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Beta-Decay Matrix Elements in Sb^{122}†*

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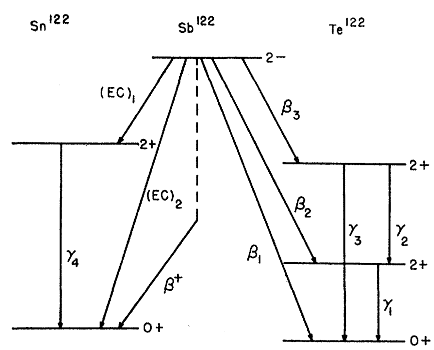
Dynamic nuclear orientation has been used to study the $2^- \leftrightarrow 2^+$ 1.42-Mev beta ray in the decay of Sb^{122}. The Sb^{122}, which was a substitutional donor atom in a silicon crystal, was oriented by saturating each of the four $\Delta(m_I+m_J)=0$ forbidden transitions. The angular distribution of the gamma ray following the beta ray was measured with two scintillation counters. The nuclear and electron relaxation times were determined by the rate of growth and decay of the nuclear orientation. The electron ($\Delta m_J=\pm1$, $\Delta m_I=0$) relaxation time was found to be (4.9±1.2) min. The nuclear relaxation can be represented as due to a combination of the modulation of the isotropic hyperfine interaction and nuclear quadrupole relaxation. For the dipole mechanism, 50 min$\leq T_N \leq$100 min and for the quadrupole mechanism, 150 min$\leq T_N \leq$1700 min. An analog computer was used to correct the initial orientation parameters for the effects of nuclear relaxation. From these, data restrictions can be placed upon the relative amounts of angular momentum carried off by the 1.42-Mev β ray. The modified B_{ij} approximation was then used to analyze this result in conjunction with the beta-gamma angular correlation. There are three sets of matrix elements which can explain the observed data. One set implies that all the antimony atoms are in the simple donor sites; the other two sets imply that only 40% of the antimony atoms are in the donor sites. The first set gives $V=-0.5\pm0.1$, $Y=-0.5\pm0.1$; the second set, $V=-4.2\pm2.0$, $Y=-1.4\pm0.5$; the third set, $V=-6.3\pm1.0$, $Y=+1.8\pm1.5$.

THIS paper reports the use of dynamic nuclear orientation to study the $2^- \leftrightarrow 2^+$ first-forbidden beta transition in Sb^{122}. There are, in general, six nuclear matrix elements which can contribute to this decay; two which carry off no angular momentum, three which carry off one unit of angular momentum, and one which carries off two units of angular momentum. In this experiment the Sb^{122} nuclei in the ground state are oriented and the angular distribution of the gamma ray following the first-forbidden beta ray is observed. The orientation parameters of the initial state can be calculated from the known method of orientation and the measured nuclear relaxation times; the orientation parameters of the daughter state are determined from the gamma-ray angular distribution. The change of orientation parameters due to the beta decay can consequently be determined. This change depends upon the angular momentum carried off by the electron-neutrino system. The method of analysis of the data is similar to that used for As^{76}. Since this has been described elsewhere,[1] only an outline of the theory will be included here.

REVIEW OF THE THEORY

A. Description of Nuclear Orientation

The decay scheme of 65-hr Sb^{122} has been extensively investigated by many workers, among whom the principal ones are Glaubman[2] and Farrelly *et al.*[3] The decay scheme which is given by the most recent set of Nuclear Data Cards[4] is shown in Fig. 1. The unique first-forbidden shape of the 1.97-Mev beta ray leads to the assignment of spin 2 and negative parity to the ground state of Sb^{122}. This is confirmed by the comparison of the experimental and theoretical K capture ratios for the Sb^{122}–Sn^{122} ground-state transition,[5] and by a previous orientation experiment.[6] The spin and parity assignments for the excited states of Te^{122} have been made from a study of the beta-gamma and



RADIATION	ENERGY	INTENSITY	LOG ft
β_1	1.97	30.0 %	8.4
β_2	1.40	62.9 %	7.6
β_3	0.74	4.0 %	7.7
β^+	~0.57	0.0063 %	—
$(EC)_1$	0.7	0.8 %	—
$(EC)_2$	1.45	.2.3 %	—
γ_1	0.564	66.4 %	EQ
γ_2	0.686	3.4 %	EQ+MD
γ_3	1.26	0.7 %	EQ
γ_4	1.14	0.73 %	EQ

Fig. 1. Nuclear energy level diagram for Sb^{122}.

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[1] F. M. Pipkin, G. E. Bradley, and R. E. Simpson, Nuclear Phys. (to be published).
[2] M. J. Glaubman, Phys. Rev. **98**, 645 (1955).
[3] B. Farrelly, L. Koerts, N. Benczer, R. van Lieshout, and C. S. Wu, Phys. **99**, 1440 (1955).
[4] Nuclear Data Cards prepared by National Research Council, p. 58-4-7.
[5] M. L. Perlman, J. P. Welker, and M. Wolfsberg, Phys. Rev. **110**, 381 (1958).
[6] F. M. Pipkin, Phys. Rev. **112**, 935 (1958).

gamma-gamma angular correlations.[7–9] The radiations γ_1 and γ_3 are pure electric quadrupole; the radiation γ_2 is a mixture of $(92\pm4)\%$ electric quadrupole and $(8\pm4)\%$ magnetic dipole. The mean life of the 564-kev state of Te^{122} has been found to be 2.2×10^{-10} sec by a coincidence technique[10] and 1.4×10^{-11} sec by a study of the Coulomb excitation of the 564-kev level.[11]

The 1.40-Mev beta ray (β_2) is a first-forbidden transition with an allowed spectrum shape. The $\beta_2-\gamma_1$ angular correlation has been measured[12] for beta rays of energy 1 to 1.4 Mev the angular correlation is

$$1+(0.099\pm0.013)P_2(\cos\theta). \tag{1}$$

Somoilov *et al.*[13] have oriented Sb^{122} by an adiabatic demagnetization experiment. In their experiment the antimony was an impurity atom in an iron lattice. In this environment there exists a magnetic field of the order of 200 to 300 kgauss at the antimony nucleus. The antimony atoms were cooled by placing the iron foil in thermal contact with a paramagnetic salt which could be cooled by demagnetization. The result of their experiment can be expressed in terms of the observed gamma-ray angular distribution,

$$W(\theta)=1-\frac{(2.2\pm0.2)\times10^{-5}}{T^2}P_2(\cos\theta), \tag{2}$$

where T is the absolute temperature.

In the experiment reported in this paper, the angular distribution of the gamma rays from dynamically oriented Sb^{122} is observed. It is desired to deduce the orientation parameters of nuclei which have reached the 564-kev level via the emission of β_2. The angular distribution of those γ_1 which follow β_2 is[14]

$$W(\theta)=1-(10/7)B_2f_2P_2(\cos\theta) -(40/3)B_4f_4P_4(\cos\theta), \tag{3}$$

where

$$f_2=\frac{1}{j_0^2}\sum_{m_0} m_0^2(a_{m_0}-\tfrac{1}{5}),$$

$$f_4=\frac{1}{j_0^4}\sum_{m_0} m_0^2\left(m_0^2-\frac{31}{7}\right)(a_{m_0}-\tfrac{1}{5}),$$

and

$$P_2(\cos\theta)=\tfrac{3}{2}(\cos^2\theta-\tfrac{1}{3}),$$
$$P_4(\cos\theta)=(35/8)[\cos^4\theta-(6/7)\cos^2\theta+(3/35)].$$

[7] F. Lindquist and J. Marklund Nuclear Phys. **4**, 189 (1957).
[8] C. F. Coleman, Nuclear Phys. **5**, 495 (1958).
[9] I. Asplund, L. G. Strömberg, and T. Wiedling, Arkiv Fysik **18**, 65 (1960).
[10] C. F. Coleman, Phil. Mag. **46**, 1135 (1955).
[11] G. M. Temmer and N. P. Heydenburg, Phys. Rev. **104**, 967 (1956).
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[13] B. N. Somoilov, V. V. Sklyarevskii, and E. P. Stepanov, Soviet Phys.–JETP **11**, 261 (1960).
[14] S. R. de Groot and H. A. Tolhoek in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), Chap. 19, Part III, p. 613.

