respectively. (It must be emphasized that if experimentally ϕ_r turns out to have a great deal more structure than is present in our choices, then the qualitative conclusions that we arrive at need have no bearing.) Equation (3) yields $-1.6<\tau(K_1^0)\Delta m<+0.4$, where the lower and higher limits are reached as the scattering length goes to zero and infinity, respectively. With this choice we are limited to positive scattering lengths since for the convergence of our integrals ϕ_r has to be finite and positive at high energy.[10] We utilized the additional freedom of Eq. (4) to fit the various $I=0$ scattering lengths which have been proposed so far[11–14] and at the same time yield $\tau(K_1^0)|\Delta m|=1.5$.[1] For negative scattering lengths the numerical search yields the following conclusions: (a) We can obtain $\tau(K_1^0)\Delta m=-1.5$ for a scattering length $a_0=-0.8\ \mu^{-1}$. The result is sensitive to the high-energy behavior of ϕ_r; 70% of the mass difference comes from center-of-mass pion momenta above 1.3 Bev/c. (b) It is impossible to obtain a possible mass difference for negative scattering lengths. For positive scattering lengths, however, the conclusions are: (a) We can obtain a negative mass difference which is again sensitive to the high-energy behavior, 50–90% of the mass difference comes from center-of-mass pion momenta above 1.3 Bev/c. (b) A positive mass difference can be obtained for $a_0=2.81\ \mu^{-1}$ but not for $a_0=1.96\ \mu^{-1}$; in this case 99% of the mass difference comes from pion momenta less than 0.5 Bev/c. Both choices for ϕ_r support the following qualitative conclusion that negative and small positive scattering lengths will give a negative Δm and a positive large scattering length can yield positive mass difference if ϕ_r is in the range of 0.5–1.8 for very large momenta (values appreciably less than this will give a negative mass difference).

All numerical work was performed on the IBM 650 at the Computation Center of the Pennsylvania State University.

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Two Pictures of the Strong-Coupling Method*

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In the strong-coupling method of the meson theory two different pictures have been used. One picture exhibits the isobaric nature of the meson-nucleon interaction by expressing the Hamiltonian in terms of the integrals of motion of the total system. It may be called the rotation picture. In the other picture the isobaric dependency comes out by splitting the total system into a free field system and a compound nucleon system, such that the interaction between them vanishes for infinite g. It may be called the splitting picture. These two pictures are compared with each other. The difference between them with regard to the scheme of the strong-coupling approximation method, especially with regard to the calculations of isobaric energy corrections and resonance scattering, is investigated.

1. INTRODUCTION

THE strong-coupling approximation method of the meson theory, introduced a long time ago by Wentzel,[1] Oppenheimer and Schwinger,[2] and Pauli and Dancoff,[3] has recently been extended to a somewhat higher degree of completeness. In course of these investigations two pictures have been introduced to carry through this line of approximation.

One of these two pictures has already been introduced by Wentzel[1] and has lately been extended by Serber and Dancoff,[4] Miyazima, Tati, and Tomonaga,[5] Kaufman,[6] Pais and Serber,[7] Nickle and Serber,[8] and Chun.[9] The main characteristic of this picture is to express the Hamiltonian by the operators of the important total integrals of motion of the system, such as the total isospin operator in the charged and the symmetric scalar fixed source theories and the total isospin and angular momentum operator together in

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the symmetric pseudoscalar case. This can be done by relating all operators of the system to coordinate systems rotating together with the meson-cloud of the nucleon in both isospin space and ordinary space. So we call this picture the rotation picture.

The other picture has already been introduced by Pauli and Dancoff[3] and Wentzel[10] and lately extended by the author.[11–13] Its main characteristic is that the total system becomes split into two partial systems, the compound-nucleon system and the free-meson system, with separate dynamical variables and separate integrals of motion. The compound-nucleon system consists of the bare nucleon and the meson cloud. This splitting is carried through in such a way to fulfill the requirement that the interaction between the two partial systems shall vanish for infinite g. This picture may be called the splitting picture. The aim of this paper is to show the relations between these two pictures and to compare the two kinds of approximation which can be obtained from them for the isobaric energy corrections and the meson-nucleon scattering.

