# Isotope Shift in Axially Asymmetric Nuclei

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The effect of axial asymmetry on isotope shift, as introduced in the theory of Davydov and Filippov, is investigated and is found to be non-negligible. An accurate experimental study of the isotope shift may therefore provide evidence in favor of or against Davydov-Filippov's theory for deformed nuclei.

Davydov and Filippov,[1] as a modification of Bohr's[2,3] theory, have assumed that medium- and heavy-weight nuclei possess axial asymmetry in order to explain systematically the first excited states of even atomic nuclei, such as the spin sequence of the excited states, the energy of these states, and the probabilities of electromagnetic transitions between them.[4,5] A number of publications[6] now exist which show that the nuclei under consideration may in fact be asymmetric, but the question demands further investigation due to the lack of rigor in these calculations. Some of the anomalies explained by the DF theory can also be reproduced in other ways without introducing axial asymmetry.[7] It is therefore of interest to look for observable effects of axial asymmetry in order to provide additional evidence in favor of or against Davydov-Filippov's model for nonaxial nuclei.

The introduction of axial asymmetry in nuclei will affect a large number of nuclear properties such as magnetic moment, moment of inertia, etc., and will also affect the isotope shift. A study of these effects may show whether axial asymmetry must be introduced to account for the anomalies observed in the medium- and heavy-weight nuclei. In this note the effect of axial asymmetry on the isotope shift is calculated.

The theory of the isotope shift in deformed nuclei with axial symmetry has been considered by Brix and Kopfermann,[8] by Ford,[9] by Wilets *et al.*,[10] and independently by Bodmer,[11] and has been recently reviewed by Breit.[12] It has been shown by these authors that the isotope shift, in the notation of Breit,[12] depends on the mean value $\langle r^{2\rho}\rangle$ weighted in proportion to the nuclear charge density.

Let the radius $r$ to the nuclear surface relative to the nuclear axes be given by

$$r=Nr_0[1+\textstyle\sum_\mu \alpha_\mu Y_2^\mu(\theta,\phi)], \qquad (1)$$

where

$$\alpha_0=\beta\cos\gamma;\quad \alpha_1=\alpha_{-1}=0,$$
$$\alpha_2=\alpha_{-2}=(1/\sqrt{2})\beta\sin\gamma,$$

where $N$ is a normalization constant and the other symbols have their usual meaning. The normalization constant $N$ is evaluated by the requirement of incompressibility of nuclear matter and is given by

$$N^3\left[1+\frac{3}{4\pi}\beta^2+\frac{1}{4\pi}\frac{2}{7}\left(\frac{5}{4\pi}\right)^{\frac{1}{2}}\beta^3\cos\gamma(1-4\sin^2\gamma)\right]=1. \qquad (2)$$

The ratio of the isotope shift [I.S.] for the deformed volume to that for the spherical volume is

$$[\text{I.S.}]_{\text{def}}/[\text{I.S.}]_{\text{sp}}=\left(\int r^{2\rho}d\mathbf{r}\right)\Big/\left(\int r^{2\rho}d\mathbf{r}\right)_{\beta=\gamma=0}$$
$$=N^{2\rho+3}\left[\int r^{2\rho+2}(1+\alpha_0Y_2^0+\alpha_2Y_2^2+\alpha_2Y_2^{-2})^{2\rho+3}drd\Omega\right]\Big/\left(4\pi\int r^{2\rho+2}dr\right)_{\beta=\gamma=0}.$$

Evaluating the integrals and keeping the terms to within the order $\beta^3$, one gets

$$\frac{[\text{I.S.}]_{\text{def}}}{[\text{I.S.}]_{\text{sp}}}=\left[1+\frac{\rho(2\rho+3)}{4\pi}\beta^2+\frac{\rho(2\rho+3)^2}{12\pi}\frac{2}{7}\left(\frac{5}{4\pi}\right)^{\frac{1}{2}}\beta^3\times\cos\gamma(1-4\sin^2\gamma)\right]. \qquad (3)$$

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6 A. Davydov and G. Filippov, Nuclear Phys. **10**, 654 (1959); G. Filippov, J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 1316 (1960) [translation: Soviet Phys.—JETP **11**, 949 (1960)]. References to the earlier literature are given in these papers.

7 M. Jean, Nuclear Phys. **21**, 142 (1960). It should be pointed out that the DF theory gives better agreement with experiments than the work of Jean, which is based on a purely phenomenological model. Also, some of the predictions of the DF theory such as the equality of the sum of the energies of the first two $2^+$ levels and the energy of the first level $3^+$ can be obtained exactly in Jean's work only in the two limiting cases $k\to 0$ and $k\to\infty$.

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10 L. Wilets, D. L. Hill, and K. W. Ford, Phys. Rev. **91**, 1488 (1953).