The parameters B_2 and B_4 are attenuation factors which depend upon the angular momentum carried off by the electron-neutrino system. If α_0, α_1, and α_2 represent the relative probabilities that the beta ray carries off angular momenta, 0, 1, and 2, respectively, then

$$B_2=1-(\alpha_1/2)-(17\alpha_2/14), \tag{4}$$

$$B_4=1-(5\alpha_1/3)-(5\alpha_2/7), \tag{5}$$

where $\alpha_0+\alpha_1+\alpha_2=1$. The principal object of this experiment is to determine B_2 and B_4 and from them α_0, α_1, and α_2. In describing the orientation it is sometimes advantageous to speak of the signals $S(\theta)=W(\theta)-1$ and the nuclear polarization

$$f_1=\frac{1}{j_0}\sum_{m_0} m_0(a_{m_0}-\tfrac{1}{5}).$$

The angular distribution of γ_3 will be given by an identical expression with B_2 and B_4 replaced by B_2' and B_4', the parameters which characterize β_3. In a similar fashion γ_4 can be characterized by B_2'' and B_4''. The angular distribution of γ_2 will be[15]

$$W(\theta)=1+\frac{1}{21}\left[-21a_1^2+126\left(\frac{5}{21}\right)^{\frac{1}{2}}a_1a_2+\frac{45}{7}a_2^2\right] \times f_2B_2'P_2(\cos\theta)-\frac{80}{21}a_2^2f_4B_4'P_4(\cos\theta), \tag{6}$$

where a_1 and a_2 are the reduced matrix elements for the magnetic dipole and electric quadrupole components of γ_2 normalized so that

$$a_1^2+a_2^2=1.$$

For Sb^{122},[7–9]

$$a_2/a_1=\delta=+3.5\pm0.4.$$

The angular distribution for those γ_1 which follow γ_2 is

$$W(\theta)=1-(10/7)[(a_1^2/2)-(3a_2^2/14)]f_2B_2'P_2(\cos\theta) -(40/3)[-(2a_1^2/3)+(2a_2^2/7)]f_4B_4'P_4 \times(\cos\theta). \tag{7}$$

If it is assumed that all the γ rays are counted with the same efficiency, then the intensity of the gamma rays given in Fig. 1 can be used to compute the total angular distribution. The result of this calculation is

$$W(\theta)=1-(10/7)[0.897B_2-0.030B_2'+0.010B_2''] \times f_2P_2(\cos\theta)-(40/3)[0.897B_2-0.032B_4' +0.010B_4'']f_4P_4(\cos\theta). \tag{8}$$

B. Initial Orientation Parameters

In order to find the initial orientation parameters, we must consider the mechanism of orientation and the

[15] C. D. Hartogh, H. A. Tolhoek, and S. R. de Groot, Physica **20**, 1310 (1954).

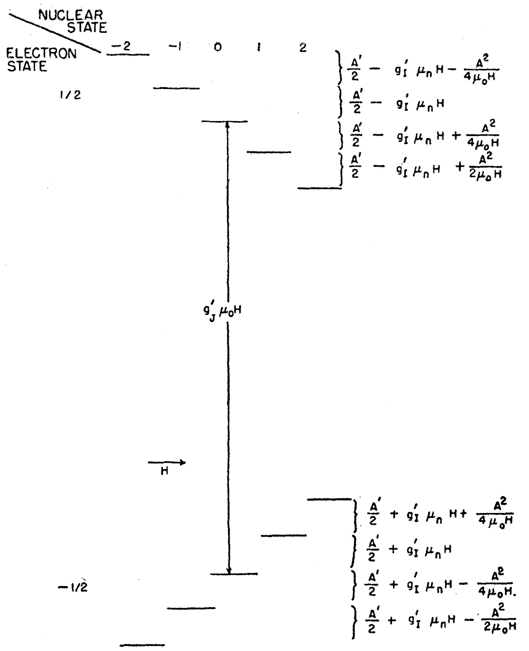

FIG. 2. Energy levels of the Sb^{122} donor atom in a silicon crystal. The arrow with an H indicates the order in which the transitions occur with fixed oscillator frequency and increasing magnetic field.

manner in which the nuclear relaxation can change the orientation parameters. The energy levels of the antimony donor atom are shown in Fig. 2. The orientation is produced by the saturation of the $\Delta(m_I+m_J)=0$ forbidden transitions. This process of production of the orientation is shown schematically in Fig. 3. If the only relaxation were due to electron spin translations ($\Delta m_J=\pm 1$, $\Delta m_I=0$), then saturation of the four forbidden transitions would result in the equilibrium values of f_2 and f_4 shown in Table I. Nuclear relaxation will cause the f_2 and f_4 to depart from these values.

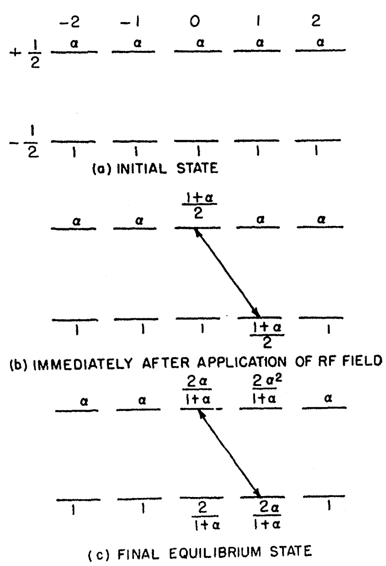

FIG. 3. Schematic representation of the formation of the nuclear orientation by saturation of a forbidden transition. The symbol α stands for $e^{-(2\mu_0 H/kT)}$.

If the relative populations of the various magnetic substates are $P(m_J,m_I)$, where

$$\sum_{m_I,m_J} P(m_J,m_I)=1,$$

then the change in time of these populations can be described by the set of equations[6]

$$\dot{P}_{m_J,m_I}=\sum_{m_I',m_J'} W(m_J,m_I|m_J',m_I') \times\{\exp[-(\mu_0 H/kT)(m_J-m_J')]P(m_J',m_I') -\exp[-(\mu_0 H/kT)(m_J'-m_J)]P(m_J,m_I)\}. \quad (9)$$

In this expression $W(m_J,m_I|m_J',m_I')$ are coefficients which determine the relaxation rate. For an atomic spin of $\frac{1}{2}$ they can be expressed in terms of the Clebsch-Gordan coefficients. For the analysis of the antimony

TABLE I. The f_2 and f_4 parameters for saturation of the four $\Delta(m_I+m_J)=0$ forbidden transitions. It has been assumed that there is no nuclear relaxation and that the time of application of the radiofrequency field is much longer than the electron relaxation time.

