

currently under discussion would be classed as dynamical. If, however, one concentrates on the $\pi\Delta$ system then the Dalitz-Tuan model makes the V^* an independent particle, whereas the model based on global symmetry rests on an attractive force between the π and the Δ .¹⁶

The chief reason for writing this paper is our impression, perhaps erroneous, that many workers in the strong-interaction field do not realize that in the S -matrix framework any distinction can be made between different types of resonance elastic scattering. The distinction emphasized here, even if the words used in the description do not appear appropriate to all readers, is subject to experimental test. A closing observation that

we find difficult to resist is that to date most such tests point *away* from the notion of independent-particle scattering. It is plausible, therefore, that *none* of the strongly interacting particles are *completely* independent but that each is a dynamical consequence of interactions between others. In such a situation, when the entire S matrix is considered, there would be no arbitrary dimensionless (coupling) constants and presumably only one dimensional constant to establish the scale of masses. However, when one concentrates on a small part of the over-all problem, asking about the elastic interaction of a particular pair of particles, the discussion presented here should still be meaningful.

Radiative Decay of the Neutral K Meson: $K^0 \rightarrow \gamma + \gamma^\dagger$

J. DREITLEIN* AND H. PRIMAKOFF

Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania

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The consequences of the particle mixture theory of the neutral K meson are investigated for the rare radiative decay mode: $K^0 \rightarrow \gamma + \gamma$. The two photon decay rates of the K_1^0 , K_2^0 mesons are estimated as $\approx 1.3 \times 10^6 \text{ sec}^{-1}$ (Cabibbo and Ferrari) and $\approx \{1.6 \times 10^6 / (g_{\Sigma\pi K^2}/4\pi)\} \text{ sec}^{-1} \approx 10^6 \text{ sec}^{-1}$. It is shown that a time-dependent net circular polarization of each of the two photons results from the interference between the K_1^0 and K_2^0 channels feeding the 2γ state. The correlated linear polarizations of the two photons also exhibit a similar time-dependent behavior. The possibility of experimental detection of the effects discussed, from which the sign as well as the magnitude of the K_1^0 , K_2^0 mass difference can be determined, is very briefly explored.

I

SOME unusual properties of the neutral K meson complex were first predicted by Gell-Mann and Pais¹: the double lifetime behavior, and by Pais and Piccioni²: the regeneration phenomenon. Such properties have since been observed experimentally,^{3,4} and theoretically have been shown to hold independently of the failure of parity conservation and of charge conjugation conservation in decays induced by the weak interactions.^{5,6}

One further phenomenon peculiar to the neutral

K mesons is the predicted time dependence of the rate of appearance of the neutral K -derived leptons,⁷ an oscillatory effect occurring with a frequency given by the mass difference $\Delta m \equiv m_1 - m_2$ between the K_1^0 and K_2^0 . Analogous oscillatory effects associated with the regeneration phenomenon and the double lifetime behavior have been used⁴ to fix the order of magnitude of $|\Delta m|$ as $|\Delta m| \approx 1/\tau(K_1^0)$.

In this note we point out another curious neutral K phenomenon which is encountered in the rare radiative decay mode: $K^0 \rightarrow \gamma + \gamma$.

II

With respect to the strong interactions, the neutral K mesons are best described by one non-Hermitian field—that of the K^0 and \bar{K}^0 mesons—while, with respect to the weak interactions, two Hermitian fields—those of the K_1^0 and K_2^0 mesons—best characterize the neutral K meson decays. Immediately after a K^0 is created the corresponding state can be viewed as a coherent mixture of a K_1^0 and a K_2^0 :

⁷ S. B. Treiman and R. Sachs, Phys. Rev. **103**, 1545 (1956); see also S. B. Treiman and S. Weinberg, *ibid.* **116**, 239 (1959) for a discussion of analogous effects in the rate of appearance of neutral K derived 3π states.

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¹ M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).

² A. Pais and O. Piccioni, Phys. Rev. **100**, 1487 (1955); see also M. L. Good, *ibid.* **106**, 591 (1957).

³ K. Lande, E. Booth, J. Impeduglia, L. Lederman, and W. Chinowsky, Phys. Rev. **103**, 1901 (1956).

⁴ F. Muller, R. W. Birge, W. B. Fowler, R. H. Good, W. Hirsch, R. P. Matsen, L. Oswald, W. M. Powell, H. White, and O. Piccioni, Phys. Rev. Letters **4**, 418 (1960) and *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960); R. W. Birge, R. P. Ely, W. M. Powell, H. Huzita, W. F. Fry, J. A. Gaides, V. Natali, R. B. Willman, and U. Camerini, *ibid.*

⁵ R. Gatto, Phys. Rev. **106**, 168 (1957).

