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Rigidity of the Inertial Moment of Large Interacting Many-Fermion Systems in Perturbation Theory*

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As a result of an addendum and a correction to the author's previous work, the vanishing of interaction effects on the inertial moment of a large many-fermion system, moving under periodic boundary conditions, in the second order of particle-particle coupling is established. The result is independent of potential form. A proof extending the theorem to all orders is given.

I. INTRODUCTION

IN this note we re-examine a previous calculation of the author[1] on the interaction correction to the inertial moment of a large many-fermion system moving under periodic boundary conditions in the second order of particle-particle coupling. As a consequence of an addendum and a correction to that calculation,[1] we now find that interaction effects *vanish* in the first two orders of perturbation theory *independent of potential form*. This analysis is described in detail below, where the notation used follows closely that of our preceding communications on this subject.[1,2] By making use of some of our previous results, together with the proper extension of the theorem of Brueckner and Amado[3] to all orders of perturbation theory, we are able to prove the vanishing of interaction effects on the inertial moment to all orders.

Section II is given over to a re-examination of our previous work.[1] We take up the general proof in Sec. III.

II. VANISHING OF INTERACTION EFFECTS IN SECOND ORDER

We begin by restating our previous results in the second order of particle-particle coupling,[4]

$$\lim_{L\to\infty}\frac{\Delta\mathcal{I}_{xy}}{(\mathcal{I}_{xy})_{\text{rigid}}}=\frac{M}{k_F}\left(\frac{dV_p^{(2)}}{dk_F}-\frac{dV_h^{(2)}}{dk_F}\right)-\{W_p^{(2)}(k_F)+W_h^{(2)}(k_F)\},\quad(1)$$

with the following *addendum* and *correction*[5]:

(a) The diagonal terms $O(\lambda^2\xi^0)$ correcting H_0, which result from $\mathcal{S}$-ordering $H_2|_{(\xi=0)}$, produce *propagator changes* in the usual ground-state energy of second order in particle-particle coupling. (These were omitted in I.) It is then easy to show that the effect of these propagator changes,[6]

$$\tfrac{1}{2}\lambda^2\frac{\partial^2}{\partial\lambda^2}\left\langle\Phi_0\left|H_V\frac{1}{-(H_T+\lambda^2h_0)}H_V\right|\Phi_0\right\rangle_L\Bigg|_{(\lambda=0)},\quad(2)$$

is to just compensate the terms,

$$-\{W_p^{(2)}(k_F)+W_h^{(2)}(k_F)\},$$

in Eq. (1). Thus, it follows that

$$\lim_{L\to\infty}\frac{\Delta\mathcal{I}_{xy}}{(\mathcal{I}_{xy})_{\text{rigid}}}=\frac{M}{k_F}\left(\frac{dV_p^{(2)}}{dk_F}-\frac{dV_h^{(2)}}{dk_F}\right).\quad(3)$$

Before turning to a discussion of the *correction* to our previous work, we shall recast Eq. (3) into a more suggestive form, by noting that the difference, $V_p^{(2)}(k_F)-V_h^{(2)}(k_F)=-\delta^{(2)}\epsilon(k_F)$, is just the negative of the single-particle energy calculated from the proper diagrams[7] in the second order of perturbation theory.[8] Thus we have for our previous results, modified by the addendum (a),

$$\lim_{L\to\infty}\frac{\Delta\mathcal{I}_{xy}}{(\mathcal{I}_{xy})_{\text{rigid}}}=-\frac{M}{k_F}\frac{d}{dk_F}[\delta^{(2)}\epsilon(k_F)].\quad(4)$$

(b) The arguments by which the terms $O(r^{-2})$,[9]

$$\begin{aligned}\frac{8\xi^2}{\Omega^2}\sum_{\substack{r\mathbf{klst}\\(l>k_F;\,s,t<k_F)}}f(\mathbf{k}+\tfrac{1}{2}\mathbf{r}_>,\mathbf{k}-\tfrac{1}{2}\mathbf{r}_<)\Bigg\{&\mathcal{V}_{\mathbf{st};\,\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{l}-\frac{1}{2}\mathbf{r}}\mathcal{V}_{\mathbf{st};\,\mathbf{k}-\frac{1}{2}\mathbf{r},\mathbf{l}+\frac{1}{2}\mathbf{r}}\left[\frac{f(\mathbf{l}+\frac{1}{2}\mathbf{r}_>,\mathbf{l}-\frac{1}{2}\mathbf{r}_<)}{\epsilon_{\mathbf{k}+\frac{1}{2}\mathbf{r}}+\epsilon_{\mathbf{l}+\frac{1}{2}\mathbf{r}}-\epsilon_{\mathbf{s}}-\epsilon_{\mathbf{t}}}-\frac{f(\mathbf{l}-\frac{1}{2}\mathbf{r}_>,\mathbf{l}+\frac{1}{2}\mathbf{r}_<)}{\epsilon_{\mathbf{k}+\frac{1}{2}\mathbf{r}}+\epsilon_{\mathbf{l}-\frac{1}{2}\mathbf{r}}-\epsilon_{\mathbf{s}}-\epsilon_{\mathbf{t}}}\right]\\&-\mathcal{V}_{\mathbf{s}-\frac{1}{2}\mathbf{r},\mathbf{t};\,\mathbf{k}-\frac{1}{2}\mathbf{r},\mathbf{l}}\mathcal{V}_{\mathbf{s}+\frac{1}{2}\mathbf{r},\mathbf{t};\,\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{l}}\left[\frac{f(\mathbf{s}+\frac{1}{2}\mathbf{r}_>,\mathbf{s}-\frac{1}{2}\mathbf{r}_<)}{\epsilon_{\mathbf{k}+\frac{1}{2}\mathbf{r}}+\epsilon_{\mathbf{l}}-\epsilon_{\mathbf{s}-\frac{1}{2}\mathbf{r}}-\epsilon_{\mathbf{t}}}-\frac{f(\mathbf{s}-\frac{1}{2}\mathbf{r}_>,\mathbf{s}+\frac{1}{2}\mathbf{r}_<)}{\epsilon_{\mathbf{k}+\frac{1}{2}\mathbf{r}}+\epsilon_{\mathbf{l}}-\epsilon_{\mathbf{s}+\frac{1}{2}\mathbf{r}}-\epsilon_{\mathbf{t}}}\right]\\&-\mathcal{V}_{\mathbf{s},\mathbf{t}-\frac{1}{2}\mathbf{r};\,\mathbf{k}-\frac{1}{2}\mathbf{r},\mathbf{l}}\mathcal{V}_{\mathbf{s},\mathbf{t}+\frac{1}{2}\mathbf{r};\,\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{l}}\left[\frac{f(\mathbf{t}+\frac{1}{2}\mathbf{r}_>,\mathbf{t}-\frac{1}{2}\mathbf{r}_<)}{\epsilon_{\mathbf{k}+\frac{1}{2}\mathbf{r}}+\epsilon_{\mathbf{l}}-\epsilon_{\mathbf{s}}-\epsilon_{\mathbf{t}-\frac{1}{2}\mathbf{r}}}-\frac{f(\mathbf{t}-\frac{1}{2}\mathbf{r}_>,\mathbf{t}+\frac{1}{2}\mathbf{r}_<)}{\epsilon_{\mathbf{k}+\frac{1}{2}\mathbf{r}}+\epsilon_{\mathbf{l}}-\epsilon_{\mathbf{s}}-\epsilon_{\mathbf{t}+\frac{1}{2}\mathbf{r}}}\right]+\tfrac{1}{2}(\mathcal{V}\to\mathcal{V}')\Bigg\},\end{aligned}\quad(5a)$$