11 A. R. Bodmer, Proc. Phys. Soc. (London) **47**, 622 (1954).

12 G. Breit, Revs. Modern Phys. **30**, 507 (1958). The $\beta$ used here is $(4\pi/5)^{\frac{1}{2}}$ times the $\beta$ used by Breit.

The first two terms are the same as those obtained by Wilets *et al.*[10] and by Breit[12] for axially symmetric nuclei. Following the work of Wilets *et al.*,[10] or Breit,[12] it can be shown that the difference between two such shifts for different isotopes of the same element may be written as

$$\frac{\Delta\delta W(\beta,\gamma)}{\Delta\delta W_r}=\frac{3}{10}A(2\rho+3)\left(\frac{\partial F(\beta,\gamma)}{\partial N}\right)_Z, \quad (4)$$

where

$$F(\beta,\gamma)=\left[\frac{5}{4\pi}\beta^2+\frac{2}{21}(2\rho+3)\left(\frac{5}{4\pi}\right)^{\frac{3}{2}}\beta^3 \times\cos\gamma(1-4\sin^2\gamma)\right]. \quad (5)$$

It may be of interest to calculate the effect of axial asymmetry for one particular case, namely for the isotopes of $_{64}$Gd for which the change in $\beta$ between isotopes as determined from the intrinsic nuclear deformation[13] is found to be large, and the value of the asymmetry parameter $\gamma$ also varies between 0° and 30°. The values of $\beta$ for the isotopes $_{64}Gd^{154}$, $_{64}Gd^{156}$, and $_{64}Gd^{158}$ are found to be approximately 0.48, 0.65, and 0.73, respectively.[14] Unfortunately, no measurements are available on the intrinsic quadrupole moment of $_{64}Gd^{152}$. The work of Ford[15] indicates that between 88 and 90 neutrons there is a marked increase in nuclear deformation. Based on the work of Ford, the value of $\beta$ is expected to be 0.24 for $_{64}Gd^{152}$ and will be assumed as such for the present calculation. The values of the asymmetry parameter $\gamma$ have been determined, without reference to $\beta$, from the expression

$$\sin^2 3\gamma=9x/2(x+1)^2, \quad (6)$$

where $x=E_2(2^+)/E_1(2^+)$, the ratio of the energies of the first two $2^+$ excited states[16]; these values of $\gamma$ are found to be 30°, 14°,[17] 11°, and 0° for $Gd^{152}$, $Gd^{154}$, $Gd^{156}$, and $Gd^{158}$, respectively. The variations in $F(\beta,\gamma)$ with and without axial asymmetry for the isotope shifts of $Gd^{154}-Gd^{152}$, $Gd^{156}-Gd^{154}$, and $Gd^{158}-Gd^{156}$ are found to be 6.0%, 0.1%, and 14.0%, respectively. Similar variations are expected in isotopes like $Sm^{150,152,154}$, $Os^{186,188,190}$, $Hg^{198,200,202}$, and $W^{182,184,186}$. The effect is therefore by no means small, and should be observable if the error involved in the experimental data on isotope shifts can be reduced to ±3% instead of the present value of ±20%.[18]

The question may be asked whether the effect being discussed would be masked by the effect of nuclear compressibility. The recent work of Meligy *et al.*[19] indicates that the observed isotope shifts in nuclei can be well accounted for by the volume and deformation effects if the nucleus is considered as a sphere in which the charge distribution has the general shape determined by the electron-nucleus scattering experiments. This work indicates that the condition of incompressibility in nuclear matter is well satisfied, in agreement with the calculations of Brueckner and Gammel[20] who find the value of the compressibility ~170 Mev.[21] The effect of axial asymmetry on isotope shift is found not to be masked by such a high value of the compressibility.[22]


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[17] The value of $\gamma=14°$ for $Gd^{154}$ as given here is different from that given by DF ($\gamma=21°$) because DF use $E_2(2^+)/E_1(2^+)=3.01$ instead of the correct value, 8.114.

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[19] A. Meligy *et al.*, Nuclear Phys. **16**, 99 (1960).

[20] K. A. Brueckner and J. L. Gammel, Phys. Rev. **109**, 1023 (1958).

[21] The large discrepancy between the experimentally observed isotope shift and the theoretically predicted values based on the incompressible nuclear model in the work of Wilets, Hill, and Ford (reference 10) could have been due to the large nuclear radius used by these authors. A smaller radius will lead to better agreement between theory and experiment and imply a larger value of the compressibility coefficient, in agreement with the work of Brueckner and Gammel (reference 20). The author is grateful to Professor K. W. Ford for bringing up this point and for his general comments on the manuscript.

[22] *Note added in proof*. After the revised version of the present paper had been submitted for publication, a paper by D. P. Grechukhin [Nuclear Phys. **24**, 576 (1961)] has appeared in which a similar suggestion is made but not carried out in de tail