⁶ T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. **106**, 340 (1957).

$$\begin{aligned}
|K_1^0\rangle &= \frac{|K^0\rangle + PC|K^0\rangle}{\sqrt{2}}, \\
|K_2^0\rangle &= \frac{|K^0\rangle - PC|K^0\rangle}{\sqrt{2}i}, \\
|K^0\rangle &= \frac{|K_1^0\rangle + i|K_2^0\rangle}{\sqrt{2}}, \\
PC|K^0\rangle &= P|\bar{K}^0\rangle = \frac{|K_1^0\rangle - i|K_2^0\rangle}{\sqrt{2}},
\end{aligned}
\tag{1}$$

where PC is the product of the parity and the charge conjugation operators. With the assumption of PC invariance (T invariance by the TPC theorem⁸) the dominant channels for the K_1^0 and the K_2^0 decays, and their mean lives,⁹ are:

$$\begin{aligned}
K_1^0 &\rightarrow \left\{ \begin{array}{l} \pi^+ + \pi^- \\ \pi^0 + \pi^0 \end{array} \right\}; \quad (\lambda_1)^{-1} \equiv \tau(K_1^0) \\
&= (1.00 \pm 0.04) \times 10^{-10} \text{ sec},
\end{aligned}
\tag{2a}$$

$$\begin{aligned}
K_2^0 &\rightarrow \left\{ \begin{array}{l} \pi^+ + \pi^- + \pi^0 \\ \pi^0 + \pi^0 + \pi^0 \\ \pi^\pm + \mu^\mp + \nu \\ \pi^\pm + e^\mp + \nu \end{array} \right\}; \quad (\lambda_2)^{-1} \equiv \tau(K_2^0) \\
&= (6.1 \pm 1.4) \times 10^{-8} \text{ sec}.
\end{aligned}
\tag{2b}$$

Both the K_1^0 and the K_2^0 can decay into a photon pair by "internal annihilation" of the π mesons¹⁰:

$$K_1^0 \rightarrow \left\{ \begin{array}{l} \pi^+ + \pi^- \\ \pi^0 + \pi^0 \end{array} \right\} \rightarrow \gamma + \gamma,
\tag{3a}$$

$$K_2^0 \rightarrow \left\{ \begin{array}{l} \pi^+ + \pi^- + \pi^0 \\ \pi^0 + \pi^0 + \pi^0 \end{array} \right\} \rightarrow \gamma + \gamma,
\tag{3b}$$

as illustrated in Fig. 1. Since a weak interaction is necessary as one step leading into the 2γ channel, we expect:

$$\begin{aligned}
\frac{[\text{Rate}(K_1^0 \rightarrow \gamma + \gamma)]/\lambda_1}{[\text{Rate}(K_2^0 \rightarrow \gamma + \gamma)]/\lambda_2} &\approx \frac{(1/137)^2}{(1/137)^2} \\
&\times \frac{(\text{two-body phase space volume})}{(\text{three-body phase space volume})} \\
&\approx (1/137)^2 \times 300,
\end{aligned}$$

⁸ See, e.g., G. Luders, Ann. Phys. 2, 1 (1957).

⁹ See, e.g., W. H. Barkas and A. H. Rosenfeld, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960).

¹⁰ Since a virtual weak interaction is involved in the 2γ decay and since PC is considered a good quantum number for all decays (2π , 3π , $\pi\mu\nu$ or $\pi e\nu$, 2γ), it is appropriate to discuss the 2γ decays in terms of the K_1^0 , K_2^0 fields.

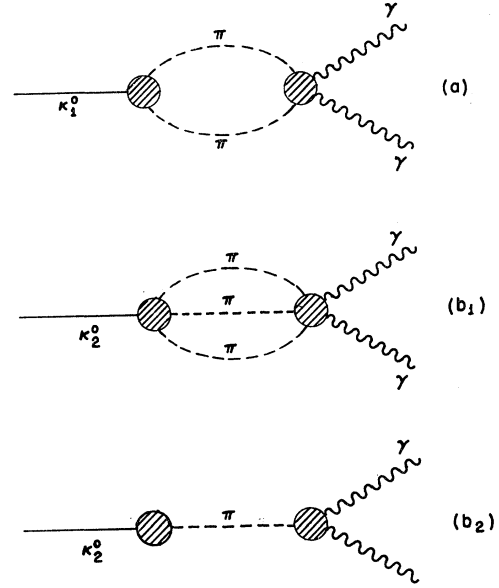


FIG. 1. Some typical processes leading to the two-photon decay of the neutral K .

