Double Pion Photoproduction in the 1.2-Bev Region

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Energy and angular distributions of π^-'s produced by the reaction $\gamma+p\rightarrow p+\pi^++\pi^-$, are calculated and the results are compared with the CalTech measurements at (1230 ± 50) Mev. The calculation starts from the static theory for the ($N\pi\gamma$) system with the source function $v(p)=\xi^2/(\xi^2+p^2)$, $\xi=4.7\ \mu$, which is of the form proposed by Fubini and Thirring in their analysis of the low-energy π-N phenomena, and thought of as a substitute for the form factor of the ($NN\pi$) vertex. The kinematical effects of the nucleon recoil, which is numerically very important, are also taken into account so as to cover, as much as possible, the drawback of the static theory. The contributions from the nucleon core current, (γ-3π) vertex, and the final-state π-π interaction are neglected. The physical reasoning of these approximations is briefly discussed.

The results of the calculation agree satisfactorily with the CalTech data and show that, in this energy region, processes other than Drell's peripheral one are still important, as well as the latter.

RECENTLY, detailed measurements on the energy and angular distributions of the π^- produced by the reaction

$$\gamma+p\rightarrow p+\pi^++\pi^-, \tag{1}$$

have been reported by the CalTech synchrotron group.[1] According to this report, the prediction of the Drell formula[2] (one pion-exchange contribution via the pion-current interaction) agrees qualitatively with the experimental results, but is too small (by about a factor of 2) in absolute cross sections. However, this quantitative disagreement can easily be understood: The Drell formula has originally been proposed for the reaction induced by an extremely energetic incident beam. Forward production of *very high energy* pions by a *highly energetic* beam would be dominated by Drell's peripheral interaction. However, at the energy region measured by the CalTech group (mean energies of the incident γ and the produced π^- being, respectively ~650 and ~480 Mev in the c.m. system), the contributions from the other interactions would not necessarily be negligible.

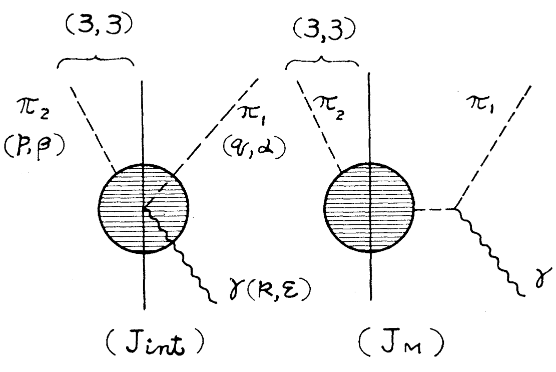

FIG. 1. Feynman diagrams for $\gamma+N\rightarrow N+\pi_1+\pi_2$.

From the viewpoint of the static theory,[3] the electromagnetic interaction Hamiltonian of the pion-nucleon system is composed of the interaction current part J_{int}, the pion current part J_M, and the nucleon core part J_N. Furthermore the γ-3π vertex part $J_{3\pi}$ may also exist.[4] Among these, we retain only J_{int} and J_M as the first approximation,[5] and taking into account only the Feynman diagrams shown in Fig. 1, calculate the differential cross sections of reaction (1) by the static theory technique. We take

$$J\approx J_{\text{int}}+J_M, \tag{2}$$

$$J_{\text{int}}=ef\int d\mathbf{q}(4\omega_q k)^{-\frac{1}{2}}[\tau_2(a_q{}^{(1)}+a_{-q}{}^{(1)\dagger}) -\tau_1(a_q{}^{(2)}+a_{-q}{}^{(2)\dagger})](\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon})v(\mathbf{k}+\mathbf{q}), \tag{3}$$

and

$$J_M=-ie\sum_{\alpha,\beta}\epsilon_{3\alpha\beta}\int d\mathbf{q}(8\omega_q\omega_{k+q}k)^{-\frac{1}{2}}(\mathbf{q}\cdot\boldsymbol{\varepsilon}) \times(a_q{}^{(\alpha)}+a_{-q}{}^{(\alpha)\dagger})(a_{-k-q}{}^{(\beta)}+a_{k+q}{}^{(\beta)\dagger}), \tag{4}$$

where $\mathbf{k}$ and $\boldsymbol{\varepsilon}$ are the momentum and the polarization vector of the photon, and $a_q{}^{(\alpha)}$ and $a_q{}^{(\alpha)\dagger}$ are the

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[2] S. D. Drell, Phys. Rev. Letters **5**, 278 and 342 (1960).

