Modified Analysis of Nucleon-Nucleon Scattering. IV. *p-p* Scattering between 9.68 and 98 Mev*

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Proton-proton scattering experiments at 9.68, 18.2, 19.8, 25.63, 39.4, 46, 66, 68.3, 95, and 98 Mev are analyzed under the assumption that the higher partial waves are correctly represented by the one-pion exchange contribution (OPEC). Although the data do not determine a unique phase shift set at any energy, the theoretically reasonable requirement that the $^1D_2$ phase be positive and the $^3P_2-{}^3F_2$ coupling parameter be negative at 68 and 98 Mev singles out the following solutions (nuclear-bar phase shifts in degrees):

Energy	$^1S_0$	$^1D_2$	$^3P_0$	$^3P_1$	$^3P_2$	ϵ_2
68.3 Mev	30.45°	2.62°	18.59°	−10.49°	6.69°	−2.38°
95 Mev	22.18°	3.87°	14.24°	−11.98°	11.17°	−2.78°

This solution type can be qualitatively followed to both lower and higher energies. Such an extension (a) has been shown by Riazuddin to be required by triplet nucleon-nucleon dispersion relations at 4 Mev, (b) is consistent with the best solutions at both 210 and 310 Mev, (c) is qualitatively similar to the requirements of the best phenomenological and semiphenomenological potential models, and (d) carries the signature of the P phases required for consistency with the final-state interaction in the photodisintegration of the deuteron. An attempt to tie solutions at 9.68, 25.63, and 39.4 Mev together using a three-parameter P-phase energy dependence derived by Fubini and Stanghellini, with two of the parameters determined by single pion exchange, was qualitatively consistent but quantitatively unsuccessful.

Although on the above grounds, we believe that this is the physically correct solution type in this energy range, the reader is warned that the solution is experimentally not unique, and that the phase shifts can be varied by a few degrees in a correlated way without doing undue violence to the data. On both counts, it is highly desirable that the triple scattering experiments needed for refining these values be carried through. That only two such experiments at a single angle are needed has recently been shown by Iwadare.

I. INTRODUCTION

THE proposal that the phase shift analysis of nucleon-nucleon scattering experiments be modified to include the one-pion exchange contribution (OPEC) for high angular momentum states,[1] has had considerable success in reducing the allowable *p-p* phase shift sets at 310 Mev[2] and at 210 Mev.[3] In the present paper we investigate whether it also has a significant effect on the analysis of *p-p* scattering experiments in the 10–98 Mev range.

Energy-dependent fits for the whole region up to 310 Mev have been undertaken by Breit *et al.*,[4] and by Stapp *et al.*[5] We are concerned about the possibility that the arbitrariness in the choice of particular forms for the energy dependence may be forcing the acceptance of a restricted subset of the solutions allowed at individual energies, and, if so, whether this restriction is physically significant. We have therefore confined ourselves in this paper to the analysis of experiments at the individual energies indicated. Comparison with the over-all fits is deferred to a later paper.

II. 9.68 TO 39.4 Mev

If the singlet scattering amplitude is assumed known, and the only triplet states present are $^3P_0$, $^3P_1$, and $^3P_2$, Clementel and Villi have shown[6] that the differential cross section can be fitted by three uniquely determined constants. The three P phase shifts are then determined by trigonometric equations which have four roots, so there is an intrinsic phase shift ambiguity unless additional information is used. At 40 Mev, it was found[7] that the Clementel-Villi equations have complex roots unless the $^1S_0$ phase is unreasonably small *and* the $^1D_2$ phase is unreasonably large. Further, even if these values are accepted, the phase shift solution predicts a polarization of at least 15%. The variation of *p-p* polarization with energy has been explored at energies above 46 Mev,[8] and any reasonable extrapolation to 39.4 Mev would lead one to expect a maximum polarization of (1±1)% at that energy. It is therefore safe to conclude that triplet scattering at 39.4 Mev includes states other than P waves. We have reanalyzed the data[9] using phenomenological S and P waves assuming that all higher partial waves are given by OPEC, and find that, although a phase shift ambiguity persists, the $^1S_0$ phase for all good solutions lies between 45° and 47°. The values of the S phase so indicated are considerably higher than the 38.3° phase shift predicted