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[1] R. M. Rockmore, Phys. Rev. **120**, 1933 (1960), hereafter referred to as I.
[2] R. M. Rockmore, Phys. Rev. **116**, 469 (1959); **118**, 1645 (1960).
[3] We refer to the theorem proved (to lowest order in particle-particle coupling) in the Appendix to R. D. Amado and K. A. Brueckner, Phys. Rev. **115**, 778 (1959).
[4] We have corrected a trivial error in sign in Eq. (4.9) of I.
[5] See the antepenultimate sentence of the third section of I.
[6] The subscript L denotes the linked part; $|\Phi_0\rangle$ denotes the unperturbed ground-state vector.
[7] J. M. Luttinger and J. C. Ward, Phys. Rev. **118**, 1417 (1960).
[8] K. A. Brueckner and D. T. Goldman, Phys. Rev. **117**, 207 (1960).
[9] See Eqs. (4.3)–(4.6) in I.

and

$$\frac{8\xi^2}{\Omega^2}\sum_{\substack{r\mathbf{klst}\\(s,t>k_F;\,l<k_F)}} f(\mathbf{k}+\tfrac{1}{2}\mathbf{r}_>,\,\mathbf{k}-\tfrac{1}{2}\mathbf{r}_<)\Bigg\{\mathcal{V}_{\mathbf{st};\,\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{l}-\frac{1}{2}\mathbf{r}}\mathcal{V}_{\mathbf{s},\mathbf{t};\,\mathbf{k}-\frac{1}{2}\mathbf{r},\mathbf{l}+\frac{1}{2}\mathbf{r}}\left[\frac{f(\mathbf{l}+\frac{1}{2}\mathbf{r}_>,\,\mathbf{l}-\frac{1}{2}\mathbf{r}_<)}{\epsilon_{\mathbf{s}}+\epsilon_{\mathbf{t}}-\epsilon_{\mathbf{k}-\frac{1}{2}\mathbf{r}}-\epsilon_{\mathbf{l}-\frac{1}{2}\mathbf{r}}}-\frac{f(\mathbf{l}-\frac{1}{2}\mathbf{r}_>,\,\mathbf{l}+\frac{1}{2}\mathbf{r}_<)}{\epsilon_{\mathbf{s}}+\epsilon_{\mathbf{t}}-\epsilon_{\mathbf{k}-\frac{1}{2}\mathbf{r}}-\epsilon_{\mathbf{l}+\frac{1}{2}\mathbf{r}}}\right]$$

$$-\mathcal{V}_{\mathbf{s}+\frac{1}{2}\mathbf{r},\mathbf{t};\,\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{l}}\mathcal{V}_{\mathbf{s}-\frac{1}{2}\mathbf{r},\mathbf{t};\,\mathbf{k}-\frac{1}{2}\mathbf{r},\mathbf{l}}\left[\frac{f(\mathbf{s}+\frac{1}{2}\mathbf{r}_>,\,\mathbf{s}-\frac{1}{2}\mathbf{r}_<)}{\epsilon_{\mathbf{s}+\frac{1}{2}\mathbf{r}}+\epsilon_{\mathbf{t}}-\epsilon_{\mathbf{k}-\frac{1}{2}\mathbf{r}}-\epsilon_{\mathbf{l}}}-\frac{f(\mathbf{s}-\frac{1}{2}\mathbf{r}_>,\,\mathbf{s}+\frac{1}{2}\mathbf{r}_<)}{\epsilon_{\mathbf{s}-\frac{1}{2}\mathbf{r}}+\epsilon_{\mathbf{t}}-\epsilon_{\mathbf{k}-\frac{1}{2}\mathbf{r}}-\epsilon_{\mathbf{l}}}\right]$$

$$-\mathcal{V}_{\mathbf{s},\mathbf{t}+\frac{1}{2}\mathbf{r};\,\mathbf{k}+\frac{1}{2}\mathbf{r},\mathbf{l}}\mathcal{V}_{\mathbf{s},\mathbf{t}-\frac{1}{2}\mathbf{r};\,\mathbf{k}-\frac{1}{2}\mathbf{r},\mathbf{l}}\left[\frac{f(\mathbf{t}+\frac{1}{2}\mathbf{r}_>,\,\mathbf{t}-\frac{1}{2}\mathbf{r}_<)}{\epsilon_{\mathbf{s}}+\epsilon_{\mathbf{t}+\frac{1}{2}\mathbf{r}}-\epsilon_{\mathbf{k}-\frac{1}{2}\mathbf{r}}-\epsilon_{\mathbf{l}}}-\frac{f(\mathbf{t}-\frac{1}{2}\mathbf{r}_>,\,\mathbf{t}+\frac{1}{2}\mathbf{r}_<)}{\epsilon_{\mathbf{s}}+\epsilon_{\mathbf{t}-\frac{1}{2}\mathbf{r}}-\epsilon_{\mathbf{k}-\frac{1}{2}\mathbf{r}}-\epsilon_{\mathbf{l}}}\right]+\tfrac{1}{2}(\mathcal{V}\rightarrow\mathcal{V}')\Bigg\}, \quad (5b)$$