2. RELATION BETWEEN THE TWO PICTURES

a. Starting Point

We explain this relation first by means of the charged scalar fixed source theory, which is the case most completely treated until now. The Hamiltonian is

$$H=\tfrac{1}{2}\sum_{\rho}\int\{p_{\rho\mathbf{k}}{}^{*}p_{\rho\mathbf{k}}+\epsilon^{2}q_{\rho\mathbf{k}}{}^{*}q_{\rho\mathbf{k}}\}d\mathbf{k}+\frac{g}{2\pi}\sum_{\rho}\tau_{\rho}\int v(k)q_{\rho\mathbf{k}}d\mathbf{k},\quad(1)$$

where $q_{\rho\mathbf{k}}$ and $p_{\rho\mathbf{k}}$ are the Fourier transforms of the canonical total meson field operators with the momentum vector $\mathbf{k}$, energy $\epsilon=(k^2+\kappa^2)^{\frac{1}{2}}$, and isospin space vector index ρ ($\rho=1, 2$), τ_ρ is the isospin vector operator of the bare nucleon, and $v(k)$ the Fourier transform of the source function.

Then the isospin space direction of the operator

$$\int u_{\mathbf{k}}q_{\rho\mathbf{k}}d\mathbf{k}\quad(2)$$

plays the role of a starting point of the method. An important part of the transformations of Pais and Serber was initially carried through without any specification of the c-number function $u_{\mathbf{k}}$. But in this paper we want to restrict ourselves to the case that $u_{\mathbf{k}}$ shall be the shape-function of the meson cloud as it has been introduced in II and (A), (B), (C) [8,11–13]; the formulations given in these papers are most suitable to see the relations between the two pictures of the strong-coupling theory. The direction of (2) in the isospin space is then the direction of the self-field vector.

b. Rotating Picture

The procedure of the rotation picture is to introduce a new coordinate system in the isospin-space of which the $1^{\sim}$ axis has the direction of (2), and to relate all operators to that new coordinate system. Then the integrated-over-$u_{\mathbf{k}}$ part of the new meson field operator components is the amount of (2) or the canonical conjugate of this amount, and is in the $1^{\sim}$ direction. But there is still an integrated-over-$u_{\mathbf{k}}$ part of the canonical conjugate field operator which is in the $2^{\sim}$ direction. This is just the part containing the total field isospin operator as the operator which produce that rotation of the total field given by the angle φ between the 1 and $1^{\sim}$ axes of the old and new systems. This part must be separated in order to bring out the dependence of the total Hamiltonian on the total isospin operator. But this separation has the consequence that the field operators in the $2^{\sim}$ direction fulfill the commutation relations

$$i[\tilde{p}_{2\mathbf{k}},\tilde{q}_{2\mathbf{k}'}]=\delta(\mathbf{k}-\mathbf{k}')-\frac{u_{\mathbf{k}}u_{\mathbf{k}'}}{\int|u_{\mathbf{k}}|^{2}d\mathbf{k}},\quad(2\mathrm{a})$$

and no longer fulfill the canonical commutation relations, which remain valid in the $1^{\sim}$ direction only. The new field operators do no longer contain any φ dependence; this has been removed by means of the rotation and the last separation. Therefore all the quantities composing the Hamiltonian measured in the rotating coordinate system commute with the isospin operator of the total system, which for this reason can be replaced by the quantum numbers of the total isospin or charge. All details of this procedure can be seen in the paper of Pais and Serber.[7] We can collect the single steps of their procedure into the formula

$$\begin{aligned}q_{\rho\mathbf{k}}&=\sum_{\rho'}s_{\rho'\rho}\tilde{q}_{\rho'\mathbf{k}},\\ p_{\rho\mathbf{k}}&=\sum_{\rho'}s_{\rho'\rho}\tilde{p}_{\rho'\mathbf{k}}+u_{\mathbf{k}}\frac{\langle s_{2\rho}(I-\Sigma-\frac{1}{2}\tau_3)\rangle}{\int u_{\mathbf{k}}\tilde{q}_{1\mathbf{k}}d\mathbf{k}},\end{aligned}\quad(3)$$

where

$$s_{\rho'\rho}\equiv\begin{pmatrix}\cos\varphi & \sin\varphi\\ -\sin\varphi & \cos\varphi\end{pmatrix}\quad(4)$$

is the orthogonal transformation leading from the fixed to the rotating coordinate system, $s_{1\rho}$ being the direction of (2), and where I is the total isospin operator, τ_3 the 3-component of the bare nucleon isospin operator and

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Σ the isospin operator formed by the $\tilde{q}_{\rho\mathbf{k}}$, which is called by Nickle and Serber the free-field isospin operator. The total Hamiltonian is then expressed by the new operators according to (3) and (4).

