_1$, $\beta_3\gamma_1$, $\beta_4\gamma_1$, $\beta_1\gamma_2$, $\beta_2\gamma_2$, $\beta_3\gamma_2$, $\beta_4\gamma_2$) simultaneously. The simultaneous registration of the coincidence pairs $\beta_i\gamma_1$ and $\beta_i\gamma_2$ made it possible to correct accurately for the presence of competing β-γ cascades. Assume a decay scheme as shown in Fig. 3. If the β channel i accepts β particles of an

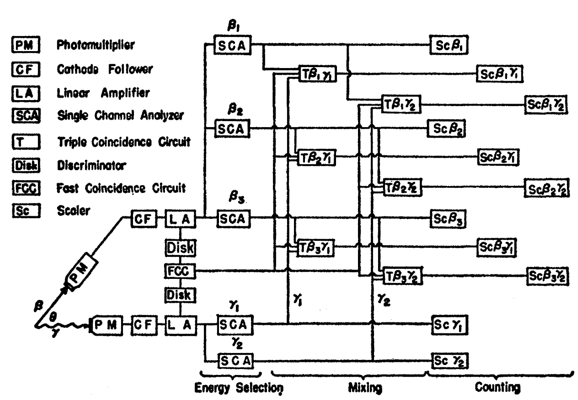

FIG. 2. Block diagram of multichannel β-γ coincidence spectrometer.

[23] G. Alaga, K. Alder, A. Bohr, and B. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. fys. Medd. **29**, No. 9 (1955), and G. Alaga, Phys. Rev. **100**, 432 (1955).
[24] T. Kotani, Phys. Rev. **114**, 795 (1959).
[25] Z. Matumoto, M. Morita, and M. Yamada, Bull. Kobayasi Inst. Phys. Research **5**, 210 (1955).
[26] R. M. Steffen, Phys. Rev. **118**, 763 (1960).

energy which is smaller than the maximum energy of the spectrum β', the measured coincidence rate $\beta_i\gamma_1$ contains also contributions from the β' spectrum.[27] A simultaneous measurement of $\beta_i\gamma_2$ determines the $\beta'-\gamma_2$ directional correlation, from which the $\beta'-\gamma_1$ directional correlation can be calculated, if the spins I_1 and I'_1 and the multipolarity of the gamma radiation γ_2 which is not observed in the $\beta_i\gamma_1$ measurement, are known. The correction applied in this manner is relatively free from errors as it is determined under the same conditions as the main measurement.

The nuclides chosen for the investigation of the energy dependence of the β-γ directional correlation were K^{42}, Sb^{122}, and Au^{198}. They were produced by exposing the separated stable isotopes K^{41} (98%) and Sb^{121} (97%) and the naturally pure isotope Au^{197} to the 10^{13} n/cm² sec neutron flux in the Argonne CP5 reactor.

The K^{42} and Sb^{122} sources were prepared by evaporation on an aluminized Mylar foil of 800-μg/cm² thickness. The foils were supported by a very thin aluminum ring of 2-in. diameter. The thicknesses of the K^{42} and Sb^{122} sources were approximately 200 μg/cm². The details of the β-γ directional correlation measurements on Au^{198} have been described previously.[26] The Au^{198} measurements are included here for comparison purposes only.

The coincidence data were corrected for chance coincidences, γ-γ coincidences and competing β-γ coincidences due to lower energy β branches. The corrected beta-gamma coincidence data measured at a particular β energy W and at different angles θ were fitted by a least-squares fit to a function of the form [see Eq. (6)]:

$$W_{\beta\gamma}''(\theta,W)=A_0''(W)+A_2''(W)P_2(\cos\theta),$$

and the experimental anisotropy factor $A_2'(W) = A_2''(W)/A_0''(W)$ was determined. The "true" anisotropy factor $A_2(W)$ was then computed taking into account finite solid angle and finite source size corrections. The corrections for backscattering of the electrons in the β detector were considered and were found to be rather small in the energy ranges involved in the present experiments.

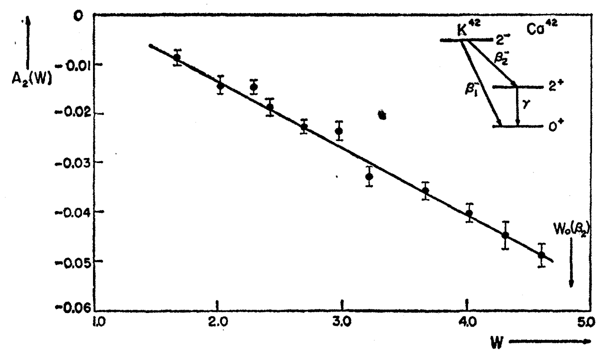

FIG. 4. Experimental anisotropy factor $A_2(W)$ of the K^{42} β-γ directional correlation involving the first-forbidden nonunique 1.98-Mev β transition (β_2 in the decay scheme) as function of β energy W.