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[5] Cutkosky and Zachariasen have already investigated double pion photoproduction in the low-energy region, using only these two classes of interaction: R. E. Cutkosky and F. Zachariasen, Phys. Rev. **103**, 1108 (1956). The reasoning behind their approximation is that, in the low-energy region, the most dominant reaction would be that in which one of the pions is produced in the S state and the other in the P state. [Within the framework of the static theory, the nucleon core current J_N can be responsible only for the magnetic dipole absorption, and accordingly cannot induce the (S,P) production process. Moreover, in the low-energy region, the effect of the (γ-3π) vertex would be negligible.] Thus, as for the contribution from the pion current J_M, they have taken into account only the S-wave part of the pion production. In the present work, we have also retained only J_{int} and J_M. However, corresponding to the difference of the energy regions in question, the physical basis of the approximation is quite different, and thus we have *not* restricted the angular momentum state of the pion produced by J_M.

destruction and creation operators of the pion, $\mathbf{q}$ and α being its momentum and charge index. v is a Fourier transform of the source function (the cutoff function). Questions as to the applicability of the "static" theory to such an energy region ($\sim$1.2 Bev lab) might be raised. Therefore, in the actual calculation, we take into account the kinematical effect of the nucleon recoil as much as possible. We shall discuss this point further at the end of this note.

The reasons for our first approximation are as follows:

(A) The isovector part of J_N is proportional to $\tau_3(\boldsymbol{\sigma}\cdot[\mathbf{k}\times\boldsymbol{\varepsilon}])$. Thus, the contribution of this part to the matrix element for reaction (1) is proportional to the static-theory matrix element for the reaction

$$\pi+N\rightarrow N+\pi_1+\pi_2, \tag{5}$$

and the proportionality constant is of the order of e. However according to Rodberg,[6] the latter is very small compared to the experimental cross section for reaction (5). This suggests that also the contribution of the isovector part of J_N to reaction (1) would be very small. The contribution of the isoscalar part of J_N is naturally expected to be smaller than that of the isovector part.

(B) The quantitative success of the static theory for the low-energy photoproduction of pions suggests that the coupling parameter λ of the (γ-3π) vertex would be quite small.[7] In fact, the contribution of $J_{3\pi}$ to the reaction (1) has also turned out to be very small,[8] unless the enhancement of λ (for the particular energies of the pions) by the π-π resonance[9] be taken into account. We think, that at least as concerns the energy and angular distributions of a *particular one* of the produced pions, the effect of the π-π resonance would not be as important. On similar reasoning, we discard the effect of the final-state π-π interactions.

(C) For an incident photon energy of the order of 1 Bev, the energies of the produced pions relative to the nucleon cannot be as large. Therefore, as for the rescattering of the second pion π_2 (Fig. 1), it is sufficient to retain only the (3,3) effects. Moreover the rescattering of the first pion π_1 can be ignored. The "π_1" produced by J_{int} is an S-wave pion, and that produced by J_M is "distant" from the nucleon core.

According to the static theory, the desired matrix element is

$$\begin{aligned}M(q\alpha,p\beta;k\epsilon)=4\pi eI_{\alpha\beta}\Delta(p)(8\omega_p\omega_qk)^{-\frac{1}{2}}\{&[3(\mathbf{p}\cdot\boldsymbol{\varepsilon})\\ -(\boldsymbol{\sigma}\cdot\mathbf{p})(\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon})]&+\beta_q(\mathbf{q}\cdot\boldsymbol{\varepsilon})(kq)^{-1}(1-\beta_q\cos\theta_{qk})^{-1}\\ &\times[3(\mathbf{p}\cdot\mathbf{k}-\mathbf{q})-(\boldsymbol{\sigma}\cdot\mathbf{p})(\boldsymbol{\sigma}\cdot\mathbf{k}-\mathbf{q})]\},\end{aligned} \tag{6}$$

with

$$\Delta(p)=p^{-3}[v(\mathbf{k}-\mathbf{q})/v(p)]\sin\delta_{33}(p)\exp[i\delta_{33}(p)].$$

$(\mathbf{q},\alpha)$ and $(\mathbf{p},\beta)$ are the momenta and charge indices of π_1 and π_2, respectively, and $I_{\alpha\beta}$ is the isotopic spin factor ($I_{-+}=1, I_{+-}=-1/3$). β_q ($\equiv q/\omega_q$) is the velocity of π_q.