Transition	f_2	f_4
$(-\frac{1}{2}, -1)\rightarrow(\frac{1}{2}, -2)$	$\frac{3}{20}\tanh\frac{\mu_0 H}{kT}$	$\frac{3}{140}\tanh\frac{\mu_0 H}{kT}$
$(-\frac{1}{2}, 0)\rightarrow(\frac{1}{2}, -1)$	$\frac{1}{20}\tanh\frac{\mu_0 H}{kT}$	$-\frac{3}{70}\tanh\frac{\mu_0 H}{kT}$
$(-\frac{1}{2}, 1)\rightarrow(\frac{1}{2}, 0)$	$-\frac{1}{20}\tanh\frac{\mu_0 H}{kT}$	$\frac{3}{70}\tanh\frac{\mu_0 H}{kT}$
$(-\frac{1}{2}, 2)\rightarrow(\frac{1}{2}, 1)$	$-\frac{3}{20}\tanh\frac{\mu_0 H}{kT}$	$-\frac{3}{140}\tanh\frac{\mu_0 H}{kT}$

it will be assumed that the following relaxation mechanisms are present:

(a) Pure electron relaxation ($\Delta m_I=0$, $\Delta m_J=\pm 1$).

$$W(m_J,m_I|m_J'm_I')=W_1\delta_{m_I,m_I'}(1-\delta_{m_J,m_J'}). \quad (10)$$

(b) Nuclear relaxation through modulation of the isotropic hyperfine interaction.

$$W(m_J,m_I|m_J',m_I') =W_2(I1m_I'\mu|I1Im_I)^2 \times(1-\delta_{m_J,m_J'})\delta_{m_I+m_J,m_I'+m_J'}. \quad (11)$$

(c) Nuclear quadrupole relaxation.

$$W(m_J,m_I|m_J',m_I') =W_3(I2m_I'\mu|I2Im_I)^2\delta_{m_J,m_J'}(1-\delta_{m_I,m_I'}). \quad (12)$$

For a general mixture of relaxation mechanisms, the behavior of the nuclear populations as a function of time will be rather complicated and some form of

computer is useful in solving the equations. There are two simple cases, however, which can easily be solved and which serve as a guide in determining the relaxation times. In both of these cases it is assumed the the electron relaxation time is much shorter than the nuclear relaxation time. For the formation of the orientation.

$$f_2(t)=f_2(\infty)(1-\tfrac{1}{2}e^{-\lambda_R t}),\quad f_4(t)=f_4(\infty)(1-\tfrac{1}{2}e^{-\lambda_R t}), \tag{13}$$

when $\lambda_R=1/2T_S$ and T_S is the electron relaxation time as it is customarily defined.[16] For the decay of the orientation,

$$f_1(t)=f_1(0)e^{-\lambda_1 t},\quad f_2(t)=f_2(0)e^{-\lambda_2 t},\quad f_4(t)=f_4(0)e^{-\lambda_4 t}, \tag{14}$$

where λ_1, λ_2, and λ_4 depend upon the relaxation mechanism. For a dipole relaxation mechanism, of which the hyperfine relaxation is an example,

$$\lambda_1=\tfrac{1}{12}W_2\,\mathrm{sech}(\mu_0H/kT),\quad \lambda_2=\tfrac{1}{4}W_2\,\mathrm{sech}(\mu_0H/kT),\quad \lambda_4=\tfrac{5}{6}W_2\,\mathrm{sech}(\mu_0H/kT), \tag{15}$$

For the quadrupole mechanism,[17]

$$\lambda_1=W_3/2,\quad \lambda_2=(17/14)W_3,\quad \lambda_4=(5/7)W_3. \tag{16}$$

Thus by observing the decay of the orientation, the relaxation time can be measured and the relaxation mechanism discovered.

To solve the more general problem it is convenient to use an analog computer. The equations for the relaxation can be duplicated by a system of condensers and resistors. The circuit for the electron relaxation plus the modulation of the isotropic hyperfine interaction is shown in Fig. 4; that for the electron relaxation plus the quadrupole relaxation in Fig. 5. In this analog the charge on the condensers corresponds to the populations of the levels. In terms of the resistors and capacitors,

$$\lambda_1=\frac{1}{3}\left(\frac{1}{1+\alpha}\right)\frac{1}{R_2C},\quad \lambda_2=\frac{1}{2}\left(\frac{1}{1+\alpha}\right)\frac{1}{R_2C},\quad \lambda_4=\frac{5}{3}\left(\frac{1}{1+\alpha}\right)\frac{1}{R_2C}, \tag{17}$$

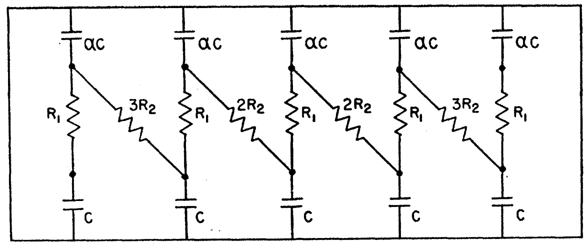

FIG. 4. Analog circuit for nuclear relaxation due to modulation of isotropic hyperfine interaction plus electron spin relaxation. The symbol α is an abbreviation for $e^{-(2\mu_0H/kT)}$.

for Fig. 4, and

$$\lambda_1=\frac{7}{6}\frac{1}{R_3C},\quad \lambda_2=\frac{17}{6}\frac{1}{R_3C},\quad \lambda_4=\frac{5}{3}\frac{1}{R_3C}, \tag{18}$$

for Fig. 5. The detailed circuit which was used to analyze the experiment is described elsewhere.[1]

EXPERIMENTAL PROCEDURE

After neutron activation in the materials testing reactor at Arco, Idaho, the sample was annealed for five hours at 1100–1200°C to heal the radiation damage. It was placed in the rectangular microwave cavity and the wave guide assembly was inserted into the helium filled Dewar. Two integral discriminators were used with each NaI(Tl) scintillation detector, one set below the 564-kev gamma-ray photopeak and the other set just above this photopeak. A vacuum was pumped on the helium to reduce the temperature to 1.15°C. A well-coupled cavity mode was located and the

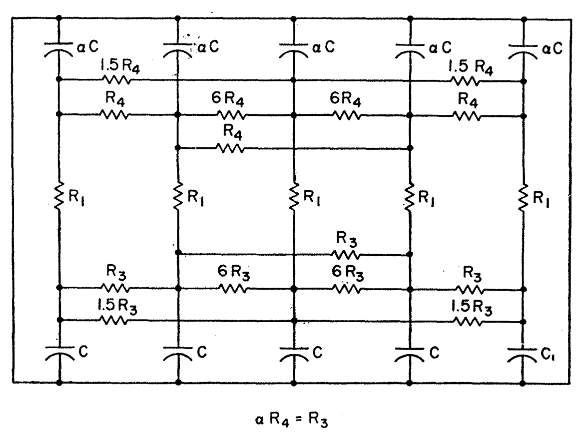

FIG. 5. Analog circuit for nuclear relaxation due to nuclear quadrupole interaction plus electron spin relaxation. The symbol α is an abbreviation for $e^{-(2\mu_0H/kT)}$.

[16] J. W. Culvahouse and F. M. Pipkin, Phys. Rev. **109**, 319 (1958).

[17] The expressions quoted for λ_2 and λ_4 in references 6 and 16 are interchanged.