so that

$$\frac{[\text{Rate}(K_2^0 \rightarrow \gamma + \gamma)]}{[\text{Rate}(K_1^0 \rightarrow \gamma + \gamma)]} \approx 300\lambda_2/\lambda_1 = 0.5.$$

Actually, an estimate by Cabibbo and Ferrari¹¹ yields:

$$\text{Rate}(K_1^0 \rightarrow \gamma + \gamma) \cong \left[\frac{1}{4}(1/137)^2\right]\lambda_1 = 1.3 \times 10^5 \text{ sec}^{-1},$$

while an estimate that we have made in the Appendix gives

$$\begin{aligned}
\text{Rate}(K_2^0 \rightarrow \gamma + \gamma) &\approx \{(1.6 \times 10^5)/(g_{\Sigma n K^2}/4\pi)\} \text{ sec}^{-1} \\
&\approx 10^5 \text{ sec}^{-1} = (6 \times 10^{-3})\lambda_2,
\end{aligned}$$

so that

$$\begin{aligned}
\frac{[\text{Rate}(K_2^0 \rightarrow \gamma + \gamma)]}{[\text{Rate}(K_1^0 \rightarrow \gamma + \gamma)]} \\
\approx (5 \times 10^2)\lambda_2/\lambda_1 = 0.8.
\end{aligned}$$

It is therefore clear that the 2γ decay rates of K_1^0 and K_2^0 should be the same to within an order of magnitude.

The major interest in the radiative decay modes of K_1^0 , K_2^0 lies, however, not in the exact numerical values of the corresponding 2γ decay rates but rather in the unusual time-dependent interference phenomena in the 2γ channel. To discuss these interference phenomena we note that PC invariance necessitates that the resultant two-photon state is characterized by $PC = +1$ if it originates from a K_1^0 decay and by $PC = -1$ if it originates from a K_2^0 decay. The most general Lorentz-invariant and gauge-invariant transition matrix elements for the 2γ decay of K_1^0 and K_2^0

¹¹ N. Cabibbo and E. Ferrari, Nuovo cimento 18, 928 (1960).

are then

$$\begin{aligned}
 T_{1;\rho} &\equiv \langle \gamma\gamma; \rho | T_1 | K_1^0 \rangle = \left\langle \gamma\gamma; \rho \left| g_1 e^{i\beta_1} \int \{ [\mathbf{E}(x)]^2 - [\mathbf{H}(x)]^2 \} \varphi_1(x) d^3x \right| K_1^0 \right\rangle, \\
 T_{2;\rho} &\equiv \langle \gamma\gamma; \rho | T_2 | K_2^0 \rangle = \left\langle \gamma\gamma; \rho \left| g_2 e^{i\beta_2} \int 2\mathbf{E}(x) \cdot \mathbf{H}(x) \varphi_2(x) d^3x \right| K_2^0 \right\rangle, \\
 (T_{2;\rho} / |T_{2;\rho}|) &= (T_{1;\rho} / |T_{1;\rho}|) e^{i\delta_\rho}
 \end{aligned} \tag{4}$$

where $|\gamma\gamma; \rho\rangle$ is a two-photon state characterized by the polarization ρ ; $\varphi_1 = \varphi_1^\dagger = +(PC)\varphi_1(PC)^{-1}$ and $\varphi_2 = \varphi_2^\dagger = -(PC)\varphi_2(PC)^{-1}$ are Hermitian field amplitudes associated with K_1^0 , K_2^0 ; g_1 and g_2 are taken as real and positive; and $\beta_2 - \beta_1 \equiv \beta$, always simply related to δ_ρ —see Eqs. (11), (16) below—allows properly for the phase difference between the matrix elements $T_{2;\rho}$ and $T_{1;\rho}$.¹² We note that:

$$\begin{aligned}
 (PC)(\mathbf{E}^2 - \mathbf{H}^2)(PC)^{-1} &= +(\mathbf{E}^2 - \mathbf{H}^2), \\
 (PC)(\mathbf{E} \cdot \mathbf{H})(PC)^{-1} &= -\mathbf{E} \cdot \mathbf{H},
 \end{aligned} \tag{5}$$

and so the two photons from K_1^0 emerge in a $J=0$, $L=0$, $S=0$ state while the two photons from K_2^0 are created in a $J=0$, $L=1$, $S=1$ state.