A main requirement of the strong-coupling theory is mostly considered to be that it diagonalizes the coupling term of the original Hamiltonian. This has been done in (II) by means of the unitary transformation

$$S=\exp\left[\tfrac{1}{2}i\tau_3\tan^{-1}\left(\int v(k)\tilde{q}_{2\mathbf{k}}d\mathbf{k}\Big/\int v(k)\tilde{q}_{1\mathbf{k}}d\mathbf{k}\right)\right], \quad (5)$$

which rotates the bare nucleon isospin to the direction of the vector $\int v(k)\tilde{q}_{\rho\mathbf{k}}d\mathbf{k}$ in the coupling term, and affects also the $\tilde{p}_{\rho\mathbf{k}}$ operators.

Then the next step in (I) and (II) is to introduce the translation

$$\tilde{q}_{1\mathbf{k}}\rightarrow\tilde{q}_{1\mathbf{k}}+u_{\mathbf{k}}, \quad (6)$$

which doesn't affect either the other field operators or the operators I and Σ because of

$$\int u_{\mathbf{k}}\tilde{p}_{2\mathbf{k}}d\mathbf{k}=\int u_{\mathbf{k}}\tilde{q}_{2\mathbf{k}}d\mathbf{k}=0. \quad (7)$$

Finally, the terms of the resultant Hamiltonian containing no $\tilde{q}_{\rho\mathbf{k}}$ and $\tilde{p}_{\rho\mathbf{k}}$ operators are collected as the static part of the Hamiltonian. By minimizing this static part, Nickle and Serber[8] get a determination of $u_{\mathbf{k}}$ with a result of which the lowest order approximation in $1/g$ is

$$u_{\mathbf{k}}=\frac{g}{2\pi}\frac{v(k)}{\epsilon^2}. \quad (8)$$

In the remaining $\tilde{q}_{\rho\mathbf{k}}$, $\tilde{p}_{\rho\mathbf{k}}$-dependent part of the Hamiltonian the quadratic or bilinear terms of order g^0 are collected together as a free-field part of the Hamiltonian which is diagonalized by introducing creation and destruction operators:

$$\tilde{q}_{1\mathbf{k}}=(2\epsilon)^{-\frac{1}{2}}(\tilde{a}_{1\mathbf{k}}+\tilde{a}_{1\mathbf{k}}^*);\quad \tilde{p}_{1\mathbf{k}}=i\left(\frac{\epsilon}{2}\right)^{\frac{1}{2}}(\tilde{a}_{1\mathbf{k}}^*-\tilde{a}_{1\mathbf{k}}),$$

$$\tilde{q}_{2\mathbf{k}}=\int(2\epsilon_0)^{-\frac{1}{2}}(v_{\mathbf{kk}_0}^*\tilde{a}_{2\mathbf{k}_0}+v_{-\mathbf{kk}_0}\tilde{a}^*_{2\mathbf{k}_0})d\mathbf{k}_0; \quad (9)$$

$$\tilde{p}_{2\mathbf{k}}=\int i\left(\frac{\epsilon_0}{2}\right)^{-\frac{1}{2}}(v_{\mathbf{kk}_0}\tilde{a}_{2\mathbf{k}_0}^*-v_{\mathbf{kk}_0}^*\tilde{a}_{2\mathbf{k}_0})d\mathbf{k},$$

with

$$v_{\mathbf{kk}_0}=\delta(\mathbf{k}-\mathbf{k}_0)+\frac{v(k)v_{0\mathbf{k}_0}}{\epsilon^2-\epsilon_0^2\mp i\alpha};$$
$$v_{0\mathbf{k}_0}=-v(k)\Big/\epsilon_0^2\int\frac{[v(k)]^2}{\epsilon^2(\epsilon^2-\epsilon_0^2\mp i\alpha)}d\mathbf{k}, \quad (10)$$

fulfilling the relation

$$\int u_{\mathbf{k}}v_{\mathbf{kk}_0}d\mathbf{k}=0, \quad (11)$$

in agreement with (7). So the Hamiltonian can finally be written in the form

$$H=H_{\text{stat}}+H_{\text{free}}+\tilde{H}^i, \quad (12)$$

$$H_{\text{stat}}=\frac{I^2-I\tau_3+\frac{1}{4}}{2V}+\frac{g^2}{2a}(\tau_1+\tfrac{1}{2});$$

$$\frac{1}{a}=\frac{2}{\pi}\int\frac{k^2[v(k)]^2}{\epsilon^2}dk;\quad V=\int|u_{\mathbf{k}}|^2d\mathbf{k}, \quad (13)$$

$$H_{\text{free}}=\sum_\rho\int\langle\tilde{a}_{\rho\mathbf{k}_0}^*\tilde{a}_{\rho\mathbf{k}_0}\rangle d\mathbf{k}_0. \quad (14)$$

The terms of $\tilde{H}^i$ can be seen in detail in (I) and (II).