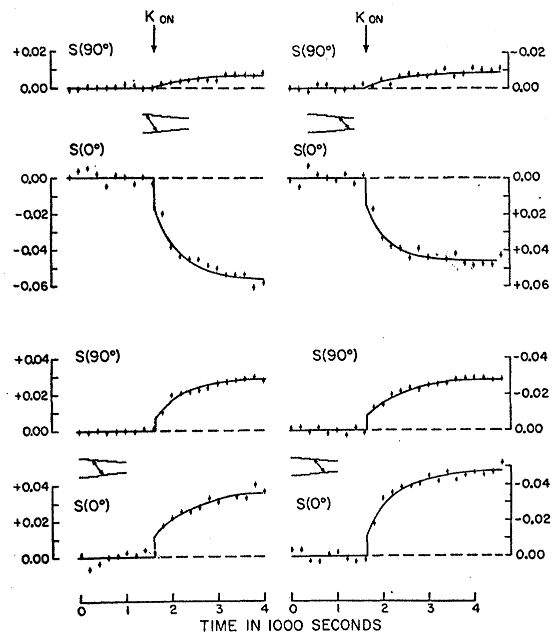

FIG. 6. A plot of the counting rates versus time for both 90° and 0° positions and for each forbidden transition. The curves are just drawn through the points and do not represent a theoretical prediction. The silicon sample (S-2) had a concentration of approximately 3×10^{16} donors/cm^3 and was irradiated for a 48-hr period in a flux of 10^{14} thermal neutrons/sec cm^2. It had never been irradiated before.

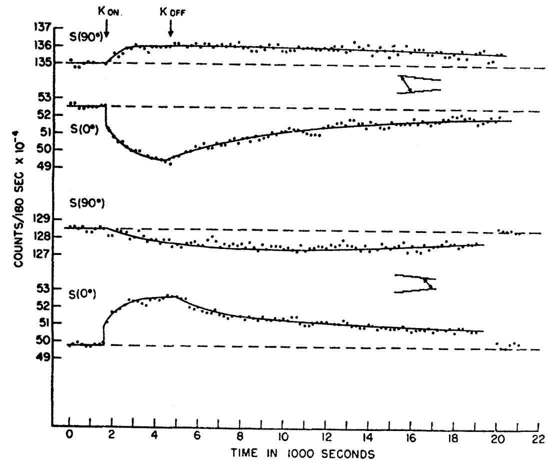

FIG. 7. The long runs which were used to determine the relaxation times for S-2. The change in counting rate at the end shows where the magnet was turned off to relax the sample.

positions of the four forbidden lines were calculated. At this point the klystron was turned off and the magnetic field at the sample was reduced to zero for a five minute period to establish initially an isotropic nuclear population. The magnetic field was turned on and adjusted so that one of the forbidden transitions was in resonance. The counting rates were then recorded for a period sufficient to establish a good reference level and the klystron was turned on. The klystron was left on until the counting rates no longer appeared to be changing. At this point the klystron was turned off and the decay of the orientation observed.

Three sources were used in this experiment. The first (S-1) was doped with 6×10^{16} antimony atoms/cm^3 and it was irradiated for a period of 44 hr in a flux of 10^{14} thermal neutrons/cm^2 sec. The second (S-2) was doped with 3×10^{16} Sb/cm^3 and it was irradiated for 48 hr in a flux of 10^{14} thermal neutrons/cm^2 sec. The third (S-3) was the old (S-1) source and it was ir-

TABLE II. Data on the S-3 Sb source. This crystal had a concentration of 6×10^{16} donors/cm^2 and was irradiated for a period of 96 hr in a flux of 10^{14} thermal neutrons/cm^2 sec. The initial signals refer to the change in the orientation immediately after the radio-frequency field is turned on and before the slow rise. According to the simple theory which neglects nuclear relaxation these initial signals should be equal to one-half the final signals.

Transition	Run	H in gauss	Temp. in °K	Initial signals in % $S(90°)$	Initial signals in % $S(0°)$	Final signals in % $S(90°)$	Final signals in % $S(0°)$
$(-\frac{1}{2}, 2)\rightarrow(\frac{1}{2}, 1)$	5–28	8006	1.12	0.0 ±0.2	2.41±0.20	−0.77±0.2	4.78±0.24
	5–62	8483	1.12	0.0 ±0.2	1.94±0.10	−0.21±0.2	4.65±0.13
	5–74	7898	1.12	−0.11±0.11	2.91±0.20	−0.43±0.2	4.82±0.25
$(-\frac{1}{2}, -1)\rightarrow(\frac{1}{2}, -2)$	5–54	8367	1.15	0.0 ±0.2	−2.89±0.20	+0.97±0.18	−5.56±0.30
	5–20	7866	1.12	0.0 ±0.2	−3.16±0.20	+0.63±0.10	−6.18±0.37
	5–70	7759	1.15	0.0 ±0.15	−2.13±0.16	+0.91±0.15	−5.24±0.16
$(-\frac{1}{2}, 1)\rightarrow(\frac{1}{2}, 0)$	5–48	8464	1.15	−1.48±0.17	−2.95±0.17	−2.46±0.18	−4.68±0.18
	5–60	8464	1.12	−1.36±0.12	−3.07±0.12	−2.64±0.12	−4.92±0.12
	5–58	8464	1.12	−1.52±0.20	−3.07±0.12	−2.64±0.24	−5.17±0.12
	5–24	7959	1.12	−1.24±0.24	−2.68±0.12	−1.96±0.24	−4.50±0.12
$(-\frac{1}{2}, 0)\rightarrow(\frac{1}{2}, -1)$	5–44	8415	1.15	1.81±0.11	2.04±0.15	3.69±0.11	2.89±0.23
	5–34	7912	1.15	0.77±0.10	0 ±0.10	2.92±0.10	1.96±0.10
	5–78	7806	1.15	2.25±0.10	1.68±0.10	3.23±0.10	2.84±0.10
$(\frac{1}{2}, 0)\rightarrow(-\frac{1}{2}, 0)$	5–64	8436	1.12	0.28±0.12	0.0 ±0.20	1.118±0.12	0.0 ±0.20
	5–76	7830	1.15	0.0 ±0.15	0.0 ±0.20	1.138±0.15	0.713±0.20
$(\frac{1}{2}, -1)\rightarrow(-\frac{1}{2}, -1)$	5–80	7781	1.15	0.0 ±0.20	0.0 ±0.20	0.099±0.20	1.392±0.20

radiated again for 96 hr in a flux of 10^{14} thermal neutrons/cm^2 sec. A period of two months elapsed between the two irradiations.

ANALYSIS OF THE DATA

In order to determine the true angular distribution, the counting rates must be corrected for counts due to Sb^{124}, for the counting rate loss, and for the finite solid angle subtended by the counters. The fraction of the counts due to Sb^{124} was determined by observing the counting rate in the two channels over a period of ten days to two weeks. The counting rates versus time were then plotted on semilog paper and the fraction of the source which was Sb^{124} was determined. Independent measurements on Sb^{124} alone showed that there were no observable (>0.5%) radio-frequency-produced anisotropies which would confuse the measurements. The data were also corrected for the decay of the Sb^{122}. Figure 6 shows one run for each of the forbidden transitions for source S-2. Figure 7 shows two of the long runs used to measure the nuclear relaxation times for source S-2. Data taken on source S-3 are shown in Table II. These data illustrate the reproducibility of the measurements.

A numerical integration was used to correct for the finite solid angle of the counters. These correction factors can be expressed as attenuation coefficients S_2, S_4 in the angular distribution

$$W(\theta)=1-(10/7)S_2B_2f_2P_2(\cos\theta) - (40/3)S_4B_4f_4P_2(\cos\theta). \quad (19)$$

The values of S_2 and S_4 for the various sources are shown in Table III.