were discarded in I, are *not correct.* In fact, we shall shortly demonstrate that they just compensate the correction given by Eq. (4), thereby yielding the desired result,[10]

$$\lim_{L\to\infty}\frac{\Delta\mathcal{G}_{xy}}{(\mathcal{G}_{xy})_{\text{rigid}}}=0,$$

in the second order of perturbation theory. It will simplify matters considerably to take the limit $r\rightarrow 0$ wherever possible in (5a) and (5b) and to make use of the symmetry property

$$f(-\mathbf{k}-\tfrac{1}{2}\mathbf{r}_>,\,-\mathbf{k}+\tfrac{1}{2}\mathbf{r}_<)=f(\mathbf{k}+\tfrac{1}{2}\mathbf{r}_>,\,\mathbf{k}-\tfrac{1}{2}\mathbf{r}_<),$$

so that (5a) and (5b) take the form

$$\frac{8\xi^2}{\Omega^2}\sum_{\substack{r\mathbf{klst}\\(l>k_F;\,s,t<k_F)}}\frac{f(\mathbf{k}+\frac{1}{2}\mathbf{r}_>,\,\mathbf{k}-\frac{1}{2}\mathbf{r}_<)}{\epsilon_{\mathbf{k}}+\epsilon_{\mathbf{l}}-\epsilon_{\mathbf{s}}-\epsilon_{\mathbf{t}}}\{f(\mathbf{l}+\tfrac{1}{2}\mathbf{r}_>,\,\mathbf{l}-\tfrac{1}{2}\mathbf{r}_<)[\mathcal{V}_{\mathbf{st};\,\mathbf{kl}}{}^2-\mathcal{V}_{\mathbf{st};\,\mathbf{k},-\mathbf{l}}{}^2]$$

$$-f(\mathbf{s}+\tfrac{1}{2}\mathbf{r}_>,\,\mathbf{s}-\tfrac{1}{2}\mathbf{r}_<)[\mathcal{V}_{\mathbf{st};\,\mathbf{kl}}{}^2-\mathcal{V}_{-\mathbf{s},\mathbf{t};\,\mathbf{kl}}{}^2]-f(\mathbf{t}+\tfrac{1}{2}\mathbf{r}_>,\,\mathbf{t}-\tfrac{1}{2}\mathbf{r}_<)[\mathcal{V}_{\mathbf{st};\,\mathbf{kl}}{}^2-\mathcal{V}_{\mathbf{s},-\mathbf{t};\,\mathbf{k},\mathbf{l}}{}^2]+\tfrac{1}{2}(\mathcal{V}\rightarrow\mathcal{V}')\}, \quad (6a)$$

and

$$\frac{8\xi^2}{\Omega^2}\sum_{\substack{r\mathbf{klst}\\(s,t>k_F;\,l<k_F)}}\frac{f(\mathbf{k}+\frac{1}{2}\mathbf{r}_>,\,\mathbf{k}-\frac{1}{2}\mathbf{r}_<)}{\epsilon_{\mathbf{s}}+\epsilon_{\mathbf{t}}-\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{l}}}\{f(\mathbf{l}+\tfrac{1}{2}\mathbf{r}_>,\mathbf{l}-\tfrac{1}{2}\mathbf{r}_<)[\mathcal{V}_{\mathbf{st};\,\mathbf{kl}}{}^2-\mathcal{V}_{\mathbf{st};\,\mathbf{k},-\mathbf{l}}{}^2]$$

$$-f(\mathbf{s}+\tfrac{1}{2}\mathbf{r}_>,\,\mathbf{s}-\tfrac{1}{2}\mathbf{r}_<)[\mathcal{V}_{\mathbf{st};\,\mathbf{kl}}{}^2-\mathcal{V}_{-\mathbf{s},\mathbf{t};\,\mathbf{kl}}{}^2]-f(\mathbf{t}+\tfrac{1}{2}\mathbf{r}_>,\,\mathbf{t}-\tfrac{1}{2}\,\mathbf{l}_<)[\mathcal{V}_{\mathbf{st};\,\mathbf{kl}}{}^2-\mathcal{V}_{\mathbf{s},-\mathbf{t};\,\mathbf{kl}}{}^2]+\tfrac{1}{2}(\mathcal{V}\rightarrow\mathcal{V}')\}. \quad (6b)$$

Since[1]

$$\lim_{r\to 0} f(\mathbf{a}+\tfrac{1}{2}\mathbf{r}_>,\,\mathbf{a}-\tfrac{1}{2}\mathbf{r}_<)=\frac{Ma_y}{r}\eta(\mathbf{r}\cdot\mathbf{a}/a)\delta(a-k_F),$$

expressions (6a) and (6b) can be further reduced to

$$\frac{8\xi^2}{\Omega^2}\sum_{r\mathbf{k}}\frac{M^2}{r^2}\delta(k-k_F)\eta(\mathbf{r}\cdot\mathbf{k}/k)\Bigg\{\sum_{\mathbf{l}}\delta(l-k_F)\eta(\mathbf{r}\cdot\mathbf{l}/l)k_yl_y\left[\sum_{s,t<k_F}\frac{\mathcal{V}_{\mathbf{st};\,\mathbf{kl}}{}^2-\mathcal{V}_{\mathbf{s},\mathbf{t};\,\mathbf{k},-\mathbf{l}}{}^2}{\epsilon_k+\epsilon_l-\epsilon_s-\epsilon_t}\right]_{(k,l=k_F)}$$

$$-\sum_{\mathbf{s}}\delta(s-k_F)\eta(\mathbf{r}\cdot\mathbf{s}/s)k_ys_y\left[\sum_{\mathbf{l}>k_F;\,t<k_F}\frac{\mathcal{V}_{\mathbf{st};\,\mathbf{kl}}{}^2-\mathcal{V}_{-\mathbf{s},\mathbf{t};\,\mathbf{kl}}{}^2}{\epsilon_{\mathbf{k}}+\epsilon_{\mathbf{l}}-\epsilon_{\mathbf{s}}-\epsilon_{\mathbf{t}}}\right]_{(k,s=k_F)}$$

$$-\sum_{\mathbf{t}}\delta(t-k_F)\eta(\mathbf{r}\cdot\mathbf{t}/t)k_yt_y\left[\sum_{\mathbf{l}>k_F;\,s<k_F}\frac{\mathcal{V}_{\mathbf{st};\,\mathbf{kl}}{}^2-\mathcal{V}_{\mathbf{s};-\mathbf{t};\,\mathbf{kl}}{}^2}{\epsilon_{\mathbf{k}}+\epsilon_{\mathbf{l}}-\epsilon_{\mathbf{s}}-\epsilon_{\mathbf{t}}}\right]_{(k,t=k_F)}\Bigg\}+\tfrac{1}{2}(\mathcal{V}\rightarrow\mathcal{V}'), \quad (7a)$$