V. EXPERIMENTAL RESULTS AND DISCUSSION

The experimentally determined anisotropy factors $A_2(W)$ of the β-γ directional correlations of K^{42}, Sb^{122}, and Au^{198} as a function of the β energy are shown in Figs. 4–6. If the ξ approximation is valid the anisotropy factors $A_2(W)$ should be proportional to $\lambda_2(Z,W)p^2/W$; thus $R(W)=A_2(W)/[\lambda_2(Z,W)p^2/W]$ is expected to be independent of the β energy W. Figures 7 and 8 show the behavior of $R(W)$ for the β-γ directional correlations of K^{42}, Sb^{122}, and Au^{198}.

K^{42}

The curve of $R(W)$ decreases slowly with increasing β energy W (solid line in Fig. 7). The deviation from a horizontal straight line (dashed curve) is of the order of 40% over the energy range measured. The deviation, however, is about as large as expected from the application of the ξ approximation to this β decay: $(W_0-1)/\xi \approx 0.6$. It is interesting to note that the shape factor

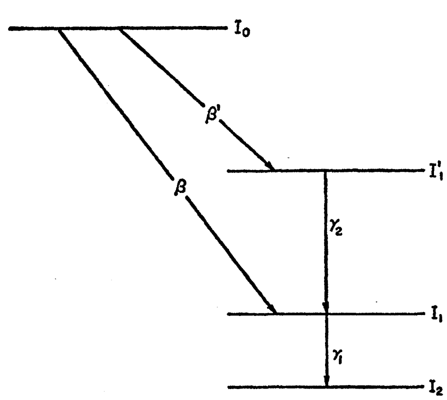

FIG. 3. Decay scheme with several β-γ cascades.

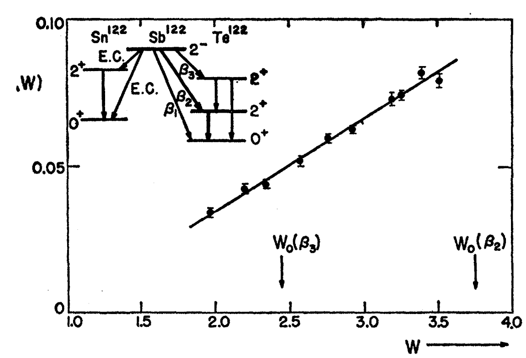

FIG. 5. Experimental anisotropy factor $A_2(W)$ of the Sb^{122} β-γ directional correlation involving the first-forbidden nonunique 1.40-Mev β transition (β_2 in the decay scheme) as function of β energy W.

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[27] For simplicity we assume that the two γ channels respond only to γ_1 and γ_2, respectively.

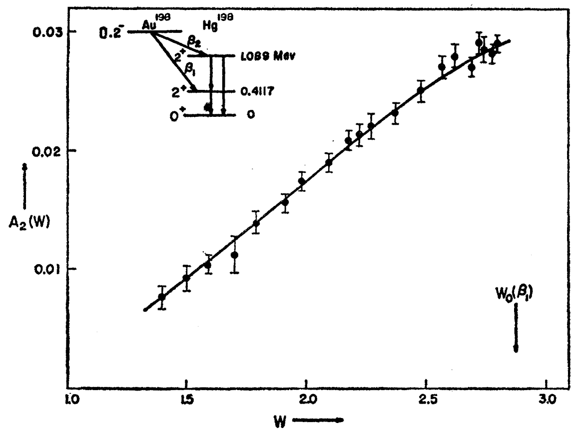

FIG. 6. Experimental anisotropy factor $A_2(W)$ of the Au^{198} β-γ directional correlation involving the first-forbidden nonunique 0.96-Mev β transitions (β_1 in the decay scheme) as function of β energy W.