While the linear polarization properties of the two emitted photons are obvious from inspection of the expressions $\mathbf{E}^2 - \mathbf{H}^2$ and $\mathbf{E} \cdot \mathbf{H}$, the circular polarization properties are more transparent if one uses the identities

$$\begin{aligned}
 \mathbf{E}^2 - \mathbf{H}^2 &= \frac{1}{2}[(\mathbf{E} + i\mathbf{H})^2 + (\mathbf{E} - i\mathbf{H})^2], \\
 2\mathbf{E} \cdot \mathbf{H} &= [(\mathbf{E} + i\mathbf{H})^2 - (\mathbf{E} - i\mathbf{H})^2]/2i,
 \end{aligned} \tag{6}$$

since the field amplitudes $(\mathbf{E} + i\mathbf{H})$, $(\mathbf{E} - i\mathbf{H}) = (\mathbf{E} + i\mathbf{H})^\dagger$ create left, right circularly polarized photons, respectively. It is clear from Eqs. (4) and (6) that the numbers of left and of right circularly polarized photons generated from *either* a pure K_1^0 beam *or* a pure K_2^0 beam are equal. Also, by Eq. (4), the planes of linear polarization of the two photons emitted in an individual K_1^0 , K_2^0 decay tend to be parallel, perpendicular. However, a pure K_1^0 beam or a pure K_2^0 beam is never directly created in a strong interaction. Instead, a time-dependent coherent mixture of a K_1^0 and a K_2^0 results as an aftermath of the birth of a K^0 [see Eq. (1)], and so an investigation of this mixing effect on the 2γ decay rate and on the photon polarization is in order.

III

To discuss the time dependence of the various 2γ decay phenomena associated with an initially produced

K^0 beam, we consider the time-dependent state vector $|\Psi(t)\rangle$,⁷

$$\begin{aligned}
 |\Psi(t)\rangle &= (1/\sqrt{2})\{e^{-\lambda_1 t/2}|K_1^0\rangle + e^{-\lambda_2 t/2}e^{i(\Delta m)t}i|K_2^0\rangle\}e^{-im_1 t} \\
 &\equiv |\Phi(t)\rangle e^{-im_1 t}, \tag{7a}
 \end{aligned}$$

for which, using also Eq. (1),

$$\begin{aligned}
 |\Psi(0)\rangle &= |\Phi(0)\rangle = |K^0\rangle; \\
 |\langle K^0 | \Psi(t) \rangle|^2 &= \frac{1}{4}|e^{-\lambda_1 t/2} + e^{-\lambda_2 t/2}e^{i(\Delta m)t}|^2; \\
 |\langle \bar{K}^0 | \Psi(t) \rangle|^2 &= \frac{1}{4}|e^{-\lambda_1 t/2} - e^{-\lambda_2 t/2}e^{i(\Delta m)t}|^2.
 \end{aligned} \tag{7b}$$

The transition matrix element at time t which leads the neutral K into the 2γ channel can now be approximately calculated by treating the relatively slowly varying state $\Phi(t)$ as time independent. Using Eqs. (4) and (7a), we have for this matrix element:

$$\begin{aligned}
 \langle \gamma\gamma; \rho | T | \Phi(t) \rangle \\
 = (1/\sqrt{2})(T_{1;\rho}e^{-\lambda_1 t/2} + iT_{2;\rho}e^{-\lambda_2 t/2}e^{i(\Delta m)t}), \tag{8}
 \end{aligned}$$

so that the corresponding rate of appearance of neutral K derived photons at proper time t becomes:

$$\begin{aligned}
 \Gamma_\rho(t) &= 2\pi \left[\frac{(m_1/2)^2 (4\pi/2)}{2(2\pi)^3} \right] |\langle \gamma\gamma; \rho | T | \Phi(t) \rangle|^2 \\
 &= (m_1^2/32\pi) \{ |T_{1;\rho}|^2 e^{-\lambda_1 t} + |T_{2;\rho}|^2 e^{-\lambda_2 t} \\
 &\quad - 2|T_{1;\rho}||T_{2;\rho}|e^{-(\lambda_1 + \lambda_2)t/2} \sin[(\Delta m)t + \delta_\rho] \}. \tag{9}
 \end{aligned}$$

The total probability of the transition: neutral $K \rightarrow \gamma + \gamma; \rho$, i.e., the $2\gamma; \rho$ branching ratio of the neutral K , is therefore

$$\begin{aligned}
 P_\rho &\equiv \int_0^\infty \Gamma_\rho(t) dt = \Gamma_{1;\rho}/\lambda_1 + \Gamma_{2;\rho}/\lambda_2 \\
 &\quad - 2(\Gamma_{1;\rho}\Gamma_{2;\rho})^{1/2} \frac{\left[(\Delta m) \cos \delta_\rho + \left(\frac{\lambda_1 + \lambda_2}{2} \right) \sin \delta_\rho \right]}{(\Delta m)^2 + [(\lambda_1 + \lambda_2)/2]^2}, \tag{10a}
 \end{aligned}$$

where

$$\begin{aligned}
 \sum_\rho \Gamma_{i;\rho} &\equiv \sum_\rho \frac{m_1^2}{32\pi} |T_{i;\rho}|^2 = \text{Rate}(K_i^0 \rightarrow \gamma + \gamma); \\
 i &= 1, 2. \tag{10b}
 \end{aligned}$$