[10] We must apologize to those who insisted on this result, although their arguments had no rigorous basis.

and

$$\frac{8\xi^2}{\Omega^2}\sum_{r\mathbf{k}}\frac{M^2}{r^2}\delta(k-k_F)\eta(\mathbf{r}\cdot\mathbf{k}/k)\Bigg\{\sum_{\mathbf{l}}\delta(l-k_F)\eta(\mathbf{r}\cdot\mathbf{l}/l)k_yl_y\Bigg[\sum_{s,t>k_F}\frac{\mathcal{V}_{\mathbf{st};\,\mathbf{kl}}{}^2-\mathcal{V}_{\mathbf{st};\,\mathbf{k},-\mathbf{l}}{}^2}{\epsilon_{\mathbf{s}}+\epsilon_{\mathbf{t}}-\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{l}}}\Bigg]_{(k,l=k_F)}$$

$$-\sum_{\mathbf{s}}\delta(s-k_F)\eta(\mathbf{r}\cdot\mathbf{s}/s)k_ys_y\Bigg[\sum_{l<k_F,\ t>k_F}\frac{\mathcal{V}_{\mathbf{st};\,\mathbf{kl}}{}^2-\mathcal{V}_{-\mathbf{s},\mathbf{t};\,\mathbf{kl}}{}^2}{\epsilon_{\mathbf{s}}+\epsilon_{\mathbf{t}}-\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{l}}}\Bigg]_{(k,s=k_F)}$$

$$-\sum_{\mathbf{t}}\delta(t-k_F)\eta(\mathbf{r}\cdot\mathbf{t}/t)k_yt_y\Bigg[\sum_{l<k_F,\ s>k_F}\frac{\mathcal{V}_{\mathbf{st};\,\mathbf{kl}}{}^2-\mathcal{V}_{\mathbf{s},-\mathbf{t};\,\mathbf{kl}}{}^2}{\epsilon_{\mathbf{s}}+\epsilon_{\mathbf{t}}-\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{l}}}\Bigg]_{(k,t=k_F)}\Bigg\}+\tfrac{1}{2}(\mathcal{V}\to\mathcal{V}'). \quad (7b)$$

To show that the sums of terms, (7a) and (7b), can be transformed into the negative of the effective mass correction given by (4), an extension of the gradient theorem of Brueckner and Amado,[3]

$$\left[\frac{d}{dk_1}\int_{k_2<k_F}d\mathbf{k}_2\langle\mathbf{k}_1\mathbf{k}_2|v|\mathbf{k}_1\mathbf{k}_2\rangle\right]_{(k_1=k_F)}=-\int d\omega_{12}[(\mathbf{k}_1\cdot\mathbf{k}_2)\langle\mathbf{k}_1\mathbf{k}_2|v|\mathbf{k}_1\mathbf{k}_2\rangle]_{(k_1,k_2=k_F)}, \quad (8)$$

to higher orders in perturbation theory is required. Introducing the variable[11] $\mathbf{s}=(\mathbf{k}_1-\mathbf{k}_2)/2$ and writing $\langle\mathbf{k}_1\mathbf{k}_2|v|\mathbf{k}_1\mathbf{k}_2\rangle=v(s)$ as they do,[3] one finds that the following simplified derivation of (8),

$$\left[\frac{d}{dk_1}\int_{k_2<k_F}d\mathbf{k}_2v(s)\right]_{(k_1=k_F)}=8\frac{d}{dk_1}\int d\mathbf{s}\eta(k_F-|\mathbf{k}_1-2\mathbf{s}|)v(s)\Big|_{(k_1=k_F)}$$

$$=-8\int d\mathbf{s}\delta(k_F-|\mathbf{k}_1-2\mathbf{s}|)\frac{(k_1{}^2-2\mathbf{s}\cdot\mathbf{k}_1)}{k_Fk_1}v(s)\Big|_{(k_1=k_F)}$$

$$=-\int d\mathbf{k}_2\delta(k_F-k_2)\frac{(k_F{}^2-2\mathbf{s}\cdot\mathbf{k}_1)}{k_F{}^2}v(s)\Big|_{(k_1=k_F)}$$

$$=-\int d\omega_{12}[(\mathbf{k}_1\cdot\mathbf{k}_2)\langle\mathbf{k}_1\mathbf{k}_2|v|\mathbf{k}_1\mathbf{k}_2\rangle]_{(k_1,k_2=k_F)}, \quad (9)$$

points the way to such a generalization. For example, in the case of the second-order single-particle potential,

$$V^{(2)}(k_1)=\sum_{k_2,k_3<k_F;\ k_4>k_F}v^{(2)}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4), \quad (10)$$

we must introduce the variables, $\mathbf{s}_2=\frac{1}{2}(\mathbf{k}_1-\mathbf{k}_2)$, $\mathbf{s}_3=\frac{1}{2}(\mathbf{k}_1-\mathbf{k}_3)$, $\mathbf{s}_4=\frac{1}{2}(\mathbf{k}_1-\mathbf{k}_4)$, so that we may write

$$v^{(2)}(k_1)=\left[8\frac{L^3}{(2\pi)^3}\right]^3\int\int\int d\mathbf{s}_2d\mathbf{s}_3d\mathbf{s}_4\eta(k_F-|\mathbf{k}_1-2\mathbf{s}_2|)\eta(k_F-|\mathbf{k}_1-2\mathbf{s}_3|)\eta(|\mathbf{k}_1-2\mathbf{s}_4|-k_F)v^{(2)}(\mathbf{s}_2,\mathbf{s}_3,\mathbf{s}_4). \quad (11)$$

