$g(\nu)$ is not Gaussian. For N^{14} the difference between $(\nu_{ms})_{exp}$ and ν_{ms} for $\nu_m=0$ is only 4%. The true linewidths have been set at

$$\nu_{ms}(14)=24\pm4 \text{ cps},$$
$$\nu_{ms}(15)<4 \text{ cps}.$$

The true linewidth of N^{15} is probably less than 1 cps due to lack of an efficient relaxation mechanism. From these data and with the assumption of Lorentzian line shapes, the relaxation times are

$$T_2(14)=1/(\sqrt{3}\pi\nu_{ms})=0.007\pm0.001 \text{ sec},$$
$$T_2(15)>0.05 \text{ sec}.$$

The relaxation T_1 has been determined for N^{14} by the method of progressive saturation. The value found is

$$T_1(14)=0.004\pm0.002 \text{ sec}.$$

Due to the long T_1 for N^{15} it was not possible to use the method of progressive saturation. Experiments to measure relaxation times T_1 and T_2 for both isotopes by means of spin echo techniques are being planned.

Note added in proof. These measurements are now in progress and reveal a very interesting and complex behavior for the relaxation of liquid nitrogen. In addition to the T_1 detected here, there is a second one about ten times longer in the isotopic mixture. These are presumably associated with N^{14} in the $N^{14}-N^{14}$ and $N^{14}-N^{15}$ molecules. The resonance with the longer relaxation time was almost totally saturated relative to the other resonance throughout the cw work reported here and so was not noticed. A full discussion will be submitted for publication shortly.[9]

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[9] P. C. Canepa and T. A. Scott (to be published).

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Stripping Analysis of the $Be^9(Li^6,\alpha)B^{11}$ Reaction*

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An analysis of the angular distributions of α particles from the $Be^9(Li^6,\alpha)B^{11}$ reactions has been carried out using a simple "lump" stripping model. Both the normal stripping mode and that exchange mode commonly called "heavy-particle stripping" have been incorporated in the analysis. The model assumes that the Li^6 and the Be^9 nuclei may be represented by the two-cluster configurations "alpha particle plus deuteron" and "alpha particle plus He^5" respectively. The angular distribution calculated for the first excited state reaction provides a satisfactory fit to previously published experimental results at a laboratory bombarding energy of 3.25 Mev. The theoretical angular distribution for the ground state reaction is less successful but does show the principle features of the experimentally observed angular distribution.

I. INTRODUCTION

ANGULAR distributions of α particles from the $Be^9(Li^6,\alpha)B^{11}$ reactions leading to the ground and the first-excited state of B^{11} have recently been published[1] for laboratory bombarding energies ranging from 2 to 4 Mev. It was suggested in that report that the character of the angular distributions indicated some direct-interaction mechanism for the reaction. An analysis based on an elementary form of stripping theory has therefore been attempted. Both the normal or "projectile stripping" mode and that exchange mode called "heavy-particle stripping"[2] or "target stripping" have been included in the analysis, as has the interference term between the two modes. No compound nucleus contribution has been included.

II. CLUSTER MODEL OF THE INTERACTING NUCLEI

In calculating the scattering amplitude for the projectile-stripping mode it is assumed that the Li^6 projectile may be represented by the two-cluster configuration $\alpha(1)+d$, where d indicates a deuteron. The internal structures of both clusters are neglected. $\alpha(1)$ and d are taken to be in a relative S state.[3] In the projectile-stripping mode the deuteron is stripped from the Li^6 and captured by the Be^9 target nucleus, leaving $\alpha(1)$ as the outgoing α particle. For the target-stripping mode the Be^9 target is assumed to be described at the time of interaction by the two-cluster configuration $\alpha(2)+He^5$. Again the internal structures of the two clusters are

* This work was supported in part by the joint program of the U. S. Atomic Energy Commission and the Office of Naval Research.

[1] J. J. Leigh and J. M. Blair, Phys. Rev. **121**, 246 (1961).

[2] L. Madansky and G. E. Owen, Phys. Rev. **99**, 1608 (1955); G. E. Owen and L. Madansky, *ibid.* **105**, 1766 (1957).

[3] J. L. Gammel, B. J. Hill, and R. M. Thaler, Phys. Rev. **119**, 267 (1960).

neglected and they are taken to be in a relative S state. In the target-stripping mode the He^5 cluster is stripped from the Be^9 and captured by the Li^6 projectile, leaving $\alpha(2)$ as the outgoing α particle.