¹² Since the energy of certain of the intermediate 2π , 3π configurations in Eqs. (3) can equal the mass of the initial K_1^0 , K_2^0 , the T_1 , T_2 transition operators possess nonvanishing "absorptive parts" so that $T_1 \neq T_1^\dagger$, $T_2 \neq T_2^\dagger$. As a result β_1 , $\beta_2 \neq 0, \pi$, whence in general, $\beta_2 - \beta_1 \equiv \beta \neq 0, \pi$. This behavior contrasts with that found in transitions mediated in lowest order by a Hermitian primitive or effective Hamiltonian, H_{int} , since in this case: $T = H_{\text{int}} = H_{\text{int}}^\dagger = T^\dagger$.

We now proceed to study the dependence of $\Gamma_\rho(t)$ and P_ρ on the polarization ρ of the two-photon state. For two left- or two right-circularly polarized photons, Eqs. (4) and (6) give:

$$T_{1;ll} = g_1 e^{i\beta_1} \times \left\langle \gamma\gamma; ll \left| \frac{1}{2} \int [\mathbf{E}(x) + i\mathbf{H}(x)]^2 \varphi_1(x) d^3x \right| K_1^0 \right\rangle, \quad (11a)$$

$$T_{2;ll} = g_2 e^{i\beta_2} e^{-i\pi/2} \times \left\langle \gamma\gamma; ll \left| \frac{1}{2} \int [\mathbf{E}(x) + i\mathbf{H}(x)]^2 \varphi_2(x) d^3x \right| K_2^0 \right\rangle,$$

$$\delta_{ll} = \beta_2 - \beta_1 - \frac{1}{2}\pi \equiv \beta - \frac{1}{2}\pi,$$

$$T_{1;rr} = g_1 e^{i\beta_1} \times \left\langle \gamma\gamma; rr \left| \frac{1}{2} \int [\mathbf{E}(x) - i\mathbf{H}(x)]^2 \varphi_1(x) d^3x \right| K_1^0 \right\rangle, \quad (11b)$$

$$T_{2;rr} = g_2 e^{i\beta_2} e^{i\pi/2} \times \left\langle \gamma\gamma; rr \left| \frac{1}{2} \int [\mathbf{E}(x) - i\mathbf{H}(x)]^2 \varphi_2(x) d^3x \right| K_2^0 \right\rangle,$$

$$\delta_{rr} = \beta_2 - \beta_1 + \frac{1}{2}\pi \equiv \beta + \frac{1}{2}\pi,$$

while the relations:

$$\begin{aligned} \langle \gamma\gamma; ll | (\mathbf{E} + i\mathbf{H})^2 | 0 \rangle &= \langle \gamma\gamma; ll | P^{-1}P(\mathbf{E} + i\mathbf{H})^2 P^{-1}P | 0 \rangle \\ &= \langle \gamma\gamma; rr | (\mathbf{E} - i\mathbf{H})^2 | 0 \rangle; \\ P &= \text{parity operator}; \\ \langle 0 | \varphi_1(x) | K_1^0 \rangle &= \langle 0 | \varphi_2(x) | K_2^0 \rangle, \end{aligned}$$

ensure the equalities:

$$\frac{T_{1;ll}}{g_1 e^{i\beta_1}} = \frac{T_{1;rr}}{g_1 e^{i\beta_1}} = \frac{T_{2;ll}}{g_2 e^{i\beta_2} e^{-i\pi/2}} = \frac{T_{2;rr}}{g_2 e^{i\beta_2} e^{i\pi/2}}. \quad (12)$$