Then differentiation of the left-hand side of (11) with respect to k_1 (which we take to be a parametric differentiation of the right-hand side) and the subsequent equating of k_1 to k_F yields the required theorem (in second order).[12] Note that each self-energy term gives rise in this way to three surface terms with the proper sign (in second order).[13] In anticipation of our later use of this theorem, we remark that it provides an interesting connection between elements of the scattering matrix (on the Fermi surface) and the proper self-energy in the same order of perturbation theory. Moreover, the scattering-matrix elements may be taken to be those of pair-pair scattering in the

[11] One makes the usual assumption of a translationally invariant potential.
[12] The conservation of momentum, i.e., the fact that $\mathbf{s}_2+\mathbf{s}_3=\mathbf{s}_4$, for example, does not introduce any difficulties.
[13] It is instructive to compare (7a) and (7b) with Eqs. (4.11) and (4.12), respectively, in I.

limit that the small net momentum of the pair, $\mathbf{r}$, goes to zero, so that both particle and hole "lie" on the Fermi surface. (Of course, this is not the only possible interpretation.)

After the verification that the terms (7a) correspond to $[(d/dk)V_p^{(2)}(k)]_{(k=k_F)}$, and that those of (7b) correspond to $[(d/dk)V_h^{(2)}(k)]_{(k=k_F)}$, it remains to examine the combinations of matrix elements which occur in (7a,b). Taking the combination $\mathcal{V}_{\mathrm{st;\ kl}}^2-\mathcal{V}_{\mathrm{st;\ k,-l}}^2$, as an example, one finds that it enables us to extend averages over the *entire* Fermi surface, which the generalized gradient theorem demands. That is,

$$\sum_{r\mathbf{kl}}\frac{1}{r^2}\eta(\mathbf{r}\cdot\mathbf{k}/k)\eta(\mathbf{r}\cdot\mathbf{l}/l)k_yl_y\left[\sum_{s,t<k_F}\frac{\mathcal{V}_{\mathrm{st;\ kl}}^2-\mathcal{V}_{\mathrm{st;\ k,-l}}^2}{\epsilon_{\mathrm{k}}+\epsilon_{\mathrm{l}}-\epsilon_{\mathrm{s}}-\epsilon_{\mathrm{t}}}\right]_{(k,l=k_F)}$$

goes over into

$$\sum_{r\mathbf{kl}}\frac{1}{r^2}\eta(\mathbf{r}\cdot\mathbf{k}/k)k_yl_y\left[\sum_{s,t<k_F}\frac{\mathcal{V}_{\mathrm{st;\ kl}}^2}{\epsilon_{\mathrm{k}}+\epsilon_{\mathrm{l}}-\epsilon_{\mathrm{s}}-\epsilon_{\mathrm{t}}}\right]_{(k,l=k_F)}, \tag{12}$$

on making the substitution $\mathbf{l}\rightarrow-\mathbf{l}$ in the second term. The latter expression may then be written as

$$\sum_{r>0}\frac{1}{r^2}\sum_{\mathbf{k}}k_yl_y\left[\sum_{s,t<k_F}\frac{\mathcal{V}_{\mathrm{st;\ kl}}^2}{\epsilon_{\mathrm{k}}+\epsilon_{\mathrm{l}}-\epsilon_{\mathrm{s}}-\epsilon_{\mathrm{t}}}\right]_{(k,l=k_F)},$$

on symmetrizing the sum over r in (12). One now makes use of the fact that at the Fermi surface,

$$\langle\mathbf{kl}|w^{(2)}|\mathbf{kl}\rangle=\sum_{s,t<k_F}\frac{\mathcal{V}_{\mathrm{st;\ kl}}^2}{\epsilon_{\mathrm{k}}+\epsilon_{\mathrm{l}}-\epsilon_{\mathrm{s}}-\epsilon_{\mathrm{t}}}\bigg|_{(k,l=k_F)},$$

can only depend on the angle between $\mathbf{k}$ and $\mathbf{l}$, and makes the spherical harmonic decomposition

$$\langle\mathbf{kl}|w^{(2)}|\mathbf{kl}\rangle=\textstyle\sum_l w_l^{(2)}(k,l)P_l(\cos\vartheta_{kl}).$$

Then by steps identical to those taken at the close of Sec. III of reference 3, one obtains

$$\sum_{r>0}\frac{1}{r^2}\sum_{\mathbf{kl}}k_yl_y\langle\mathbf{kl}|w^{(2)}|\mathbf{kl}\rangle\bigg|_{(k,l=k_F)}=\frac{2}{3}\sum_{r>0}\frac{1}{r^2}\frac{k_F^2}{(2\pi)^5}\int d\omega_{kl}(\mathbf{k}\cdot\mathbf{l})[\langle\mathbf{kl}|w^{(2)}|\mathbf{kl}\rangle]_{(k,l=k_F)}. \tag{13}$$

III. PROOF OF THE CANCELLATION OF INTERACTION EFFECTS TO ALL ORDERS

To show that this cancellation of interaction effects is altogether general, i.e., that it holds to all orders in perturbation theory independent of potential form, we must focus our attention on the complete expression for the cranking moment of an interacting system (moving under periodic boundary conditions),

$$\mathcal{I}_{xy}=-\frac{\partial^2}{\partial\lambda^2}\left\langle\Phi_0\left|(H_v-\lambda H_{ixy})\frac{1}{-H_T+\lambda H_{ixy}+i\epsilon}(H_v-\lambda H_{ixy})\right|\Phi_0\right\rangle_L\bigg|_{(\lambda=0)}, \tag{14}$$

$$\begin{aligned}=&-2\left\langle\Phi_0\left|H_{ixy}\frac{1}{-H_T+i\epsilon}H_{ixy}\right|\Phi_0\right\rangle_L-2\left\langle\Phi_0\left|H_v\frac{1}{-H_T+i\epsilon}H_{ixy}\frac{1}{-H_T+i\epsilon}H_{ixy}\right|\Phi_0\right\rangle_L\\&-2\left\langle\Phi_0\left|H_{ixy}\frac{1}{-H_T+i\epsilon}H_{ixy}\frac{1}{-H_T+i\epsilon}H_v\right|\Phi_0\right\rangle_L-2\left\langle\Phi_0\left|H_v\frac{1}{-H_T+i\epsilon}H_{ixy}\frac{1}{-H_T+i\epsilon}H_{ixy}\frac{1}{-H_T+i\epsilon}H_v\right|\Phi_0\right\rangle_L.\end{aligned} \tag{15}$$