$\alpha(1)$ and $\alpha(2)$ are taken to have the spin and parity ($J^\pi=0^+$) of free α particles. With the above S-state assumptions and the known spin and parity of the ground states of Li^6 ($J^\pi=1^+$) and Be^9 ($3/2^-$), this implies that the spin and parity of d are those of a free deuteron (1^+), and that the He^5 cluster (which is unbound in the free state) has the spin and parity ($3/2^-$) of the He^4+n state of least excitation.

Both Coulomb and nuclear interactions between the projectile P (Li^6) and the target T (Be^9), and between the residual nucleus R (B^{11}) and the outgoing particle $\alpha(1)$ or $\alpha(2)$, are neglected. These are serious simplifications at the low energies of interest here. It is well known, however, that the two types of interactions frequently result only in a smoothing of the stripping angular distributions without shifting the angular positions of their extrema. Such an effect also seems to appear in the present analysis.

III. DIFFERENTIAL CROSS SECTION EXPRESSION

Under the usual stripping assumptions, the scattering amplitude may be written as[4]

$$\mathcal{T}_0=\mathcal{T}_0^{(P)}+\mathcal{T}_0^{(T)}, \tag{1}$$

where

$$\mathcal{T}_0^{(P)}=\int\psi_f^\dagger(x)V_{d\alpha(1)}\psi_i(x)dx \tag{2}$$

is the scattering amplitude due to the projectile-stripping mode and

$$\mathcal{T}_0^{(T)}=\int\psi_f^\dagger(x)V_{H\alpha(2)}\psi_i(x)dx \tag{3}$$

is the scattering amplitude for the target-stripping mode. The wave functions $\psi_f(x)$ and $\psi_i(x)$ are those of the initial and the final states of the system and x is a collective symbol for the (vector) positions of the various nuclei and clusters involved. The factor of two in Eq. (3) arises from the two equivalent α particles in Be^9, either of which may appear as the outgoing α particle $\alpha(2)$ in the target stripping mode. $V_{d\alpha(1)}$ and $V_{H\alpha(2)}$ are the nuclear interaction potential operators describing respectively, the interaction between the two clusters of Li^6, and between the two clusters of Be^9.

In plane-wave approximation we write the initial-state asymptotic wave function as

$$\psi_i(x)=e^{i\mathbf{k}_P\cdot\mathbf{R}_P}\psi_P(r)\xi_P(J_PM_P)\times e^{i\mathbf{k}_T\cdot\mathbf{R}_T}\psi_T(\rho)\xi_T(J_TM_T). \tag{4}$$

Here $\mathbf{k}_P$ and $\mathbf{k}_T$ are the wave vectors of the motion of P and T relative to the center-of-mass of the system, and $\mathbf{R}_P$ and $\mathbf{R}_T$ are the corresponding positions. $\psi_P(r)$ and $\psi_T(\rho)$ are the S-state internal motion radial wave functions for the $\alpha+d$ system composing Li^6 and the $\alpha+He^5$ system composing Be^9. $\xi_q(J_qM_q)$ is the total angular-momentum function of particle q corresponding to total angular momentum J_q with projection M_q. For the projectile- and target-stripping modes, respectively, the asymptotic final state wave functions are

$$\psi_f(x)=e^{i\mathbf{k}_{\alpha(1)}\cdot\mathbf{R}_{\alpha(1)}}S_\alpha(j_\alpha\mu_\alpha)\times e^{i\mathbf{k}_R\cdot\mathbf{R}_R}\psi_R^{(P)}(|\mathbf{R}_d-\mathbf{R}_T|)\xi_R(J_RM_R) \tag{5}$$

and

$$\psi_f(x)=e^{i\mathbf{k}_{\alpha(2)}\cdot\mathbf{R}_{\alpha(2)}}S_\alpha(j_\alpha\mu_\alpha)\times e^{i\mathbf{k}_R\cdot\mathbf{R}_R}\psi_R^{(T)}(|\mathbf{R}_H-\mathbf{R}_P|)\xi_R(J_RM_R).$$

Here $S_\alpha(j_\alpha\mu_\alpha)$ is the spin function of the outgoing particle with spin j_α and projection μ_α. (Note, however, that $j_\alpha=\mu_\alpha=0$.) $\mathbf{k}_\alpha$ and $\mathbf{k}_R$ are the wave vectors of the motion of the outgoing α particle and the residual-nucleus relative to the center-of-mass of the system, and $\mathbf{R}_\alpha$ and $\mathbf{R}_R$ are the corresponding positions. $\mathbf{R}_H$ is the position of the He^5 cluster relative to the center-of-mass of the system. $\psi_R^{(P)}$ and $\psi_R^{(T)}$ are the radial wave functions of the residual nucleus, describing it in terms of the configurations Be^9+d and Li^6+He^5, respectively.