Equations (9)–(12) yield for the net correlated circular polarization of the two photons:

$$\begin{aligned} \Pi(t) &\equiv \frac{\Gamma_{ll}(t) - \Gamma_{rr}(t)}{\Gamma_{ll}(t) + \Gamma_{rr}(t)} \\ &= \frac{2[\kappa(1-\kappa)]^{\frac{1}{2}} e^{-(\lambda_1+\lambda_2)t/2} \cos[(\Delta m)t + \beta]}{\kappa e^{-\lambda_1 t} + (1-\kappa)e^{-\lambda_2 t}}, \quad (13a) \end{aligned}$$

where

$$\begin{aligned} \kappa &\equiv \frac{g_1^2}{g_1^2 + g_2^2} \\ &= \frac{\text{Rate}(K_1^0 \rightarrow \gamma + \gamma)}{\text{Rate}(K_1^0 \rightarrow \gamma + \gamma) + \text{Rate}(K_2^0 \rightarrow \gamma + \gamma)}. \quad (13b) \end{aligned}$$

Equations (13) illustrate the unusual phenomenon of a time-dependent net correlated circular polarization of the two photons. Similarly, Eqs. (10)–(12) give for the time-integrated net excess of left-left circularly polarized pairs of photons:

$$P \equiv \frac{P_{ll} - P_{rr}}{P_{ll} + P_{rr}} = \frac{2[\kappa(1-\kappa)]^{\frac{1}{2}} \{[(\lambda_1+\lambda_2)/2] \cos\beta - \Delta m \sin\beta\} / \{(\Delta m)^2 + [(\lambda_1+\lambda_2)/2]^2\}}{\kappa/\lambda_1 + (1-\kappa)/\lambda_2}. \quad (14)$$

Further, because the only possible two-photon polarization states arising from a spin-zero neutral K meson are ll and rr , $\Pi(t)$ and P also give the time-dependent net circular polarization of *each* photon and the time-integrated net excess of left circular *individual* photons. For neutral K mesons of definite velocity v , the time-dependent net circular polarization will vary with distance d traveled between birth and radiative death according to Eqs. (13) with $t = d(1-v^2)^{1/2}/v$.

Besides a time-dependent net circular polarization there will also be a time dependence in the correlation of the planes of linear polarization of the two photons. In this case the polarization ρ of the two-photon state

is characterized by a specification of the angle ϕ between the linear polarization vectors \mathbf{e}' , \mathbf{e}'' of the two photons, viz.:

$$\cos\phi = \mathbf{e}' \cdot \mathbf{e}''; \quad \sin\phi = (\mathbf{e}' \times \mathbf{e}'') \cdot \mathbf{k}' / |\mathbf{k}'|. \quad (15)$$

The matrix elements $T_{1;\phi}$, $T_{2;\phi}$ of Eq. (4) are then related by

$$T_{1;\phi}/g_1 e^{i\beta_1} \cos\phi = T_{2;\phi}/g_2 e^{i\beta_2} \sin\phi; \quad \delta_\phi = \beta_2 - \beta_1 \equiv \beta. \quad (16)$$

Thus, using Eqs. (16) and (9), the probability that a photon pair emitted at proper time t by a neutral K meson is characterized by the above-defined angle ϕ , is

$$p(\phi, t) = \frac{\cos^2\phi \kappa e^{-\lambda_1 t} + \sin^2\phi (1-\kappa) e^{-\lambda_2 t} - 2 \cos\phi \sin\phi [\kappa(1-\kappa)]^{\frac{1}{2}} e^{-(\lambda_1+\lambda_2)t/2} \sin[(\Delta m)t + \beta]}{\pi [\kappa e^{-\lambda_1 t} + (1-\kappa) e^{-\lambda_2 t}]}, \quad (17a)$$

with the normalization

$$\int_0^{2\pi} p(\phi, t) d\phi = 1. \quad (17b)$$

Equations (17) exhibit explicitly the limiting cases, (a): $\kappa = 1 - p(\phi, t) = (1/\pi) \cos^2\phi$, i.e., only K_1^0 decays into 2γ —photon polarization planes tend to parallelism, and, (b): $\kappa = 0 - p(\phi, t) = (1/\pi) \sin^2\phi$, i.e., only K_2^0

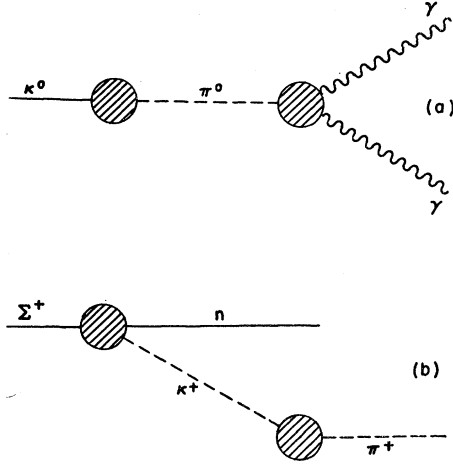


FIG. 2. Processes considered in estimating $\text{Rate}(K_2^0 \rightarrow \gamma + \gamma)$.