The electron relaxation time was determined by fitting the rise of the signals to an expression of the form

$$A-Be^{-\lambda t}.$$

The data were then analyzed into B_2f_2 and B_4f_4 components and the nuclear relaxation was investigated by fitting the decay of the B_2f_2 and B_4f_4 to curves of the form

$$B_2f_2=(B_2f_2)_0e^{-\lambda_2 t},$$
$$B_4f_4=(B_4f_4)_0e^{-\lambda_4 t}.$$

The average results of these calculations for each of the transitions and for each source is shown in Table IV.

The average rise constant for all three sources is

$$\lambda_R=(1.7\pm0.4)\times10^{-3}\ \mathrm{sec}^{-1}.$$

This implies that the electron relaxation time is

$$T_S=(4.9\pm1.2)\ \mathrm{min}.$$

From the λ_2 and λ_4 derived for each transition, W_2 and W_4 can be determined. The average value for W_2 obtained in this fashion is

$$W_2=(3.2\pm1.0)\times10^{-4}/\mathrm{sec}.$$

The average value of W_3 is

$$W_3=(1.2\pm1.0)\times10^{-4}/\mathrm{sec}.$$

The corresponding relaxation times for the nuclear polarization can be found from Eqs. (15) and (16). For the modulation of the isotropic hyperfine interaction the nuclear relaxation time is

$$50\ \mathrm{min}\leq T_N\leq100\ \mathrm{min},$$

and for the quadrupole relaxation

$$150\ \mathrm{min}\leq T_N\leq1700\ \mathrm{min}.$$

TABLE III. Solid-angle correcti on factors.

	0° counter		90° counter	
Source	S_2	S_4	S_2	S_4
S-1	0.884	0.655	0.921	0.831
S-2	0.924	0.763	0.871	0.621
S-3	0.924	0.763	0.935	0.795

TABLE IV. The average values of B_2f_2 and B_4f_4, the rise constant λ_R, and the decay constants λ_2 and λ_4 obtained from all three sources. The errors are based upon the spread of the various measurements. The total spread of the B_2f_2 and B_4f_4 values for one source was about 10%; this is taken as an estimate of their error.

Transition	Source	B_2f_2 in %	B_4f_4 in %	λ_R (10^{-3} sec^{-1})	λ_2 (10^{-5} sec^{-1})	λ_4 (10^{-5} sec^{-1})	λ_4/λ_2
$(-\frac{1}{2}, 2)\rightarrow(\frac{1}{2}, 1)$	S-1	−2.70	−0.20	1.7±0.5	10±3	43±7	4.3
	S-2	−2.39	−0.30	1.1±0.3	15±7	16±7	1.05
	S-3	−2.26	−0.22	1.9±0.5	16±3	33±3	2.04
$(-\frac{1}{2}, -1)\rightarrow(\frac{1}{2}, -2)$	S-1	2.73	0.26	2.0±0.5	...	...	...
	S-2	2.92	0.27	1.0±0.5	7.0±0.5	31±5	4.44
	S-3	2.93	0.23	1.4±0.5	16±5	29±5	1.81
$(-\frac{1}{2}, 0)\rightarrow(\frac{1}{2}, -1)$	S-1	2.55	−0.58	2.3±0.5	89.4	76.5	0.855
	S-2	2.22	−0.49	1.6±0.5	28±3	28±3	1.0
	S-3	2.19	−0.53	2.3±0.6	30±7	31±2	1.04
$(-\frac{1}{2}, 1)\rightarrow(\frac{1}{2}, -0)$	S-1	−1.81	0.64	2.1±0.5	...	...	...
	S-2	−1.32	0.64	2.1±0.5	14±3	19±2	1.36
	S-3	−0.64	0.60	1.7±0.5	21±7	31±2	1.45

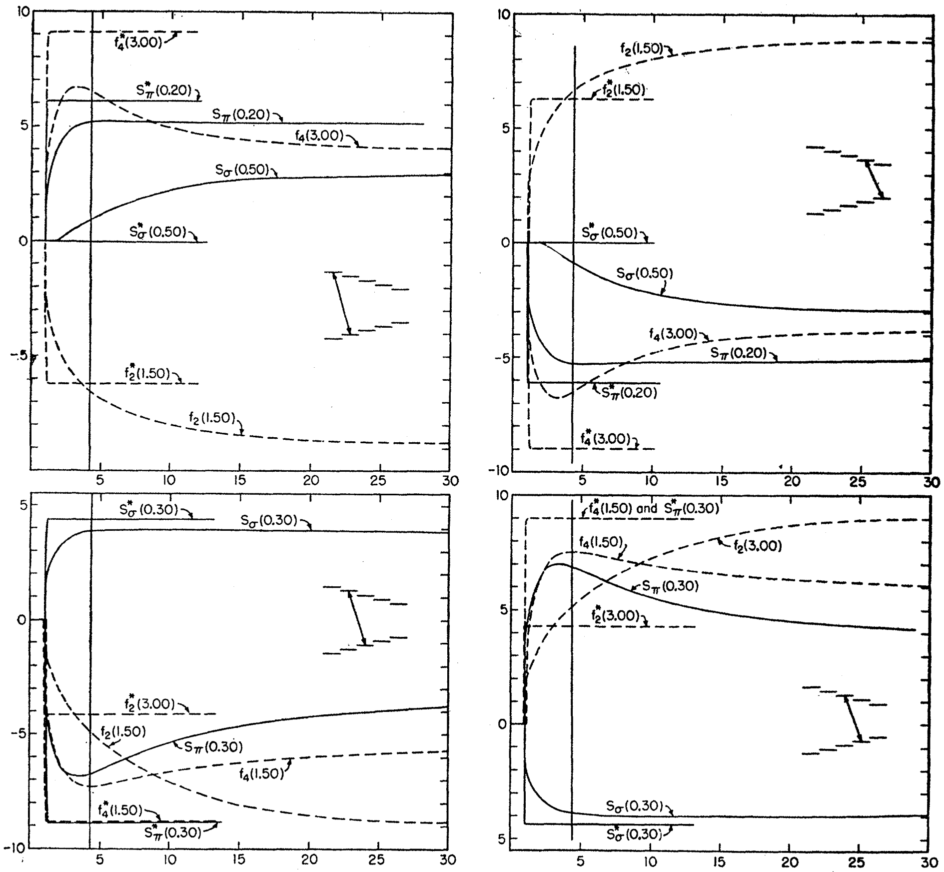

FIG. 8. A set of runs on the analog computer which shows the formation of the signals. For this run the circuit was as shown in Fig. 4, with $R_1=100$ kohms, $R_2=214$ kohms, $c=20$ mf, and $\alpha=0.4$. S_π is the signal at 0° and S_σ the signal at 90°. The starred quantities are for the computer when $R_1=0$, $R_2=\infty$. The figures in parentheses give the relative gain of the computer on the basis of full gain being 10. Full scale (+10) on the recorder corresponds to $f_2=-2$, $f_4=10$, $S(0°)=S_\pi=4$, and $S(90°)=S_\sigma=-4$. The vertical black line corresponds to the point in the experiment of which the saturating microwave field was turned off.