In these papers also the case of an I-dependent cloud function is treated, which can be seen by replacing in (8) the κ^2 of ϵ^2 by $\kappa'^2=\kappa^2-(I^2-\frac{1}{4})/V^2$. This represents the full result obtained in (II) by minimizing H_{stat}, showing the spreading out of the cloud function for decreasing g by means of isobaric excitation. This I dependency then also appears in $v_{\mathbf{kk}_0}$ which already can be seen by (II). But nevertheless these two functions remain c-number functions because I as the total isospin commutes in this rotating picture with all quantities in the Hamiltonian.

c. Splitting Picture

In the splitting picture we remain in the fixed coordinate system of the isospin space. Then the angle φ is no longer related to the total system. But by giving the direction of the self-field vector (2), φ has instead the meaning of a dynamical variable of the compound nucleon, the partial system composed by the bare nucleon and the surrounding self-field. The full expression by the original meson field variables of the total system (1) is according to (2) and (4) given by

$$s_{1\rho}=\int u_{\mathbf{k}}q_{\rho\mathbf{k}}d\mathbf{k}\Big/\left(\sum_\rho\left(\int u_{\mathbf{k}}q_{\rho\mathbf{k}}d\mathbf{k}\right)^2\right)^{\frac{1}{2}}. \quad (15)$$

Instead of the total isospin operator I the canonical conjugate to φ is then, of course, the compound-nucleon isospin operator $\mathcal{T}$ which has to be the introduced by

$$I=\mathcal{T}+I^f. \quad (16)$$

In (16),

$$I^f=\frac{1}{i}\int(a_{1\mathbf{k}_0}^*a_{2\mathbf{k}_0}-a_{2\mathbf{k}_0}^*a_{1\mathbf{k}_0})d\mathbf{k}_0 \quad (17)$$

is formed by the creation and destruction operators $a_{\rho o k o}$, $a_{\rho o k o}{}^*$ measured in the fixed isospin space system, and related to the $\tilde{a}_{\rho k}, \tilde{a}_{\rho k}{}^*$ of the rotating picture in (9) according to

$$\tilde{a}_{\rho k o}=\sum_{\rho 0} s_{\rho\rho o}a_{\rho o k o}. \tag{18}$$

By introducing (16), (17), and (18) into the formulas (3), (9), and (10) we get

$$q_{\rho k}=q_{\rho k}'+u_k s_{1\rho};$$

$$p_{\rho k}=p_{\rho k}'+u_k\frac{\langle s_{2\rho}(\mathcal{T}+I^f-I'-\frac{1}{2}\tau_3)\rangle}{\sum_{\rho'} s_{1\rho'}\int u_k q_{\rho' k'}d\mathbf{k}'} \tag{19}$$

with

$$q_{\rho k}'=\sum_{\rho 0}\int(2\epsilon_0)^{-\frac{1}{2}}(v_{kko,\rho\rho o}{}^*a_{\rho o k o}+v_{-kko,\rho\rho o}a_{\rho o k o}{}^*)d\mathbf{k}_0;$$

$$p_{\rho k}'=\sum_{\rho 0}\int i\left(\frac{\epsilon_0}{2}\right)^{\frac{1}{2}}(v_{kko,\rho\rho o}a_{\rho o k o}{}^*-v_{-kko,\rho\rho o}{}^*a_{\rho o k o})d\mathbf{k}_0, \tag{20}$$

and

$$v_{kko,\rho\rho o}=\delta(\mathbf{k}-\mathbf{k}_0)\delta_{\rho\rho o}+\frac{v(k)v_0k_o}{\epsilon^2-\epsilon_0{}^2\mp i\alpha}s_{2\rho}s_{2\rho o}, \tag{21}$$

where I' is the isospin operator formed by $q_{\rho k}'$ and $p_{\rho k}'$ of which it can be shown that $I'=\Sigma$. Instead of (2a) we then have

$$i[p_{\rho' k'}{}',q_{\rho k}{}']=\delta(\mathbf{k}'-\mathbf{k})\delta_{\rho'\rho}-\frac{u_{k'}u_k}{\int|u_k|^2d\mathbf{k}}s_{2\rho'}s_{2\rho}. \tag{22}$$