In order to evaluate the matrix elements (2) and (3) it is convenient to express the angular parts of the initial-state wave function (4) in terms of those of the final state. It is rather arbitrarily assumed at this point that the angular momenta combine according to the J-J coupling scheme, but L-S- and channel-spin coupling could also be considered. The procedure followed in expanding ψ_i closely parallels that given by Edwards[5] for a (d,n) reaction. The result for the projectile stripping mode is

$$\psi_i=e^{i\mathbf{k}_{\alpha(1)}\cdot\mathbf{R}_{\alpha(1)}}e^{i\mathbf{k}_R\cdot\mathbf{R}_R}G_P(K_1)g_{l(d)}(k_1r_d)\times\psi_T(\rho)\Gamma_P\xi_R(J_RM_R)S_\alpha(j_\alpha\mu_\alpha), \tag{6}$$

where, symbolically,

$$\Gamma_P=\sum_{J_RM_RJ_dM_d\mu_d}\langle j_d\mu_d j_\alpha\mu_\alpha|J_PM_P\rangle\times\langle j_d\mu_d l(d)m(d)|J_dM_d\rangle\langle J_TM_TJ_dM_d|J_RM_R\rangle.$$

The notation $\langle j_1m_1j_2m_2|JM\rangle$ is used for the Clebsch-Gordan coefficient corresponding to the vector addition $\mathbf{j}_1+\mathbf{j}_2=\mathbf{J}$, $m_1+m_2=M$. Other quantities in (6) are the usual form factor

$$G_P(K_1)=\int e^{i\mathbf{K}_1\cdot\mathbf{r}}\psi_P(r)d\mathbf{r},$$

and

$$g_{l(d)}=\int Y_{l(d),0}^\dagger e^{i\mathbf{k}_1\cdot\mathbf{r}_d}\sin\theta d\theta d\phi$$

$$=i^{l(d)}\{4\pi[2l(d)+1]\}^{\frac{1}{2}}j_{l(d)}(k_1r_d),$$

[4] T. Fulton and G. E. Owen, Phys. Rev. **108**, 789 (1957).

[5] S. Edwards, Jr., Phys. Rev. **113**, 1277 (1959).

in which $Y_{l(d),0}(\theta,\phi)$ and $j_{l(d)}(k_1r_d)$ are respectively, a spherical harmonic, and a spherical Bessel function. $\hbar l(d)$ is the capture orbital angular momentum of the deuteron relative to the center-of-mass of the residual nucleus. The wave vectors which appear as a result of coordinate transformations are defined by

$$\mathbf{K}_1=\mathbf{k}_\alpha-(M_\alpha/M_P)\mathbf{k}_P,$$
$$\mathbf{k}_1=\mathbf{k}_P-(M_T/M_R)\mathbf{k}_\alpha,$$

in which M_q is the mass of particle q. The position of the deuteron relative to the center-of-mass of the target nucleus is given by the vector $\mathbf{r}_d=\mathbf{R}_d-\mathbf{R}_T=\mathbf{R}_{\alpha(1)}+\mathbf{r}-\mathbf{R}_T$. In establishing (6), $\mathbf{k}_1$ is chosen to define the axis of quantization.

In evaluating the matrix element (2), the method of the "kinematical form factor"[4,6] has been used, with the result

$$\mathcal{T}_0^{(P)}=C_P\int\psi_f^\dagger(x)\psi_i(x)dx, \tag{7}$$

where

$$C_P=T_{\text{c.m.}}+\epsilon_{d\alpha(1)}-\frac{\hbar^2}{2}\left[\frac{k_P^2}{\mu_{dT}}+\frac{k_\alpha^2}{\mu_{d\alpha}}-\frac{2\mathbf{k}_P\cdot\mathbf{k}_\alpha}{M_d}\right],$$

is the kinematical form factor. $T_{\text{c.m.}}$ is the total kinetic energy of the system in the initial state, $\epsilon_{d\alpha(1)}$ is the binding energy of the deuteron in Li^6, and μ_{pq} is the reduced mass of particles p and q.