The last of the terms in (15) has previously[1] been shown to vanish in the limit $L\rightarrow\infty$, relative to the rigid moment

by virtue of momentum conservation. That is, one has

$$\left\langle \Phi_0 \left| H_v \frac{1}{-H_T+i\epsilon} H_{ixy} \frac{1}{-H_T+i\epsilon} H_{ixy} \frac{1}{-H_T+i\epsilon} \right| \Phi_0 \right\rangle_L \to \sum_{r\neq 0} \frac{1}{r^2} \left\langle \Phi_0 \left| H_v \frac{1}{-H_T+i\epsilon} \sum_{\mathbf{k}\sigma} k_y (a_{\mathbf{k}\sigma}{}^\dagger a_{\mathbf{k}+\mathbf{r},\sigma} - b_{\mathbf{k}+\mathbf{r},\sigma}{}^\dagger b_{\mathbf{k}\sigma}) \right.\right.$$

$$\left.\left. \times \frac{1}{-H_T+i\epsilon} \sum_{\mathbf{k}'\sigma'} k_y' (a_{\mathbf{k}'+\mathbf{r},\sigma'}{}^\dagger a_{\mathbf{k}'\sigma'} - b_{\mathbf{k}'\sigma'}{}^\dagger b_{\mathbf{k}'+\mathbf{r},\sigma'}) \frac{1}{-H_T+i\epsilon} H_v \right| \Phi_0 \right\rangle_L \to \sum_{r\neq 0} \frac{1}{r^2}$$

$$\times \left\langle \Phi_0 \left| H_v \frac{1}{-H_T+i\epsilon} (\mathbf{P})_y \frac{1}{-H_T+i\epsilon} (\mathbf{P})_y \frac{1}{-H_T+i\epsilon} H_v \right| \Phi_0 \right\rangle_L = 0, \quad (16)$$

where we have as usual neglected terms at most $O(r^0)$.

A similar argument will remove the *propagator corrections* in the second and third terms of (15) as well. It will be sufficient to discuss the second term alone, where[14]

$$\left\langle \Phi_0 \left| H_v \frac{1}{-H_T+i\epsilon} H_{ixy} \frac{1}{-H_T+i\epsilon} H_{ixy} \right| \Phi_0 \right\rangle_{L\ (\text{propagator corrections})} \to \sum_{r\neq 0} \frac{1}{r^2} \sum_{\mathbf{k}\sigma} k_y$$

$$\times \left\langle \Phi_0 \left| H_v \frac{1}{-H_T+i\epsilon} \Big\{ \sum_{\substack{\mathbf{k}'\sigma' \\ (\mathbf{k}'\neq\mathbf{k})}} k_y' (a_{\mathbf{k}'\sigma'}{}^\dagger a_{\mathbf{k}'+\mathbf{r},\sigma'} - b_{\mathbf{k}'+\mathbf{r},\sigma'}{}^\dagger b_{\mathbf{k}'\sigma'}) + k_y b_{\mathbf{k}\sigma} a_{\mathbf{k}+\mathbf{r},\sigma} \Big\} \frac{1}{-H_T+i\epsilon} a_{\mathbf{k}+\mathbf{r},\sigma}{}^\dagger b_{\mathbf{k}\sigma}{}^\dagger \right| \Phi_0 \right\rangle_L = 0. \quad (17)$$

The additional term, $k_y b_{\mathbf{k}\sigma} a_{\mathbf{k}+\mathbf{r},\sigma}$, in the summand in (17) is necessary since no scattering insertion can be made in the line bearing the pair insertion $a_{\mathbf{k}+\mathbf{r},\sigma}{}^\dagger b_{\mathbf{k}\sigma}{}^\dagger$.[15] Application of momentum conservation to the sum of diagrams obtained by making a scattering insertion in each internal line but the one bearing the pair insertion (these insertions lie on a cut which divides the diagram into earlier and later pieces), yields a term which just cancels

$$\sum_{r\neq 0} \frac{1}{r^2} \sum_{\mathbf{k}\sigma} k_y^2 \left\langle \Phi_0 \left| H_v \frac{1}{-H_T+i\epsilon} b_{\mathbf{k}\sigma} a_{\mathbf{k}+\mathbf{r},\sigma} \frac{1}{-H_T+i\epsilon} a_{\mathbf{k}+\mathbf{r},\sigma}{}^\dagger b_{\mathbf{k}\sigma}{}^\dagger \right| \Phi_0 \right\rangle_L .$$

Thus it remains to discuss

$$\mathcal{I}_{xy} = -2 \left\langle \Phi_0 \left| H_{ixy} \frac{1}{-H_T+i\epsilon} H_{ixy} \right| \Phi_0 \right\rangle_L$$

$$-2 \sum_{r\neq 0} \frac{1}{r^2} \sum_{\substack{\mathbf{k}\sigma\mathbf{k}'\sigma' \\ (\mathbf{k}\neq\mathbf{k}')}} k_y k_y' \left\langle \Phi_0 \left| H_v \frac{1}{-H_T+i\epsilon} (b_{\mathbf{k}'\sigma'} a_{\mathbf{k}'+\mathbf{r},\sigma'} - a_{-\mathbf{k}'-\mathbf{r},\sigma'}{}^\dagger b_{-\mathbf{k}',\sigma'}{}^\dagger) \frac{1}{-H_T+i\epsilon} a_{\mathbf{k}+\mathbf{r},\sigma}{}^\dagger b_{\mathbf{k}\sigma}{}^\dagger \right| \Phi_0 \right\rangle_L$$

$$-2 \sum_{r\neq 0} \frac{1}{r^2} \sum_{\substack{\mathbf{k}\sigma\mathbf{k}'\sigma' \\ (\mathbf{k}\neq\mathbf{k}')}} k_y k_y' \left\langle \Phi_0 \left| b_{\mathbf{k}\sigma} a_{\mathbf{k}+\mathbf{r},\sigma} \frac{1}{-H_T+i\epsilon} (a_{\mathbf{k}'+\mathbf{r},\sigma'}{}^\dagger b_{\mathbf{k}'\sigma'}{}^\dagger - b_{-\mathbf{k}'\sigma'} a_{-\mathbf{k}'-\mathbf{r},\sigma'}) \frac{1}{-H_T+i\epsilon} H_v \right| \Phi_0 \right\rangle_L , \quad (18)$$

namely, the contribution from the forward scattering, non-forward scattering, and annihilation (or creation) of pairs. We calculate these for momenta close to the Fermi surface in the limit of large volume.