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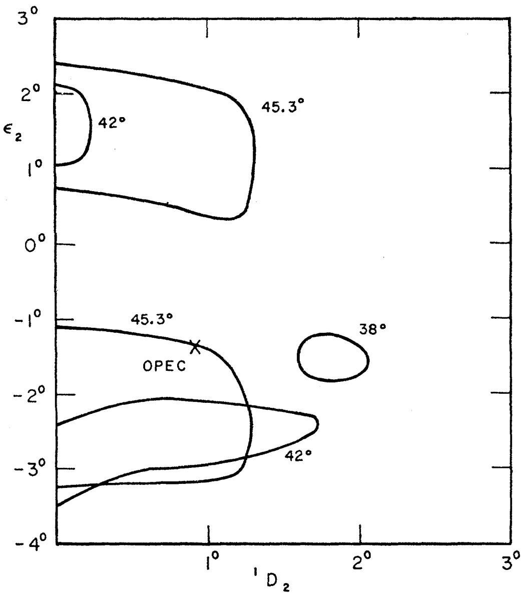

FIG. 1. Allowed regions of $^1D_2$ and ϵ_2 at 39.4 Mev for fixed $^1S_0$ phases of 38°, 42°, or 45.3° with phenomenological P phases and higher phases given by OPEC.

by the Gammel-Thaler potential at this energy[10] or by the Raphael extended effective range formula,[11] but are close to the value given by the extended effective range calculation of Noyes and Wong[12] or Cini *et al.*[13] However, Perring and Phillips[14] have recently shown that it is possible to construct hard-core potential models which give a phase shift as high as 45° at this energy, so even if established, the result could not be used to discriminate between potential model calculations and the partial wave dispersion relation.[12,13]

Because of the usefulness of a precise S phase at this energy whatever the model used, we have examined the sensitivity of the above result to departures of the $^1D_2$ phase and the $^3P_2-^3F_2$ coupling parameter from the values given by OPEC. 40 Mev is already at a high enough energy that *some* deviation from the OPEC value might reasonably be expected. We have therefore explored the χ^2 surface as a function of these two phases for fixed $^1S_0$ phases of 38°, 42°, and 45.3°. The allowed values of these parameters fall within the contours plotted in Fig. 1. We see that although the solution region is small for an S phase of 38°, rather precise limits on the allowable departure of $^1D_2$ and ϵ_2 from the OPEC value would have to be fixed in order to show that it is excluded.

At energies below 40 Mev, it is much more reasonable to assume that everything except S and P waves is adequately represented by OPEC. Unfortunately, the data are not sufficiently precise in this region to give evidence for the very small phase shifts predicted by OPEC, so we cannot hope for much improvement in the analysis. We have checked this in detail using differential cross sections at 9.68,[15] 18.2,[16] 19.8,[17] and 25.63[18] Mev, and find that the solution regions already given by one of us (MHM)[19] are not significantly changed. At first sight, one might hope that at sufficiently low energy, the ^3P phases could be taken from OPEC, but model calculations show that the centrifugal shielding is insufficient to make this true even at 4 Mev.[20] If we analyze the 18.2-Mev data using in addition the polarization measured at 16 Mev,[21] we find that the $^1S_0$ phase is more likely to be 50° or 51° than either 49° or 52°, but this uncertainty covers the region of theoretical interest.

Since the data do not determine the S phase, we have attempted to see if, for reasonable S phase, the energy variation of the P phases gives any clue. We have computed the S phase at four energies both from the usual shape independent effective range formula and from the fit to the Raphael formula,[11] and searched to find the 4 types of P-wave solutions, assuming higher partial waves are given by OPEC. The results are given in Table I. We see that qualitatively the four types can be followed from one energy to the next, except that solutions of type II tend to flop over into I or IV, but that the fluctuations from an expected smooth energy variation are large. One can conclude from this either that the S-wave energy variation we have assumed is not accurate enough to give consistent results, or that there is some systematic inconsistency between the two intermediate experiments and those at the end of the range. We remind the reader that, as noted in reference 19, phase shifts obtained from analyzing $\sigma(\theta)$ are extremely sensitive to systematic errors ("tipping") in angle.

We have also tried to avoid this possible inconsistency between different laboratories by taking only the Minnesota experiments at 9.68, 25.63, and 39.4 Mev, and trying them together by a theoretically derived energy dependence for the P phases. Fubini and Stanghellini[22] have used a technique analogous to the one used for the $^1S_0$ state by Cini *et al.*[13] to derive a three parameter formula for each of the P states. In this formula two

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TABLE I. Nuclear-bar phase shift sets for fixed $^1S_0$ phase, and $^1D_2$, ϵ_2, and higher phases taken from OPEC. (a) $^1S_0$ phase from the shape-independent effective-range approximation. (b) $^1S_0$ phase from the Coulomb-corrected Raphael formula fitted to SYM-1 at 310 Mev.