The usual stripping-theory cutoff or interaction radius R_1 was introduced by means of a δ-function interaction multiplying the potential $V_{d\alpha(1)}$. Thus, putting (5) and (6) into (2) and using (7) gives

$$\begin{aligned}\mathcal{T}_0^{(P)}&=\int\psi_R^{(P)\dagger}(r_d)V_{d\alpha(1)}G_P(K_1)g_{l(d)}(k_1r_d)\psi_T(\rho)\Gamma_P d\tau\\&\approx\Gamma_PC_PG_P(K_1)\\&\quad\times\int\psi_R^{(P)\dagger}(r_d)\delta(r_d-R_1)g_{l(d)}(k_1r_d)\psi_T(\rho)d\tau\\&\approx(i)^{l(d)}\Gamma_PC_PG_P(K_1)j_{l(d)}(k_1R_1)\Lambda_P.\end{aligned}$$

Λ_P is an overlap integral defined by

$$\Lambda_P=\{4\pi[2l(d)+1]\}^{\frac{1}{2}}\int\psi_R^{(P)\dagger}(r_d)\psi_T(\rho)d\tau,$$

the integration being over all space.

The target stripping mode is treated in a way exactly analogous to that given for the projectile-stripping mode. $\mathbf{k}_1$ is again used as the axis of quantization. No transformation of axes of angular momentum functions is necessary because comparison of the present stripping analysis with experiment shows that the capture angular momentum $\hbar l(H)$ of the He^5 cluster relative to the center-of-mass of the residual nucleus cannot reasonably take any value other than zero. The only remaining preferred axis is therefore $\mathbf{k}_1$. A more complete discussion of the various axes of quantization can be found in reference 5. Again using a δ-function approximation to introduce an interaction radius R_2, the target-stripping amplitude is found to be

$$\mathcal{T}_0^{(T)}=i^{l(H)}\Gamma_TC_TG_T(K_2)j_{l(H)}(k_2R_2)\Lambda_T,$$

with

$$\Lambda_T=2\{4\pi[2l(H)+1]\}^{\frac{1}{2}}\int\psi_R^{(T)\dagger}(\rho_H)\psi_P(r)d\tau,$$

$$C_T=T_{\text{c.m.}}+\epsilon_{H\alpha(2)}-\frac{\hbar^2}{2}\left[\frac{k_P^2}{\mu_{HP}}+\frac{k_\alpha^2}{\mu_{H\alpha}}+\frac{2\mathbf{k}_P\cdot\mathbf{k}_\alpha}{M_H}\right],$$

$$G_T(K_2)=\int\exp(-i\mathbf{K}_2\cdot\boldsymbol{\varrho})\psi_T(\rho)d\boldsymbol{\varrho},$$

$$\begin{aligned}\Gamma_T=\sum_{J_RM_RJ_HM_H\mu_H}&\langle j_\alpha\mu_\alpha j_H\mu_H|J_TM_T\rangle\\&\times\langle j_H\mu_Hl(H)m(H)|J_HM_H\rangle\\&\times\langle J_HM_HJ_PM_P|J_RM_R\rangle,\end{aligned}$$

$$\mathbf{K}_2=\mathbf{k}_\alpha-(M_\alpha/M_T)\mathbf{k}_T,$$
$$\mathbf{k}_2=\mathbf{k}_T-(M_P/M_R)\mathbf{k}_\alpha,$$
$$\boldsymbol{\varrho}_H=\mathbf{R}_{\alpha(2)}-\mathbf{R}_P-\boldsymbol{\varrho}=\mathbf{R}_H-\mathbf{R}_P.$$

$\epsilon_{H\alpha(2)}$ is the binding energy of an α particle in Be^9 and R_2 is again a cutoff radius. $\mathbf{j}_H$ is the intrinsic (spin) angular momentum of the He^5 cluster, and $\boldsymbol{\varrho}_H$ is the position of that cluster relative to the center-of-mass of the Li^6 projectile.

Conservation of parity and angular momentum requires that $l(d)$ and $l(H)$ be even integers not greater than 4 if the residual nucleus B^{11} is left in its ground state ($J^\pi=3/2^-$) and not greater than 2 if the B^{11} is left in its first excited state ($1/2^-$). It has already been pointed out that $l(H)=0$ is required by the experimental results.

The differential cross section $d\sigma/d\Omega$, which is proportional to the squared modulus of the scattering amplitude $\mathcal{T}_0$, can now be written as

$$\begin{aligned}d\sigma/d\Omega\propto&|C_PG_P(K_1)j_{l(d)}(k_1R_1)|^2\\&+|\Lambda_T\Lambda_P^{-1}C_TG_T(K_2)j_{l(H)}(k_2R_2)|^2\\&+2(-1)^{\frac{1}{2}l(d)}\delta_{n,l(d)}\Gamma_P\Gamma_T\Lambda_T\Lambda_P^{-1}C_PG_PC_TG_Tj_{l(d)}j_{l(H)},\end{aligned} \tag{8}$$

where n is any non-negative even integer.