Consider the contribution from forward pair scattering,

$$-2 \sum_{r\neq 0} \frac{1}{r^2} \sum_{\mathbf{k}\sigma} k_y^2 \langle \Psi_0 | \rho_{\mathbf{k},\mathbf{r}}{}^\sigma (E_0 - H_T + i\epsilon)^{-1} \rho_{\mathbf{k},\mathbf{r}}{}^{\sigma\dagger} | \Psi_0 \rangle, \quad (19)$$

[14] In the second and third terms of (15), only propagator corrections and pair annihilation (or creation) terms are $O(1/r^2)$ in the limit as $r \to 0$.

[15] See the discussion on diagrams of class (α) in I.

where $\rho_{\mathbf{k},\mathbf{r}}{}^{\sigma\dagger}=a_{\mathbf{k}+\mathbf{r},\sigma}{}^{\dagger}b_{\mathbf{k}\sigma}{}^{\dagger}$. In the limit of large volume and for $|r/k_F|\ll 1$, we have

$$\langle\Psi_0|\rho_{\mathbf{k},\mathbf{r}}{}^{\sigma}(E_0-H_T+i\epsilon)^{-1}\rho_{\mathbf{k},\mathbf{r}}{}^{\sigma\dagger}|\Psi_0\rangle\rightarrow G_{\mathbf{k},\mathbf{r}}{}^{\sigma}=[-W(\mathbf{k}+\mathbf{r}_>)+W(\mathbf{k}_<)+i\varepsilon]^{-1}\eta(k_F-k)\eta(|\mathbf{k}+\mathbf{r}|-k_F), \tag{20}$$

where $G_{\mathbf{k},\mathbf{r}}{}^{\sigma}$ is the physical pair-propagator, and $W(\mathbf{k}_>)$, the perturbed single-particle energy,[16]

$$W(\mathbf{k}_>)=E(\mathbf{k}_>)-i\tfrac{1}{2}\Gamma(\mathbf{k}_>). \tag{21}$$

This follows from the fact that diagrams with interactions between the particle and hole lines in the expansion of the pair-propagator (20) in perturbation theory are at most $O(L^{-3})$ relative to those with *no* interactions between the particle and hole.[16] In the limit $r\rightarrow 0$, we have

$$\lim_{r\rightarrow 0} G_{\mathbf{k},\mathbf{r}}{}^{\sigma}\rightarrow G_{\mathbf{k},\mathbf{r}}{}^{\sigma(0)}=-\frac{\mathbf{r}\cdot\hat{n}\eta(\mathbf{r}\cdot\hat{n})\delta(k-k_F)}{\mathbf{r}\cdot\mathbf{k}\{(1/k_F)[dE(k_F)/dk_F]\}}=-\eta(\mathbf{r}\cdot\hat{n})\delta(k-k_F)M^*/k_F, \tag{22}$$

where $1/M^*$ is the *exact* effective mass. Consequently, in the limit of large volume and in the limit $r\rightarrow 0$, the contribution of (19) takes the form

$$2M^*k_F\sum_{r\neq 0}\frac{1}{r^2}\sum_{\mathbf{k}\sigma}n_y{}^2\eta(\mathbf{r}\cdot\hat{n})\delta(k-k_F)=(M^*/M)(\mathcal{I}_{xy})_{\text{rigid}}. \tag{23}$$

Now in the same limits, the non-forward matrix element associated with pair scattering

$$\langle\Psi_0|\rho_{k',\mathbf{r}}{}^{\sigma'}(E_0-H_T+i\epsilon)^{-1}\rho_{\mathbf{k},\mathbf{r}}{}^{\sigma\dagger}|\Psi_0\rangle|_{(\mathbf{k}\neq\mathbf{k}')}$$

is given by the expression

$$G_{\mathbf{k}',\mathbf{r}}{}^{\sigma'}(\mathbf{k}'+\mathbf{r}_>,\mathbf{k}'_<|R|\mathbf{k}+\mathbf{r}_>,\mathbf{k}_<)\tilde{G}_{\mathbf{k},\mathbf{r}}{}^{\sigma}, \tag{24}$$

where $\tilde{G}_{\mathbf{k},\mathbf{r}}{}^{\sigma}$ satisfies the integral equation,

$$\tilde{G}_{\mathbf{k},\mathbf{r}}{}^{\sigma}=G_{\mathbf{k},\mathbf{r}}{}^{\sigma}+\sum_{\mathbf{k}'\sigma'}G_{\mathbf{k},\mathbf{r}}{}^{\sigma}\{(\mathbf{k}+\mathbf{r}_>,\mathbf{k}_<|R|\mathbf{k}'+\mathbf{r}_>,\mathbf{k}'_<)-(0|R|\mathbf{k}'+\mathbf{r}_>,\mathbf{k}'_<;\,\mathbf{k}+\mathbf{r}_>,\mathbf{k}_<)\}\tilde{G}_{\mathbf{k}',\mathbf{r}}{}^{\sigma'}. \tag{25}$$