Energy (Mev)	$^1S_0$	I $^3P_0$	I $^3P_1$	I $^3P_2$	II $^3P_0$	II $^3P_1$	II $^3P_2$	III $^3P_0$	III $^3P_1$	III $^3P_2$	IV $^3P_0$	IV $^3P_1$	IV $^3P_2$
(a) 9.68	54.74°	4.07	−3.84	1.29	7.53	−1.78	−0.66	−7.91	0.29	1.20	−0.74	3.72	−2.29
18.2	51.74°	8.31	−3.47	1.58		I		−9.08	2.32	3.26	0.14	6.93	−1.23
19.8	50.85°	0.97	−4.07	2.44		IV		−5.70	−2.12	2.58	4.61	2.98	−2.52
25.63	49.51°	8.19	−4.24	2.00		IV		−8.74	−0.89	3.49	3.95	5.38	−2.75
(b) 6.98	54.06°	5.15	−4.75	1.57	9.28	−2.55	−0.57	−9.91	0.72	1.29	−2.20	4.78	−2.61
18.2	49.52°	11.43	−5.21	1.86		I		−13.35	2.92	3.52	−12.77	9.14	−2.00
19.8	48.58°	7.82	−6.36	2.29	13.03	−3.61	−0.31	−12.92	−0.55	2.95	1.26	6.50	−4.12
25.63	45.3°	15.41	−6.73	1.98		I		−17.50	0.77	4.10	0.54	9.68	−4.55

of the parameters can be derived from single pion exchange, and the third, taken to represent the rest of the interaction, can be fitted to experiment. We have fitted this constant to the 25.63-Mev P phases, and calculated to phase shifts at 9.68 and 39.4 Mev for all the cases given above. At neither energy did we obtain an acceptable fit, so we searched for solutions using these values as a starting point. In most cases the $^3P_1$ phase did not change a great deal, but the other two changed by 25–50% or even more. We conclude that while this approach gives a qualitatively adequate description of the energy variation, it still does not allow us to resolve the phase shift ambiquity. This attempt to correlate the data at different energies, although unsuccessful, is, in one sense, encouraging. It shows that the data *are* quite restrictive if a quantitative model for the two-nucleon interaction is required to fit them.

The conclusion we reach from this study is that triple scattering experiments are required to give an *experimental* determination of the phase shifts in this energy range. Recently Iwadare[23] has explored this situation in detail, and finds that two such experiments at an appropriate angle can indeed give the required information. We refer the reader to his paper for details.

Another way to resolve the ambiguity has recently been given by Riazuddin.[24] This makes use of dispersion relations for certain combinations of the triplet amplitudes evaluated at low energy. This calculation uniquely specifies the signs of certain combinations of the P phases. Together with the measured sign of the polarization at 16 Mev, this allows him to conclude that solution type I given above is the physically correct one. To obtain the acutal values of the phases, the triple scattering experiments are, of course, still required.

III. 66 AND 68.3 Mev

The differential cross section has been measured at 68.3 Mev,[25] and the polarization at 66 Mev.[8] We have analyzed these data, given in Table II, assuming both at an energy of 68.3 Mev. Starting from random values for the phase shifts and searching on S, P, and D waves with the higher phases taken from OPEC did not lead us to solutions with physically reasonable values for the S and D phases, so that the data did not determine the phases without additional assumptions. By holding the S wave fixed at values lying between 30° and 40°, however, we were able to get sets which look vaguely like the four Clementel-Villi solutions at lower energy, but usually with poor χ^2 values. If one believes that at this energy the D phase should be greater than 1° and that ϵ_2 is negative, only one of these sets looks reasonable. We therefore took this set and searched on S, P, D, and ϵ_2, starting from various values of the S phase and OPEC values for $^1D_2$ and ϵ_2. All these searches ended up close to a single solution, which is given in Table III.

TABLE II. Differential cross section data at 68.3 Mev (reference 24) and polarization data at 66 Mev (reference 8) used to obtain the phase shift solutions given in Table III.