decays into 2γ —photon polarization planes tend to perpendicularity. In each of these cases the interference term vanishes as does the coefficient of either $e^{-\lambda_1 t}$ or $e^{-\lambda_2 t}$ so that $p(\phi, t)$, $\Gamma_\phi(t)$ decay according to a single exponential. Such a behavior is a consequence of the neutral K decay photons reflecting the PC quantum number of their parent neutral K — K_1^0 or K_2^0 —in just the way that the neutral π decay photons reflect the P or, equivalently, the PC quantum number of their parent neutral π .¹³ As expected the maximum value of the interference term occurs for $\kappa = \frac{1}{2}$ and $\phi = \pi/4$. It is also worth mentioning that the $p(\phi, t)$ of Eqs. (17) and the $\Pi(t)$ of Eqs. (13) depend upon the sign as well as the magnitude of Δm ,¹⁴ a situation which, in general, does not prevail in other K_1^0 , K_2^0 interference phenomena [thus, for example, the growth in time of the \bar{K}^0 probability:

$$|\langle \bar{K}^0 | \Psi(t) \rangle|^2 = \frac{1}{4}(e^{-\lambda_1 t} + e^{-\lambda_2 t} - 2e^{-(\lambda_1 + \lambda_2)t/2} \cos(\Delta m)t)$$

determines only the magnitude of Δm].¹⁵

The rate of appearance of the neutral K derived photons as a function of (proper) time can also be studied without regard to the polarization properties of the photons. Since the interference term in the $\Gamma_\phi(t)$ of Eq. (9) drops out in forming $\sum_\rho \Gamma_\rho(t)$ [see Eqs. (11), (12)], we have [see also Eq. (10b)]

$$\Gamma(t) \equiv \sum_\rho \Gamma_\rho(t) = [\text{Rate}(K_1^0 \rightarrow \gamma + \gamma)]e^{-\lambda_1 t} + [\text{Rate}(K_2^0 \rightarrow \gamma + \gamma)]e^{-\lambda_2 t}. \quad (18)$$

¹³ C. N. Yang, Phys. Rev. **77**, 242 (1950); J. Bernstein and K. A. Johnson, *ibid.* **109**, 189 (1958); J. Bernstein and L. Michel, *ibid.* **118**, 871 (1960); G. Feinberg, *ibid.* **120**, 640 (1960).

¹⁴ It is to be noted that the interference term in $p(\phi, t)$ is proportional to $\sin[(\Delta m)t + \beta]$, while the interference term in $\Pi(t)$ is proportional to $\cos[(\Delta m)t + \beta]$. Thus even if $\beta = \pi/2$, or $\beta = 0$, π , the determination of the sign of Δm is possible from either $\Pi(t)$ or $p(\phi, t)$.

¹⁵ As shown by I. Y. Kobzarev and L. B. Okun, J. Exptl. Theoret. Phys. (U.S.S.R.) **39**, 605 (1960) [Soviet Phys.-JETP **12**, 426 (1961)] and by N. N. Biswas, Phys. Rev. **118**, 866 (1959), the sign of Δm can also be deduced from an analysis of suitably arranged regeneration experiments.

Thus, when the photon polarization is not detected, $\Gamma(t)$ behaves as a linear combination of two exponentially decaying rates.

IV

With the advent of copious neutral K meson sources the detection of the radiative neutral K meson decay should become experimentally feasible especially since the K_2^0 amplitude appears to branch relatively strongly into the 2γ channel. In addition, according to the estimates given above, $\text{Rate}(K_1^0 \rightarrow \gamma + \gamma) \approx \text{Rate}(K_2^0 \rightarrow \gamma + \gamma)$, an approximate equality which tends to maximize the interference parameter $[\kappa(1 - \kappa)]^{1/2}$ of Eqs. (17) and (13). A spark chamber or high- Z bubble chamber measurement of the angle ϕ' between the planes of the electron-positron pairs, created by external conversion of each of the two neutral K decay photons, will then provide a means of observing the effects predicted by Eqs. (17). In fact, essentially the same experimental procedure can be used for the study of the neutral K decay photons as has previously been suggested for the neutral π decay photons,¹⁶ and the requisite calculations relating the theoretically desired distribution in ϕ [$p(\phi, t)$ of Eqs. (17)] to the directly measured distribution in ϕ' are already available in the neutral π case.¹⁶