In order to calculate the B_2 and B_4, the f_2 and f_4 must be known. The analog computer was used to determine f_2 and f_4 in terms of f_2^* and f_4^*, which are the f_2 and f_4 given by the simple no-nuclear-relaxation theory (Table I). The correction coefficients A_i are defined by

$$f_i=A_if_i^*.$$

The A_i were determined as follows: The quantities λ_2/λ_R and λ_4/λ_R were first calculated. Then it was assumed that the nuclear relaxation was due to a mixture of the modulation of the isotropic hyperfine interaction and the nuclear quadrupole interaction, and such a mixture was taken so as to reproduce the λ_2/λ_R and λ_4/λ_R values. The computer was then set

TABLE V. The time constants and their reproduced values. All time constants are expressed in terms of the rise time constant. The notation for the various transitions is as follows: π_u, $(-\frac{1}{2},2)\rightarrow(\frac{1}{2},1)$; π_l, $(-\frac{1}{2},-1)\rightarrow(\frac{1}{2},-2)$; σ_u, $(-\frac{1}{2},1)\rightarrow(\frac{1}{2},0)$; σ_l $(-\frac{1}{2},0)\rightarrow(\frac{1}{2},-1)$. The column labeled "Run" refers to the transition from which the decay data were taken and the relaxation mechanism used to set up the computer. The W_2/W_3 gives the relative contributions of dipole and quadrupole relaxation mechanisms.

Run	Quantity	Exp.	Simple theory	π_u	π_l	σ_u	σ_l	Average
S-1, π_u, $W_3=0$	λ_2	5.9	6.24	5.47	5.44	4.81	5.74	5.37
	λ_4	25.1	20.8	19.7	18.0	18.9	20.1	19.2
	λ_4/λ_2	4.3	3.33	3.61	3.31	3.25	3.50	3.5
S-2, π_u, $W_2/W_3=1.1$	λ_2	13.6	11.2	10.5	10.5	10.1	10.1	10.3
	λ_4	14.3	12.3	11.5	11.4	11.2	11.2	11.3
	λ_4/λ_2	1.05	1.10	1.10	1.09	1.11	1.11	1.10
S-2, π_l, $W_3=0$	λ_2	7.0	7.87	6.78	7.27	6.54	7.03	6.90
	λ_4	31.0	26.2	22.9	21.8	22.8	25.2	2.32
	λ_4/λ_2	4.44	3.33	3.38	3.00	3.59	3.36	3.36
S-3, π_l, $W_2/W_3=4.8$	λ_2	11.3	9.88	8.93	8.75	8.93	9.08	8.92
	λ_4	20.8	18.2	16.9	15.8	16.1	16.6	16.4
	λ_4/λ_2	1.81	1.84	1.89	1.81	1.80	1.83	1.84

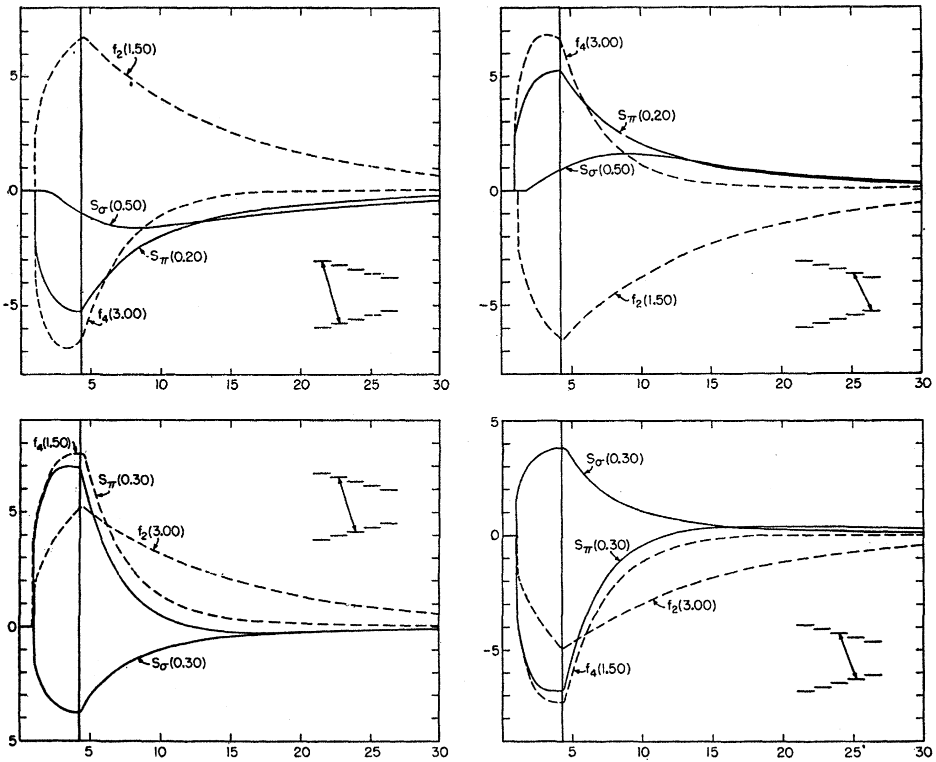

FIG. 9. A set of runs on the computer which shows the formation and decay of the signals. The circuit values are the same as in Fig. 8.

up to duplicate this situation. The various transitions were run and the f_2, f_4, $S(0°)$ and $S(90°)$ recorded. The nuclear relaxation resistors were removed and the resistors representing the electron relaxation were shorted out. The signals observed with this arrangement gave a calibration with which to determine the A_i coefficients. To set up the computer only the relaxation data for the $(-\frac{1}{2}, 2)\rightarrow(\frac{1}{2}, 1)$ and $(-\frac{1}{2}, -1)\rightarrow(\frac{1}{2}, -2)$ transition were used. In case the measured relaxation times for these two transitions differed, a computer run was made for each of them. Figure 8 shows a typical run with the computer illustrating the formation of the signals. Figure 9 shows a run illustrating not only the formation but also the decay of the signals. The point at which the A_i were determined was selected by finding that time which corresponded to the point in the experiment at which the klystron was turned off. Table V gives a comparison between the time constants observed for the various transitions and those calculated on the basis of the simple theory which assumes predominance of the electron relaxation. Table VI gives the A_i coefficients which were determined for the various transitions.

Once the A_i have been determined, it is a simple matter to calculate the values of B_2 and B_4. The results of these calculations are shown in Table VII. In order to check the consistency of the procedure, the calculated B_2 and B_4 were then used to set the computer so that it reproduced the observed signals. Figure 10 shows a comparison between the observed signals and those reproduced in this fashion. A reproduction is shown for decay constants determined from both the $(-\frac{1}{2}, 2)\rightarrow(\frac{1}{2}, 1)$ and $(-\frac{1}{2}, -1)\rightarrow(\frac{1}{2}, -2)$ transitions; the B_2 and B_4 values were the averages of those obtained from these transitions.

INTERPRETATION OF THE RESULTS

It is clear from Table VI that essentially the same result is obtained for B_4 from all the transitions and for all three sources; this is not true, however, of B_2. The value of B_2 for the $(-\frac{1}{2}, 0)\leftrightarrow(\frac{1}{2}, -1)$ transition is particularly erratic. The reason for this behavior is not known; a similar behavior was found for As^{76}. In particular, it is not clear whether this behavior indicates that all of the Sb^{122} atoms are not in the assumed donor sites or that there are some nuclear relaxation

TABLE VI. A_2 and A_4 correction factors. The notation for the various transitions is as follows: π_u, $(-\frac{1}{2}, 2)\rightarrow(\frac{1}{2}, 1)$; π_l, $(-\frac{1}{2}, -1)\rightarrow(\frac{1}{2}, -2)$; σ_u, $(-\frac{1}{2}, 1)\rightarrow(\frac{1}{2}, 0)$; σ_l, $(-\frac{1}{2}, 0)\rightarrow(\frac{1}{2}, -1)$.