68.3 Mev		66 Mev	
(1)	(2) (3)	(4)	(5) (6)
$\theta_{c.m.}$	$\sigma(\theta)$ (mb)	$\theta_{c.m.}$	$P(\theta)$
10.18°	12.96±0.104	20.4°	0.050±0.014
12.215°	7.05±0.056	25.5°	0.047±0.009
13.233°	6.14±0.049	30.5°	0.077±0.010
14.25°	5.53±0.044	35.6°	0.078±0.008
16.29°	5.20±0.042	40.7°	0.062±0.008
18.32°	5.24±0.042	45.7°	0.069±0.008
20.36°	5.50±0.044	50.8°	0.067±0.008
22.39°	5.65±0.045	55.9°	0.059±0.008
24.42°	5.81±0.047	60.9°	0.058±0.007
26.45°	5.94±0.048	65.9°	0.053±0.007
28.48°	6.11±0.049	71.0°	0.038±0.007
30.51°	6.23±0.050		
32.54°	6.28±0.050		$P/\sin\theta\cos\theta \pm \Delta P/\sin\theta\cos\theta$
34.57°	6.33±0.051	20.4°	0.153±0.043
36.61°	6.30±0.050	25.5°	0.121±0.023
40.67°	6.34±0.051	30.5°	0.176±0.023
44.71°	6.31±0.050	35.6°	0.165±0.017
50.80°	6.34±0.051	40.7°	0.125±0.016
54.83°	6.30±0.050	45.7°	0.138±0.016
60.90°	6.34±0.051	50.8°	0.137±0.016
64.93°	6.28±0.050	55.9°	0.127±0.017
70.97°	6.21±0.050	60.9°	0.137±0.016
74.99°	6.17±0.049	65.9°	0.142±0.019
81.01°	6.18±0.049	71.0°	0.123±0.023
91.04°	6.17±0.049		
101.01°	6.17±0.049		

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TABLE III. Type I nuclear-bar phase shift sets at 68.3 and 95 Mev obtained from the analysis of the data given in Tables II and IV. Similar sets found by Perring (reference 26) are given for comparison.

Energy (Mev)	Authors	$^1S_0$	$^1D_2$	$^3P_0$	$^3P_1$	$^3P_2$	ϵ_2	χ^2	χ^2 (expected)
68.3	Present work	30.45°	2.62°	18.59°	−10.49°	6.96°	−2.38°	42.0	30
68.3	Perring	33.17	2.71	16.65	−10.22	7.73	OPEC	38.0	26
95	Present work	22.18°	3.87°	14.24°	−11.98°	11.17°	−2.78°	30.84	28
98	Perring	23.52	4.04	14.05	−12.88	10.14	OPEC	16.5	22

The expected value of χ^2 is 28 as compared to values of 40–46 for phase shifts in this neighborhood. However, 16 of this value comes from the smallest angle point where the cross section is varying so rapidly as to make this discrepancy reasonable. We conclude that this is the physically reasonable solution at this energy, although it is not *experimentally* unique. A similar solution has been found by Perring,[26] which we quote for comparison. He used fewer pieces of data, and took ϵ_2 from OPEC; the difference in the phase shift values should serve as a warning that the uncertainty in these values is quite large. Again it is clear that triple scattering data are needed both to give an experimental resolution of the phase shift ambiguity and to give an accurate value of the $^1S_0$ phase.

IV. 95 AND 98 Mev

We have analyzed differential cross section and polarization measurements at 95 Mev, and depolarization[27] data at 98 Mev, assuming all measurements were at 95 Mev. The data used are given in Table IV. Searching on S, P, and D phases, and taking the rest from OPEC, we found only one solution type. Displacing the initial value from this solution and searching on ϵ_2 as well, 10 searches returned to this solution, which is given in Table III, again in comparison with a similar solution found by Perring.[26] The eleventh search found a different relative minimum with a negative D phase and positive ϵ_2 but the χ^2 is 123.7. If we also allow the F phases to vary, many solution types can be found, so this solution is "unique" only under rather strict assumptions about the higher partial waves. That the solution is only qualitative is emphasized by Fig. 2, where we plot χ^2 as a function of the pion-nucleon coupling constant. Note that the fit is improved by including ϵ_2 as a phenomenological parameter, but that the minimum as a function of G^2 comes at an unreasonably high value.

Since this work was completed, we have learned of a new analysis by Palmieri and Prenowitz[28] using more recent and more extensive data, as well as a more careful treatment of the experimental errors. They have found the same solution type that we present here, without using OPEC for the higher partial waves.