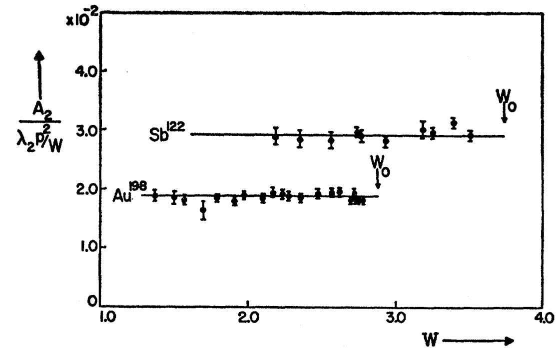

FIG. 8. The experimental ratio $R(W)=A_2(W)/(\lambda_2 p^2/W)$ for the Sb^{122} and Au^{198} β-γ directional correlations. Here $R(W)$ is independent of W over the measured energy range in agreement with the predictions of the ξ approximation.

of the 1.98-Mev β spectrum of K^{42} also seems to show a slight deviation from the constant value[28] expected for a β transition described by the ξ approximation. The deviation of the shape factor from a constant value over the same energy range as measured in the β-γ directional correlation, however, is less than 10%.

The ft value of the 1.98-Mev β transition of K^{42}, $ft=10^{7.5}$ sec, is of the magnitude expected for a first forbidden $2^- \to 2^+$ transition and does not indicate the presence of selection rule or cancellation effects.

A rough estimate of the upper limit of the $\int B_{ij}$ contribution to the 1.98-Mev transition of K^{42} may be obtained by applying the expressions of the modified B_{ij} approximation [Eq. (13)] to the measured energy dependence of $A_2(W)$. Such an analysis of the data yields $|C_A\int B_{ij}|<0.3(|Y_1|+|V_0|)$. A more accurate upper limit of $C_A\int B_{ij}$ could be obtained if the ratio $y'=V_0/V_1$ were accurately known.

In principle y' may be computed from the β-γ circular polarization correlation anisotropy A_1 which has been determined by Daniel.[29] The measured anisotropy factor

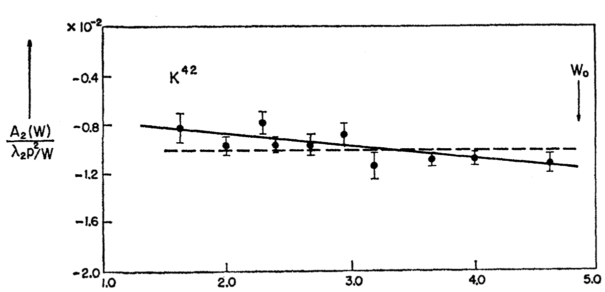

FIG. 7. The experimental ratio $R(W)=A_2(W)/(\lambda_2 p^2/W)$ for the K^{42} β-γ directional correlation. The solid line (experimental curve) shows a deviation of approximately 40% from a constant value (dashed line).

[28] A. V. Pohm, R. C. Waddell, and E. N. Jensen, Phys. Rev. **101**, 1315 (1956).
[29] H. Daniel (private communication).

$A_1=0.10+0.05$ corresponds to values of $y'=+0.1\pm0.1$ or $y'=-6.0\pm1.5$, if the expressions of the ξ approximations are used. The application of the ξ approximation to the 1.98-Mev transition of K^{42}, however, seems to be hardly justified, since the basic condition, $\xi\gg W_0-1$, is not fulfilled in this case.

More information on the matrix elements contributing to the K^{42} β decay could be obtained by investigating the β energy dependence and angular distribution of the circularly polarized 1.51-Mev γ radiation following the 1.98-Mev β transition.

Sb^{122}

The β-γ directional correlation anisotropy $A_2(W)$ of the 1.40-Mev β transition of Sb^{122} is shown in Fig. 5. The measured energy dependence of $A_2(W)$ agrees well with the predictions of the ξ approximation (Fig. 8). In fact the agreement is better than expected within the limitations of the ξ approximation. $R(W)$ is constant within less than 10% over the measured energy range although the condition for the applicability of the ξ approximation, $\xi\gg W_0-1$, is not too well satisfied in this case: $\xi=10$, $W_0-1=2.74$.

The measured shape[30,31] of the 1.40-Mev β spectrum also agrees within experimental error with the statistical shape as predicted by the ξ approximation.

The satisfactory agreement of the β-γ directional correlation with the ξ approximation justifies the evaluation of other experimental data on the 1.40-Mev β transition of Sb^{122} within the framework of the ξ approximation. The β-γ circular polarization correlation has been studied by Deutsch and Lipnik.[32] From the measured β-γ circular polarization correlation anisotropy, $A_1=-0.033\pm0.033$, the following two values of y' are extracted: $y'=+0.25$ and $y'>12$, the first one being the more probable.

The ft value of the 1.40-Mev β transition of Sb^{122},

[30] B. Farrelly, L. Koerts, N. Benczer. R. van Lieshout, and C. S. Wu, Phys. Rev. **99**, 1440 (1955).
[31] M. J. Glaubmann, Phys. Rev. **98**, 645 (1955).
[32] J. P. Deutsch and P. Lipnik, J. phys. radium **21**, 806 (1960).