Relaxation data	A_2				A_4			
	π_u	π_l	σ_u	σ_l	π_u	π_l	σ_u	σ_l
S-1, π_u	1.15	1.16	1.36	1.33	0.78	0.77	0.85	0.88
S-2, π_u	0.95	0.97	0.94	0.96	0.96	0.94	0.93	0.93
S-2, π_l	1.08	1.08	1.20	1.23	0.75	0.73	0.83	0.82
S-3, π_u	1.03	1.09	1.20	1.28	0.91	0.83	0.88	0.92
S-3, π_l	1.00	1.01	1.04	1.05	0.84	0.84	0.87	0.88

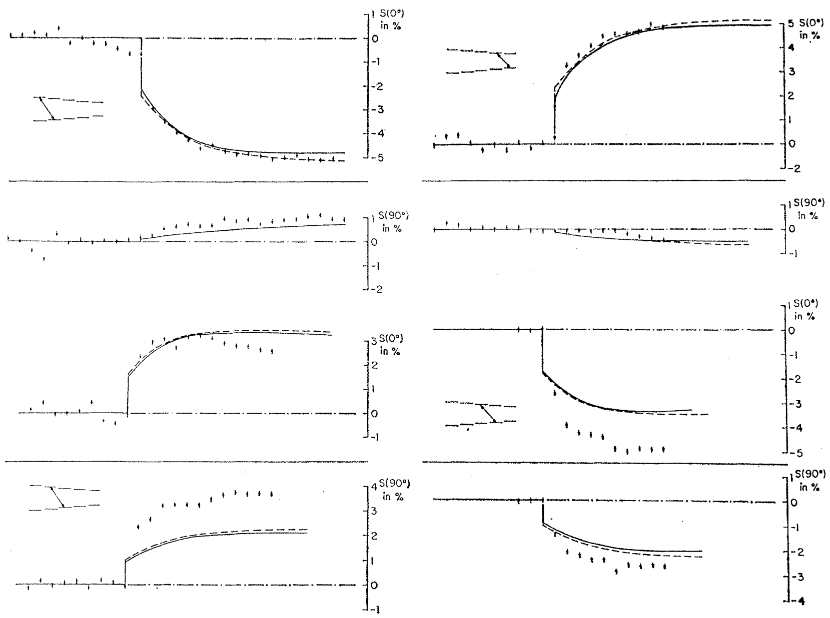

FIG. 10. A figure showing a comparison between the observed signals for S-3 and those which are reproduced by the computer. For the solid line the resistors were chosen to reproduce the average relaxation times observed for the $(-\frac{1}{2}, 2) \to (\frac{1}{2}, -1)$ transition; for the dashed line, the $(-\frac{1}{2}, -1) \to (\frac{1}{2}, -2)$ transition. The average values of the B_2 and B_4 obtained from both transitions were used.

processes in which the Sb^{122} donor interacts with other paramagnetic centers. For our analysis it will be assumed that only the B_2 values from the outer $[(-\frac{1}{2}, 2)\to(\frac{1}{2}, 1)$ and $(-\frac{1}{2}, -1)\to(\frac{1}{2}, -2)]$ transitions are reliable. On this basis we conclude that

$$B_2/B_4 = 1.2\pm0.2,$$
$$B_4 = (0.37\pm0.05)/\eta.$$

The parameter η is an orientation efficiency; it is equal to the fraction of the Sb^{122} atoms which are in the simple donor sites depicted by the theory. From this experiment

$$0.32\leq\eta\leq1.$$

To see what this implies concerning α_0, α_1, and α_2, it is instructive to plot the line

$$\frac{B_2}{B_4}=\frac{1-(\alpha_1/2)-(17\alpha_2/14)}{1-(5\alpha_1/3)-(5\alpha_2/7)}=1.2\pm0.2, \qquad (20)$$

and the set of lines

$$B_4=1-(5\alpha_1/3)-(5\alpha_2/7)=(0.37\pm0.05)/\eta$$

on the same graph. This plot is shown in Fig. 11. The shaded region gives those values of α_0, α_1, and α_2 compatible with the results of this orientation experiment.

The data from the orientation experiment of Somoilov *et al.*[13] can be used to express B_2 in terms of the unknown magnetic field H at the antimony nucleus in the iron sample. The result is

$$B_2=\{[(1.9\pm0.2)\times10^5/H]\}^2.$$

If a value of H is assumed, then a value of B_2 can be obtained. A set of lines for various values of H superimposed on the result for this experiment is shown in Fig. 12. From the two experiments we conclude that

$$170 \text{ kgauss}\leq H\leq400 \text{ kgauss}.$$

If the nuclear resonance for a stable Sb nucleus in an iron sample could be found, light would be shed on both orientation experiments.

To secure further information concerning the nuclear matrix elements, it is necessary to take a closer look

TABLE VII. The B_2 and B_4 coefficients calculated for the various runs. The correction column shows the transition whose data were used to determine the nuclear relaxation times in order to correct for the effect of nuclear relaxation on the initial f_2 and f_4 parameters. The notation for the transitions is π_u, $(-\frac{1}{2}, 2) \to (\frac{1}{2}, 1)$; π_l, $(-\frac{1}{2}, -1) \to (\frac{1}{2}, -2)$; σ_u, $(-\frac{1}{2}, 1) \to (\frac{1}{2}, 0)$; σ_l, $(-\frac{1}{2}, 0) \to (\frac{1}{2}, -1)$. The uncertainty in the determination of the B_2 and B_4 values is estimated to be 10 to 15%.

		B_2				B_4				B_2/B_4			
Source	Correction	π_u	π_l	σ_u	σ_l	π_u	π_l	σ_u	σ_l	π_u	π_l	σ_u	σ_l
S-1	π_u	0.43	0.43	0.74	1.07	0.34	0.44	0.49	0.43	1.26	0.98	1.51	2.49
S-2	π_u	0.38	0.44	0.64	1.03	0.33	0.30	0.37	0.28	1.15	1.47	1.73	3.68
S-2	π_l	0.34	0.41	0.50	0.82	0.43	0.38	0.43	0.32	0.79	1.08	1.16	2.56
S-3	π_u	0.32	0.40	0.24	0.76	0.26	0.30	0.35	0.30	1.23	1.33	0.69	2.53
S-3	π_l	0.34	0.44	0.29	0.95	0.29	0.30	0.36	0.32	1.17	1.47	0.81	2.97

at the beta-decay theory. In the notation of Kotani,[18,19] the six nuclear matrix elements which can contribute to a first-forbidden transition are

$$\eta w = C_A \int \boldsymbol{\sigma}\cdot\mathbf{r};\ \eta\xi' v = C_A \int i\gamma_5, \qquad \lambda=0$$

$$\eta u = C_A \int i\boldsymbol{\sigma}\times\mathbf{r};\ \eta\xi' y = -C_V \int i\boldsymbol{\alpha};\eta x = -C_V \int \mathbf{r}, \qquad \lambda=1$$

$$\eta z = C_A \int B_{ij}, \qquad \lambda=2.$$

The parameter λ gives the angular momentum carried off by the electron-neutrino system. Five of the matrix elements can be expressed in terms of the unique one z by choosing η so $z=1$. In the expressions for the spectrum shape, angular correlation, etc., there occur two particular combinations of matrix elements for which it is advantageous to introduce a special notation. These combinations are