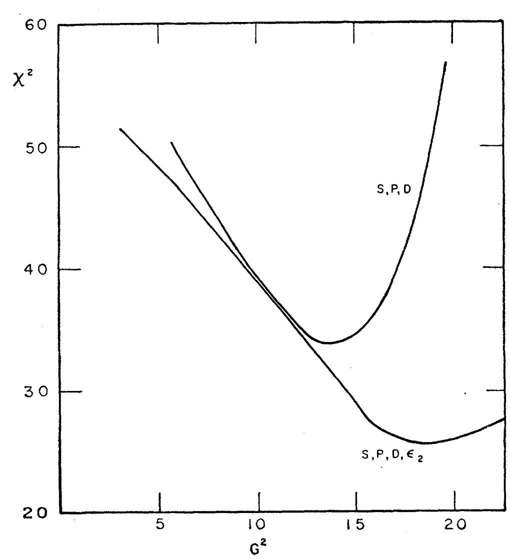

FIG. 2. Value of χ^2 vs G^2 for 5 and 6 parameter fit to 95-Mev $p-p$ scattering data.

V. CONCLUSION

We conclude that the published p-p scattering data in the range 10–98 Mev do not allow a unique *experimental* determination of the phase shifts at any single energy when supplemented only by the requirement that the higher partial waves be given by OPEC. However, the very plausible requirement that the $^1D_2$ phase have a reasonable positive value and the ϵ_2 be negative does single out one solution type at both 68 and 95 Mev. This can be followed down in energy, qualitatively, and is the same solution favored by the dispersion theoretic analysis of Riazuddin[24] at 4 Mev. It also is reasonable from the point of view of phenomenological or semiphenomenological[29] potential models, and when compared to solution 1 at 310 Mev,[2,30] and solution b at 210 Mev.[3] In addition we point out that the signature of the P phases in this solution type is the only one consistent with the final state interaction in the photo-

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TABLE IV. Harvard $p-p$ scattering experiments at 95 Mev (reference 8, 26) used to obtain the phase shift sets in Table III.[a]

$\theta_{c.m.}$ (deg)	$\sigma(\theta)$ (mb/sr)	$\Delta\sigma(\theta)$ (mb/sr)	$r(x)$	$\Delta r(x)$	$P(x)$	$\Delta P(x)$	$s(x)$	$\Delta s(x)$	$D(x)$	$\Delta D(x)$	$t(x)$	$\Delta t(x)$
20.6	4.09	0.08	0.928	0.0182	0.092	0.010	0.279	0.030	0.00	0.08	1.00	0.08
25.7	4.51	0.08	1.024	0.0182	0.111	0.008	0.284	0.021				
30.7	4.71	0.08	1.069	0.0182	0.130	0.007	0.296	0.016	0.00	0.07	1.00	0.07
35.8	4.77	0.08	1.083	0.0182	0.131	0.007	0.276	0.015				
40.9	4.68	0.08	1.062	0.0182	0.112	0.007	0.226	0.014	0.00	0.08	1.00	0.08
46.0	4.74	0.08	1.076	0.0182	0.126	0.007	0.252	0.014				
51.1	4.74	0.08	1.076	0.0182	0.115	0.007	0.235	0.014	−0.12	0.10	1.12	0.10
56.2	4.66	0.08	1.058	0.0182	0.096	0.007	0.208	0.015				
61.2	4.64	0.08	1.053	0.0182	0.099	0.007	0.234	0.017	−0.11	0.16	1.11	0.16
66.3	4.63	0.08	1.051	0.0182	0.087	0.007	0.236	0.019				
71.3	4.61	0.08	1.046	0.0182	0.069	0.008	0.227	0.026				
76.4	4.58	0.08	1.040	0.0182	0.058	0.007	0.254	0.031				
81.4	4.51	0.08	1.024	0.0182	0.038	0.007	0.257	0.047				
86.4	4.46	0.08	1.012	0.0182	0.023	0.007	0.367	0.112				

[a] The differential cross section at 90° was taken to be 4.405. This reference value has an absolute error of 5%. D was actually measured at 98 Mev. The definition of the quantities $r(x)$ and $s(x)$ and $t(x)$ are as in Ref. 30: $r(x)=\sigma(\theta)/\sigma(90°)$, $s(x)=P(x)/\sin\theta\cos\theta$, $t(x)=1-D(x)$, where all the θ's are $\theta_{c.m.}$

disintegration of the deuteron.[31,32] From a theoretical point of view, we therefore have little doubt that this is the correct solution type at low energy, but the triple scattering experiments which would settle the question are clearly desirable. Further, they are urgently needed to give reasonably accurate phase shifts for more quantitative comparison with theories of the two-nucleon interaction. That the data considered as a function of energy put serious restrictions on such models is shown by our failure to fit the 9.68, 29.63, and 39.4 data simultaneously with a simple, but theoretically reasonable, energy dependence.

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