APPENDIX

To make a rough estimate of the decay rate of a K_2^0 meson into two photons, we consider the decay processes depicted in Fig. 2. The decay process in Fig. 2(a) is envisioned as the sequential combination of the weak-interaction-induced transmutation $K_2^0 \rightarrow \pi^0$ followed by the radiative π^0 decay. Although the virtual π^0 is here very far off its mass shell, we nevertheless describe its decay by the same effective interaction Hamiltonian that is used to treat the real process $\pi^0 \rightarrow \gamma + \gamma$, namely:

$$\mathcal{H}(\pi^0 \rightarrow \gamma + \gamma) = (g/m_\pi)\phi(x; \pi^0)\mathbf{E}(x) \cdot \mathbf{H}(x).$$

We further describe the weak interaction that induces $K_2^0 \rightarrow \pi^0$ by the effective Hamiltonian:

$$\mathcal{H}(K_2^0 \rightarrow \pi^0) = Gm_\pi^2\phi(x; \pi^0)\phi(x; K_2^0).$$

We then find:

$$[\text{Rate}(K_2^0 \rightarrow \gamma + \gamma)] = G^2 \left(\frac{4(m_K/m_\pi)^3}{[(m_K/m_\pi)^2 - 1]^2} \right) [\text{Rate}(\pi^0 \rightarrow \gamma + \gamma)]. \quad (A.1)$$

In order to estimate G we assume¹⁷ that the process of Fig. 2(b) is the dominant contributor to the

¹⁶ C. N. Yang, Phys. Rev. **77**, 722 (1950); E. Karlson, Arkiv Fysik **13**, 1 (1958).

¹⁷ See G. Feldman, P. T. Matthews, and A. Salam, Phys. Rev. **121**, 302 (1961); L. Wolfenstein, *ibid.* **121**, 1245 (1961).

$\Sigma^+ \rightarrow n + \pi^+$ transition rate, an assumption consistent with the vanishing of the asymmetry parameter α_+ . We also assume that the $\Sigma n K$ relative intrinsic parity is odd and that the form factor of the $\Sigma n K$ vertex does not change appreciably as $-(p_\Sigma - p_n)^2$ increases from m_π^2 to m_K^2 . Then, relating also the $K_2^0 \rightarrow \pi^0$ amplitude to the $K^+ \rightarrow \pi^+$ amplitude through the isospinor character of the corresponding weak interaction effective Hamiltonian, we obtain:

$$[\text{Rate}(\Sigma^+ \rightarrow n + \pi^+)] = 4 \left(\frac{g_{\Sigma n K}}{4\pi} \right) G^2 \left(\frac{q}{m_n} \right)^2 q / \left[\left(\frac{m_K}{m_\pi} \right)^2 - 1 \right]^2, \quad (\text{A.2})$$

where $g_{\Sigma n K}$ is the value of the form factor of the $\Sigma n K$ vertex for $(p_\Sigma - p_n)^2 = -m_K^2$ and

$$q \equiv \left[\left(\frac{m_\Sigma^2 - m_n^2 + m_\pi^2}{2m_\Sigma} \right)^2 - m_\pi^2 \right]^{\frac{1}{2}}.$$

Elimination of G^2 from Eqs. (A.1) and (A.2) yields:

$$[\text{Rate}(K_2^0 \rightarrow \gamma + \gamma)] = - \left(\frac{m_K}{q} \right)^3 \left(\frac{m_n}{q} \right)^2 \times \frac{[\text{Rate}(\pi^0 \rightarrow \gamma + \gamma)] \times [\text{Rate}(\Sigma^+ \rightarrow n + \pi^+)]}{(g_{\Sigma n K}^2 / 4\pi)}, \quad (\text{A.3})$$

whence, using the measured values of

$$[\text{Rate}(\pi^0 \rightarrow \gamma + \gamma)], \quad [\text{Rate}(\Sigma^+ \rightarrow n + \pi^+)],^9$$

we get

$$[(\text{Rate} K_2^0 \rightarrow \gamma + \gamma)] = (1.6 \times 10^5 \text{ sec}^{-1}) / (g_{\Sigma n K}^2 / 4\pi) \approx 10^5 \text{ sec}^{-1}. \quad (\text{A.4})$$

It is encouraging to find that this estimate of $\text{Rate}(K_2^0 \rightarrow \gamma + \gamma)$ agrees in order of magnitude with the estimate made above on the basis of phase-space considerations:

$$[\text{Rate}(K_2^0 \rightarrow \gamma + \gamma)] \approx [\text{Rate}(K_2^0 \rightarrow 3\pi, \pi\mu\nu, \pi e\nu)] \times (1/137)^2 \frac{\text{two-body phase space volume}}{\text{three-body phase space volume}} \approx 2.5 \times 10^5 \text{ sec}^{-1}. \quad (\text{A.5})$$