In order that such an expression of the (π_p+N) scattering part (the black-sphere parts in Fig. 1) in terms of the phase shift and the projection operator is permissible, Eq. (6) should be referred to the c.m. frame of the (π_p+N) system.[10] Hereafter, this Lorentz frame will be referred to as the primed system, and quantities in this frame will be distinguished by primes from quantities in the total c.m. frame. Among the various kinematical effects of the nucleon recoil, the above-mentioned distinction is numerically very important, owing to the sensitive momentum dependence of $\Delta(p)$. Moreover as another effect of the recoil, Eq. (6) should be multiplied by $(E_p'+\omega_p')/E_p'$, where E_p' and ω_p' are respectively the energies of the final nucleon and π_p in the primed system. Finally, the Lorentz transformation of the matrix element from the primed system to the total c.m. frame yields

$$\begin{aligned}M(q\alpha,p\beta;k\epsilon)=&\frac{4\pi e}{(8\omega_p\omega_qk)^{\frac{1}{2}}}\left(\frac{E_p'E_k'}{E_fE_k}\right)^{\frac{1}{2}}\frac{E_p'+\omega_p'}{E_p'}I_{\alpha\beta}\Delta(p')\\ &\times\left\{[3(\mathbf{p}'\cdot\boldsymbol{\varepsilon}')-(\boldsymbol{\sigma}\cdot\mathbf{p}')(\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}')]+\frac{1}{k'q'}\frac{\beta_q'(\mathbf{q}'\cdot\boldsymbol{\varepsilon}')}{1-\beta_q'\cos\theta_{q'k'}}\right.\\ &\left.\times[3(\mathbf{p}'\cdot\mathbf{k}'-\mathbf{q}')-(\boldsymbol{\sigma}\cdot\mathbf{p}')(\boldsymbol{\sigma}\cdot\mathbf{k}'-\mathbf{q}')]\right\},\end{aligned} \tag{7}$$

where E_f is the energy of the final nucleon in the total c.m. frame.

To obtain the cross sections, it is necessary to calculate $(1/4)\sum_\epsilon \operatorname{trace}|M_{-+}+M_{+-}|^2$, where $M_{-+}\equiv M(q-,p+;k\epsilon)$ and $M_{+-}\equiv M(p+,q-;k\epsilon)$. Now, $|M_{-+}|^2$, $|M_{+-}|^2$, and $\operatorname{Re}(M_{-+}{}^*M_{+-})$ contain, respectively, the factors $I_{-+}{}^2\sin^2\delta_{33}(p')$, $I_{+-}{}^2\sin^2\delta_{33}(q'')$, and $I_{-+}I_{+-}\sin\delta_{33}(p')\sin\delta_{33}(q'')\cos[\delta_{33}(p')-\delta_{33}(q'')]$, where q'' is the momentum of π^- in the c.m. frame of the (π^-+N) system. Under the CalTech experimental condition, p' is in the (3,3) resonance region, but q'' is almost outside of that region. Therefore, also taking account of the smallness of $I_{+-}{}^2$ and $I_{-+}I_{+-}$ compared to $I_{-+}{}^2$, we can neglect the contributions from M_{+-} altogether.

Taking into account the kinematical effect of the nucleon recoil also in the statistical factors, we obtain

[6] L. S. Rodberg, Phys. Rev. **106**, 1090 (1957); Phys. Rev. Letters **3**, 58 (1959).

[7] M. Kato, Progr. Theoret. Phys. (Kyoto) **25**, 493 (1961).

[8] T. Kotani (private communication). See also D. Boccaletti and C. Gualdi, Nuovo cimento **18**, 895 (1960).

[9] For example, W. R. Frazer and J. R. Fulco, Phys. Rev. Letters **2**, 365 (1959); Phys. Rev. **117**, 1609 (1960). See also reference 4.

[10] This fact and the need for the various recoil corrections which lead to the final expression Eq. (8) have already been pointed out by Cutkosky and Zachariasen, reference 5. The correction factors proposed by them are, however, slightly different from ours in that, in the former, the quantity $E_{p'}$ has been replaced by M. This is justified only for the relatively low-energy case.