$ft = 10^{7.6}$ sec, has the expected value for a first-forbidden $2^- \to 2^+$ β transition.

An estimate of the upper limit of the $\int B_{ij}$ contribution, on the basis of the modified B_{ij} approximation and the measured energy dependence of the β-γ directional correlation, yields $C_A \int B_{ij} < 0.15|Y_1|$ if $Y' = 0.25$, and $C_A \int B_{ij} < 0.2|V_0|$, if $y' > 12$.

The small contribution of the $\int B_{ij}$ component to the 1.40-Mev γ transition of Sb^{122} is somewhat surprising in view of the fact that in the β decay of its sister nucleus Sb^{124}, the B_{ij} matrix element represents the main contribution to the nonunique first-forbidden 2.3-Mev and 1.6-Mev β transitions.[10,11,33–35] There is evidence that the large $\int B_{ij}$ contribution to the Sb^{124} β transitions is a result of the j selection rule. One might have expected that the same selection rule effect is also operative in Sb^{122}, which differs by only two neutrons from Sb^{124}.

Au^{198}

The energy dependence of the β-γ directional correlation involving the 0.96-Mev β transition of Au^{198} agrees very well with the predictions of the ξ approximation. The quantity $R(W)$ is independent of the energy W within less than 8% over the energy range from $W = 1.3$ to $W = 2.9$ (Fig. 8). In addition the shape of the 0.96-Mev β spectrum is well represented by the ξ approximation.[36] The good agreement with the ξ approximation is to be expected for Au^{198}, since the parameter ξ for $Z = 80$ ($\xi = 15$) is considerably larger than $W_0 - 1 = 1.9$.

The parameters V_0 and Y_1 are approximately equal in magnitude but of opposite sign in the 0.96-Mev β transition of Au^{198}: $y' = -1.0_{-0.6}^{+0.8}$.[26] An analysis of the β-γ directional correlation data on the basis of the modified B_{ij} approximation yields an estimate of the upper limit of the $\int B_{ij}$ contribution to the 0.96 Mev β transition of Au^{198}: $|C_A \int B_{ij}| < 0.1|Y_1|$.

The small (if any) contribution of the $\int B_{ij}$ matrix element to the 0.96-Mev β transition of Au^{198} is consistent with the ft value, $ft = 10^{7.5}$ sec, of the β transition.

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[33] R. M. Steffen, Phys. Rev. (to be published).
[34] P. Alexander and R. M. Steffen, Phys. Rev. (to be published).
[35] H. Paul, Phys. Rev. **121**, 1175 (1961).
[36] A. H. Wapstra, Nuclear Phys. **9**, 519 (1958/59).

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Magnetic Moments of 69-min Ag^{104} and 27-min Ag^{104m}†

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The hyperfine structure separations of 69-min Ag^{104} and of 27-min Ag^{104m} have been measured using the atomic beam magnetic resonance method. The results are: $\Delta\nu_{I=5}$(69-min Ag^{104}) $= 33\,500_{-1000}^{+2000}$ Mc/sec, $\Delta\nu_{I=2}$(27-min Ag^{104m}) $= 35\,000 \pm 2000$ Mc/sec. The sign of the nuclear magnetic dipole moment has been found to be positive for both states, and by use of the Fermi-Segrè formula one obtains $\mu_I(I=5) = +4.0_{-0.1}^{+0.2}$ nm, $\mu_I(I=2) = +3.7 \pm 0.2$ nm. Nuclear configurations which give these moments are discussed and we comment on the difference between Ag^{104} which shows a 2^+, 5^+ angular momentum recoupling doublet and Ag^{106} and Ag^{110} which show a 1^+, 6^+ doublet.

I. INTRODUCTION

In a previous paper[1] we described work performed at this laboratory to find the correct assignments of spins, half-lives, and γ rays to the neutron-deficient silver isotopes with mass numbers 102, 103, and 104. The isotope Ag^{104} was studied in the most detail and was found to have a ground state with $I = 5$ and a half-life of 69 min. In addition, Ag^{104} has a low-lying isomeric state with $I = 2$ and a half-life of 27 min.

We have measured the nuclear magnet dipole moment of the $I = 5$ and $I = 2$ states in Ag^{104} by the atomic-beam magnetic-resonance method. The large values of the zero-field hyperfine structure separations made it necessary to perform these measurements by observing multiple-quantum transitions, and made

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[1] O. Ames, A. M. Bernstein, M. H. Brennan, R. A. Haberstroh, and D. R. Hamilton, Phys. Rev. **118**, 1599 (1960).