$(\mathbf{k}'+\mathbf{r}_>,\mathbf{k}'_<|R|\mathbf{k}+\mathbf{r}_>,\mathbf{k}_<)$ is the matrix element for non-forward pair-scattering; in the limit $r\rightarrow 0$, it is the sum of all pair-scattering diagrams with ingoing and outgoing pairs on the Fermi surface. One obtains it by application of the generalized gradient theorem

$$\frac{d}{dk_1}\int_{k_2,\cdots,k_n<k_F;\;k_{n+1},\cdots,k_{2n}>k_F}d\mathbf{k}_2\cdots d\mathbf{k}_{2n}V^{(n)}(\mathbf{k}_1\mathbf{k}_2\cdots\mathbf{k}_{2n})\Big|_{(k_1=k_F)}$$

$$=\Big(-\sum_{i=2}^{n}+\sum_{i=n+1}^{2n}\Big)\int d\omega_{1i}\int[(\mathbf{k}_1\cdot\mathbf{k}_i)V^{(n)}(\mathbf{k}_1\mathbf{k}_2\cdots\mathbf{k}_{2n})]_{(k_1,k_i=k_F)},$$

to the sum of all *proper* self-energy diagrams. Note that as *defined* the matrix element $(\cdots|R|\cdots)$ does *not* contain any intermediate energy denominators which vanish in the limit $r\rightarrow 0$, corresponding to intermediate states with one or more pairs with net momentum r.[17] We shall write

$$\lim_{r\rightarrow 0}(\mathbf{k}+\mathbf{r}_>,\mathbf{k}_<|R|\mathbf{k}'+\mathbf{r}_>,\mathbf{k}'_<)=\langle\mathbf{k}\mathbf{k}'|\mathcal{K}|\mathbf{k}\mathbf{k}'\rangle|_{(k,k'=k_F)}. \tag{26}$$

The matrix element $(0|R|-\mathbf{k}'-\mathbf{r}_>,-\mathbf{k}'_<;\,\mathbf{k}+\mathbf{r}_>,\mathbf{k}_<)$ is the matrix element for pair-annihilation. In the limit

[16] D. F. Dubois, Ann. Phys. 7, 174 (1959).

[17] Note that by the process of disentanglement of Schrödinger diagrams, an R-matrix element with an intermediate state containing such a pair can be written as a surface integral of the product of two R-matrix elements of lower order, i.e.,

$$R^{(n)}\rightarrow\sum_{\text{Fermi surface}}R^{(n')}G^{(0)}R^{(n'')},\quad(n'+n''=n).$$

k, $k' \to k_F$, the pair-annihilation matrix element is related to the non-forward pair-scattering by the symmetry,

$$\lim_{r\to 0} (0|R|-\mathbf{k}'-\mathbf{r}_>,-\mathbf{k}'_<;\mathbf{k}+\mathbf{r}_>,\mathbf{k}_<)=\langle \mathbf{k},-\mathbf{k}'|\mathcal{K}|\mathbf{k},-\mathbf{k}'\rangle|_{(k,k'=k_F)}. \tag{27}$$

That is, we may reverse the direction of the outgoing pair lines *on the Fermi surface* in scattering graphs, to obtain the corresponding pair-annihilation graphs *on the Fermi surface.*

We consider now the terms in (18) associated with pair annihilation (or creation):

$$2\sum_{r\neq 0}\frac{1}{r^2}\sum_{\mathbf{k}\sigma\mathbf{k}'\sigma'} k_y k_y'\{-\langle\Phi_0|H_v(-H_T+i\epsilon)^{-1}\rho_{-\mathbf{k}',-\mathbf{r}}{}^{\sigma'\dagger}(-H_T+i\epsilon)^{-1}\rho_{\mathbf{k},\mathbf{r}}{}^{\sigma\dagger}|\Phi_0\rangle_L$$
$$-\langle\Phi_0|\rho_{\mathbf{k},\mathbf{r}}{}^{\sigma}(-H_T+i\epsilon)^{-1}\rho_{-\mathbf{k}',-\mathbf{r}}{}^{\sigma'}(-H_T+i\epsilon)^{-1}H_v|\Phi_0\rangle_L\}. \tag{28}$$

The doubling of terms is necessary so that the disentanglement[1] of the two ingoing (or outgoing) pairs have the proper weight. The difference in sign as compared to that of the scattering matrix elements is crucial.[18] The terms in the curly brackets in (28) may be expressed as

$$-G_{\mathbf{k}',\mathbf{r}}{}^{\sigma'}(0|R|-\mathbf{k}'-\mathbf{r}_>,\ -\mathbf{k}'_<;\ \mathbf{k}+\mathbf{r}_>,\ \mathbf{k}_<)\tilde{G}_{\mathbf{k}'\mathbf{r}}{}^{\sigma}. \tag{29}$$

The sum of (24) and (29) then yields schematically

$$[G\Delta\tilde{G}]_{\mathbf{k}',\mathbf{r};\ \mathbf{k},\mathbf{r}},$$

where (schematically)

$$\tilde{G}=G+G\Delta\tilde{G}$$

and

$$\Delta_{\mathbf{k}',\mathbf{r};\ \mathbf{k},\mathbf{r}}=(\mathbf{k}'+\mathbf{r}_>,\ \mathbf{k}'_<|R|\mathbf{k}+\mathbf{r}_>,\ \mathbf{k}_<)-(0|R|-\mathbf{k}'-\mathbf{r}_>,\ -\mathbf{k}'_<;\ \mathbf{k}+\mathbf{r}_>,\ \mathbf{k}_<).$$

These equations are *identical in structure*[19] to those of our random-phase treatment[2] of this problem; however *they are exact to all orders of perturbation theory.* In particular, expansion of the exact propagator in the limit of large volume

$$G_{\mathbf{k},\mathbf{r}}{}^{\sigma(0)}=-\eta(\mathbf{r}\cdot\hat{n})\delta(k-k_F)(M/k_F)\{1-(1-M/M^*)\}^{-1},$$

yields the terms

$$(\mathcal{I}_{xy})_{\text{rigid}}\{1+(1-M/M^*)+(1-M/M^*)^2+\ldots\},$$

which (except for the first) precisely cancel the surface integrals which result from the expansion

$$G\Delta\tilde{G}=G\Delta G+G\Delta G\Delta G+\ldots,$$

in the limit $r\to 0$. This cancellation is a consequence of the generalized gradient theorem. We therefore conclude that the inertial moment of a large, interacting many-fermion system, moving under periodic boundary conditions, has the rigid value to all orders in perturbation theory.

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[18] Again the combination $\langle \mathbf{kk}'|\mathcal{K}|\mathbf{kk}'\rangle-\langle\mathbf{k},-\mathbf{k}'|\mathcal{K}|\mathbf{k},-\mathbf{k}'\rangle$ permits us to extend averages over the *entire* Fermi surface.
[19] Specifically, Eqs. (23)–(25), and (29).