The transforms G_P and G_T were evaluated using the zero-range approximation wave functions,

$$\psi_P(r)=(\alpha/2\pi)^{\frac{1}{2}}r^{-1}e^{-\alpha r}\quad\text{and}\quad\psi_T(\rho)=(\beta/2\pi)^{\frac{1}{2}}\rho^{-1}e^{-\beta\rho},$$

with

$$(\hbar\alpha)^2=2\mu_{d\alpha}\epsilon_{d\alpha(1)}\quad\text{and}\quad(\hbar\beta)^2=2\mu_{H\alpha}\epsilon_{H\alpha(2)}.$$

The products C_PG_P and C_TG_T turn out to be nearly

[6] G. E. Owen, L. Madansky, and S. Edwards, Jr., Phys. Rev. 113, 1575 (1959).

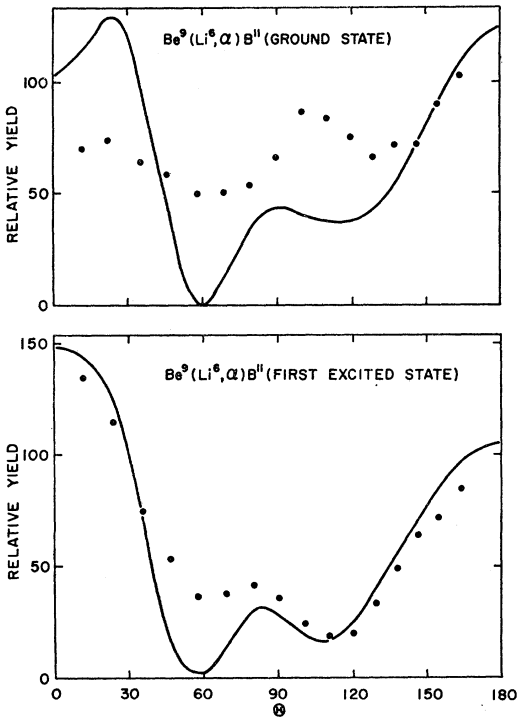

FIG. 1. Angular distributions at a laboratory bombarding energy of 3.25 Mev for the $Be^9(Li^6,\alpha)B^{11}$ reactions, leaving B^{11} in its ground state (upper plate) and in its first-excited state (lower plate). The solid lines are the theoretical results from this paper and the solid circles are the experimental points from reference 1. Θ is the center-of-mass scattering angle.

independent of the scattering angle for the experimental conditions of reference 1, so only their mean values were used in computing theoretical angular distributions.

Upon forming the product of the six Clebsch-Gordan coefficients and then doing the requisite sum, it is readily found by the usual methods of Racah algebra that $\Gamma_P\Gamma_T=0$ if $l(d)\neq 0$ and that if $l(d)=0$ then $\Gamma_P\Gamma_T=1$ or $\frac{1}{2}$ if the B^{11} is left in its ground or first excited state, respectively.

IV. RESULTS AND CONCLUSION

The differential cross section in the form of Eq. (8), with arbitrary normalizing factors, has been evaluated for the $Be^9(Li^6,\alpha)B^{11}$ reaction leading to the ground and the first excited states of B^{11}. Because of the restricted range of bombarding energies for which complete experimental angular distributions were available and because of the limited success in fitting the ground-state experimental results, the evaluation of Eq. (8) has been carried out only for a laboratory bombarding energy of 3.25 Mev. The results are shown in Fig. 1. The parameters used to obtain the curves shown in the figure are given in Table I.

TABLE I. Parameters used in Eq. (8) to calculate the theoretical angular distributions shown in Fig. 1.

	R_1 (f)	R_2 (f)	$l(d)$	$l(H)$	$\Lambda_T\Lambda_P^{-1}$
B^{11} ground state	6.0	2.75	0	0	0.28
B^{11} first-excited state	6.0	2.75	2	0	0.23

The calculated angular distribution for the first excited state reaction is considered to be in satisfactory agreement with the experimental results. The partial failure of the theory in treating the ground-state reaction does not seem to be serious in view of the grossly simplified analysis used here. Improvement might be sought in a more realistic treatment, such as a distorted-wave Born-approximation calculation[7] or a multinucleon stripping calculation.[8] The present work however does seem to demonstrate that the reaction mechanism is primarily one of stripping.

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[7] See, for example, W. Tobocman, Phys. Rev. **115**, 98 (1959) and references therein.

[8] M. el Nadi and M. el Kishin, Proc. Phys. Soc. (London) **A73**, 705 (1959); I. Manning and A. H. Aitken, Naval Research Labs. Quart. Nuclear Sci. Tech. (to be published); see, M. el Nadi, Phys. Rev. **119**, 242 (1960), and **120**, 1360 (1960).