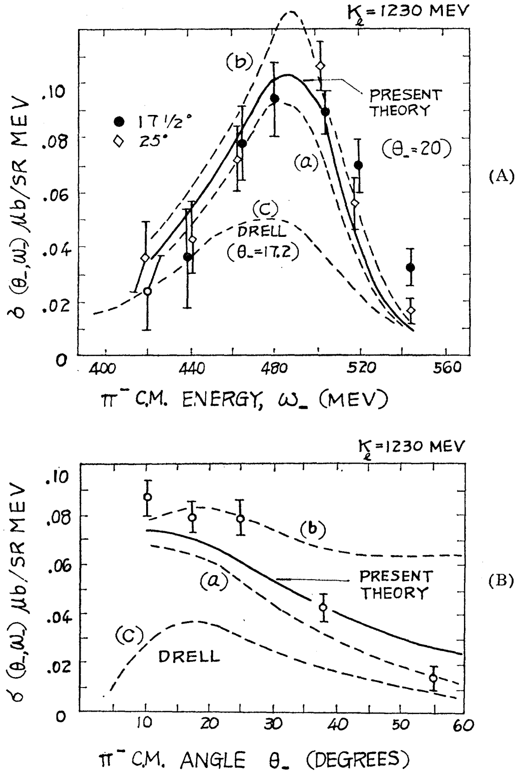

FIG. 2. (A) Energy and (B) angular distributions of π^-. The quantities are all referred to the total c.m. frame. The experimental data are taken from reference 1, and are the average cross sections for the incident photon energy range $k_{\text{lab}}=1230\pm50$ Mev. The solid curves are the results of the present calculation [Eqs. (8) and (9)] for $k_{\text{lab}}=1230$ Mev. For comparison, (a) the curves with $v(p)=\exp(-p^2/2\xi^2)$, $\xi=3\mu$, (b) those without cutoff [$v(p)\equiv1$], and (c) those quoted in reference 1 as the Drell curves, are also plotted. In Fig. 2(A), the theoretical curves [except (c)] are plotted for $\theta_-=20°$. In Fig. 2(B), our result is the average angular distribution of $\omega_-=460$-, 500-, and 520-Mev points, while the experimental points are the averages of $\omega_-=464$-, 505-, and 519-Mev points.

finally the following differential cross section:

$$\frac{d^2\sigma}{d\omega_-d\Omega_-}=\frac{e^2}{4\pi^2}\frac{E_p'+\omega_p'}{E_k+k}\frac{E_k'}{E_p'}\frac{qp'}{k}\Delta(p')^2 \times\left\{2-\frac{2q'}{k'}\frac{\beta_q'}{1-\beta_q'\cos\theta_{q'k'}}\sin^2\theta_{q'k'} +\left(\frac{\beta_q'}{1-\beta_q'\cos\theta_{q'k'}}\right)^2\frac{(\mathbf{k}'-\mathbf{q}')^2}{k'^2}\sin^2\theta_{q'k'}\right\}. \quad (8)$$

The last term in Eq. (8) corresponds to the Drell formula, except that in the latter, the angle-dependent factor $(\mathbf{k}'-\mathbf{q}')^2$ is replaced by a certain angle-independent factor. This angle-dependent factor favors the large-angle π^- production and thus the forward peak of the Drell formula diminishes relatively. However, the second term in Eq. (8) (the interference term between the contributions of J_{int} and J_M) restores the forward peak.

We choose the form of the cutoff function v in $\Delta(p')$ as

$$v(p)=\xi^2/(\xi^2+p^2),\quad \xi=4.7\,\mu. \quad (9)$$

This is the one proposed by Fubini and Thirring[11] from their analysis of the low-energy (π-N) phenomena. In Fig. 2, the numerical results calculated from Eqs. (8) and (9) are compared with the CalTech data. For comparison, the results of the other choices of the cutoff function and the curves quoted in reference 1 as the "Drell" results are also plotted. Although we have not considered the effects of excitation of the nucleon "levels" corresponding to the second and third π-N (and $\gamma\to\pi$) resonances by the incident γ, the results agree strikingly well with the experimental data. This is perhaps due to the fact that the photon energy in the CalTech measurements is considerably higher than those resonance energies. For the photon energy in or near those resonance regions, the above-mentioned effects would have to be considered.

In this paper, we have adopted the static theory technique. As far as concerns the treatment of the final (πN) system, this would not introduce a very serious error, since the energies of the final particles are not very large in our problem. Moreover, the kinematical effects of the nucleon recoil have been taken into account as much as possible, and thus the main feature of the static theory retained in our formula is the method of replacing the off-energy-shell π-N scattering amplitude by the physical one. The larger the π^- production angle, the more our π-N "scattering" deviates from the energy shell and accordingly, the method of the above-mentioned replacement becomes less reliable. This would be the main reason for the disagreement of our results with the large-angle data. To some extent this drawback would be covered by introducing the proper cutoff function, which is thought of as a substitute for the *dynamical* effect of the nucleon recoil present in the "true" π-N vertex. For further improvement of our results, it seems desirable that the cutoff function be more rapidly damped than Eq. (9). Similarly, the electromagnetic form factor of the pion may affect the J_M contribution, and thus play a certain role in improving the results. However, since we do not know at present the "true" π-N vertex and the pion form factor, the degree of the agreement of our results with the experimental data seems to be very satisfactory.

ACKNOWLEDGMENT

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[11] S. Fubini and W. E. Thirring, Phys. Rev. **105**, 1382 (1957).