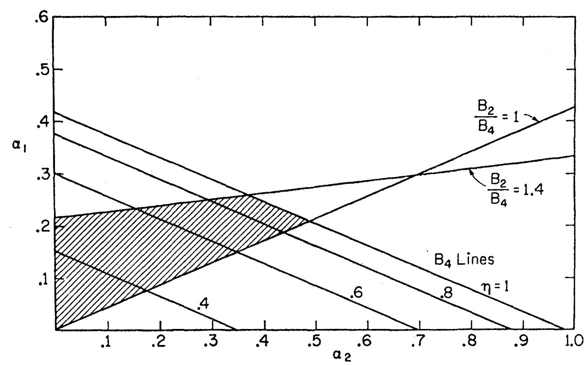

Fig. 11. A diagram showing the possible α_1 and α_2 which are compatible with this experiment. This restriction is based on the observed B_4 and the B_2/B_4 ratios for the two outer transitions.

$$V = \xi' v + \xi w, \qquad \lambda=0$$
$$Y = \xi' y - \xi(u+x), \qquad \lambda=1,$$

where $\xi = \alpha Z/2\rho$; ρ is the nuclear radius in units of the Compton wavelength, α is the fine structure constant and Z the charge on the nucleus. For Sb^{122}, $\xi=12.7$. For our analysis the modified B_{ij} approximation will be used. This assumes that

$$z\neq 0,\quad Y\neq 0,\quad V\neq 0,$$

but

$$x=u=w=0.\quad [Y,\, V<\xi].$$

In this approximation the shape factor for the beta spectrum is given by

$$C(W) = \{Y^2+V^2+[(q^2+\lambda_1 p^2/12]\},$$

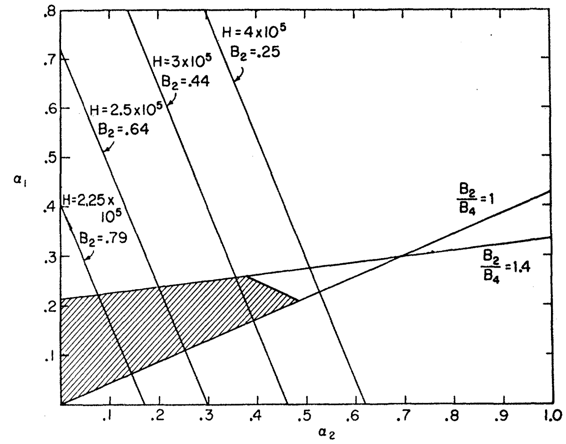

Fig. 12. A diagram showing the relationship between the orientation experiment of Somoilov *et al.* and this one. From the two experiments we conclude that for Sb in iron 170 kgauss $\leq H \leq 400$ kgauss.

where q is the neutrino momentum, p the electron momentum, and λ_1 is a relativistic parameter which has been tabulated by Kotani and Ross.[19] In the B_{ij} approximation the values for α_0, α_1, and α_2 are

$$\alpha_0 = V^2/(V^2+Y^2+0.521),$$
$$\alpha_1 = Y^2/(V^2+Y^2+0.521),$$
$$\alpha_2 = 0.521/(V^2+Y^2+0.521).$$

The normalization relationship between these parameters can be expressed in the form

$$V^2+Y^2 = 0.521[1/\alpha_2 - 1].$$

This equation together with Eq. (20) defines a locus in the V, Y plane of those points which are compatible with the observed orientation, that is, the shaded region in Fig. 11. Figure 13 shows this plot.

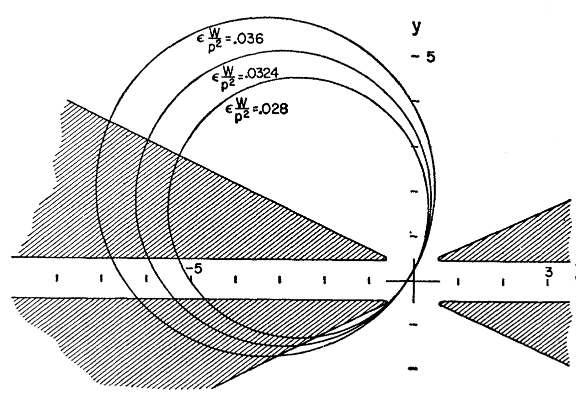

Fig. 13. A diagram showing the values of V and Y which are compatible with this orientation experiment and the observed angular correlation. For this analysis the modified B_{ij} approximation was used. *Note added in proof*. The labels for $\epsilon(w/p^2)$ on the smallest and largest circles should be interchanged.

[18] T. Kotani, Phys. Rev. **114**, 795 (1959).
[19] T. Kotani and M. Ross, Phys. Rev. **113**, 622 (1959).

In terms of the B_{ij} approximation the β-γ angular correlation is given by the expression[18]

$$1+\epsilon P_2(\cos\theta),$$

where

$$\epsilon=(p^2/W)(R_3k+eWk)/C(W),$$
$$R_3k=\lambda_2(1/56)^{\frac{1}{2}}[Y-(8/3)^{\frac{1}{2}}V],$$
$$ek=-(\lambda_1/112).$$

The relationship between Y and V given by the angular correlation can be written in the form

$$\left[V+\frac{\lambda_2}{2(21)^{\frac{1}{2}}\epsilon W/p^2}\right]^2+\left[Y-\frac{\lambda_2}{2(56)^{\frac{1}{2}}\epsilon W/p^2}\right]^2 =\frac{11\lambda_2^2}{672[\epsilon W/p^2]^2}-\frac{\lambda_1 W}{112\epsilon W/p^2}-\frac{8^2+\lambda_1 p^2}{12}.$$

The three circles corresponding to the experimentally observed value and its limits of error are shown in Fig. 13. The overlap of these circles and the area given by the orientation experiment gives three regions of possible solution. These solutions are

$$V=-6.3\pm1.0,$$
$$Y=+1.8\pm1.5,$$

and

$$V=-4.2\pm2.0,$$
$$Y=-1.4\pm0.5,$$

and

$$V=-0.5\pm0.1,$$
$$Y=-0.5\pm0.1.$$

The first two of these imply that the orientation efficiency is $\sim$40%; the third implies that the orientation is 100% efficient. The first two imply that for Sb in iron

$$H=190 \text{ kgauss};$$

the third implies that for Sb in iron

$$H=340 \text{ kgauss}.$$

DISCUSSION

At present there is insufficient evidence to decide between these three possibilities. We can only suggest additional experiments which would clarify the situation.

(1) Determination of the magnetic field at an antimony atom in an iron lattice. This could be done by a nuclear resonance method. Antimony is soluble in iron up to a concentration of a few percent. Experiments with Fe^{57} centers in such a lattice indicate that the nuclear resonance should be detectable. If this magnetic field could be found, then B_2 could be found from the orientation experiment of Shaknov and consequently the orientation efficiency in our experiment.

(2) An improved measurement of the beta-gamma angular correlation as a function of energy. This would not only narrow the wide limits of the region of intersection, but it would also give a clue to the validity of the modified B_{ij} approximation.

(3) Measurement of the correlation between the direction of the beta ray and the circular polarization of the subsequent gamma ray. This gives an independent relationship between the nuclear matrix elements.

(4) A more precise measurement of the spectrum shape to see if the beta ray has a component with a unique forbidden shape.

ACKNOWLEDGMENTS

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