In order to have $a_{\rho o k o}$, $a_{\rho o k o}{}^*$, and $s_{\rho'\rho}$, $\mathcal{T}$, $\boldsymbol{\tau}$ as operators of two independent systems the $a_{\rho o k o}$, $a_{\rho o k o}{}^*$ have to commute with the $s_{\rho'\rho}$, $\mathcal{T}$, $\boldsymbol{\tau}$. This is fulfilled and all the commutation relations of the operators in (19) and (20) are valid if $v_{kko,\rho\rho o}$ fulfills the relations

$$\sum_{\rho 0}\int v_{k'ko,\rho'\rho o}v_{kko,\rho\rho o}{}^*d\mathbf{k}_0=\delta(\mathbf{k}'-\mathbf{k})\delta_{\rho'\rho}-\frac{u_{k'}u_k}{\int|u_k|^2d\mathbf{k}}s_{2\rho'}s_{2\rho}, \tag{23}$$

$$\sum_{\rho}\int v_{kko',\rho\rho o'}{}^*v_{kko,\rho\rho o}d\mathbf{k}=\delta(\mathbf{k}_0'-\mathbf{k}_0)\delta_{\rho o'\rho o}, \tag{24}$$

as has been shown in detail in (A). Contrary to this, the $\tilde{a}_{\rho k o}$ [Eq. (18)] of the rotating picture do not commute with $\mathcal{T}$ because they contain the $s_{\rho'\rho}$. But instead, it can be seen by (18) that the $\tilde{a}_{\rho k o}$ are invariants in regard to the isospin space, so that they commute with the total isospin according to (16), as is required by the rotating picture. By means of (23) and (24) the inversion of (19), (20) can also be derived, which gives[11]

$$a_{\rho o k o}=\left(\frac{\epsilon_0}{2}\right)^{\frac{1}{2}}\sum_{\rho}\int v_{kko,\rho\rho o}q_{\rho k}d\mathbf{k}+i(2\epsilon_0)^{-\frac{1}{2}}\times\sum_{\rho}\int\langle v_{-kko,\rho\rho o}{}^*p_{\rho k}\rangle d\mathbf{k}-\left(\frac{\epsilon_0}{2}\right)^{\frac{1}{2}}u_{ko}s_{1\rho o}. \tag{25}$$

Then by means of (25), (21), (17), (16), and (15), $\mathcal{T}$ can be expressed completely by the original field operators.

So by means of the substitution (19), (20), and (21) we have split the total system into two systems: the compound nucleon system with the operators $s_{\rho'\rho}$, $\mathcal{T}$, and $\boldsymbol{\tau}$, and the free-field system with the operators $a_{\rho o k o}$ amd $a_{\rho o k o}{}^*$. Especially the total isospin of the system has been split exactly according to (16) by means of the substitution (19)–(21).

After introducing the substitution (19), (20), and a similar $\boldsymbol{\tau}$ rotation[11,12] to that given by (5), the Hamiltonian of the splitting picture has the form

$$H=H^{0c}+H^{0f}+H^i, \tag{26}$$

where

$$H^{0c}=\frac{\mathcal{T}^2-\mathcal{T}\tau_3+\frac{1}{4}}{2V}+\frac{g^2}{2a}(\bar{\tau}_1+\tfrac{1}{2}),\quad \bar{\tau}_1=(\mathbf{s}_1,\boldsymbol{\tau}), \tag{27}$$

$$H^{0f}=\sum_{\rho 0}\int\langle a_{\rho o k o}{}^*a_{\rho o k o}\rangle d\mathbf{k}_0, \tag{28}$$

and the most important terms of H^i can be seen in (A), (B), and (C). In these papers the results (8) and (21) for u_k and $v_{kko,\rho\rho o}$ are obtained by the sole requirement that the terms of the strong-coupling perturbation series shall vanish for infinite g. This replaces the three requirements in the rotating picture, namely minimizing the static part, diagonalizing the coupling term, and introducing and using the normal modes. But the spreading out of the cloud function treated in (I) and (II), as remarked after Eq. (14), cannot yet be treated in the splitting picture because u_k, being then $\mathcal{T}$ dependent, would no longer be a c-number function as in (I) and (II). For this purpose a more extended transformation than (19) and (20) would be introduced —which has not yet been done. Therefore, we compare here the two pictures only for the case without taking account of the spreading out of the cloud function.

3. ISOBARIC ENERGY CORRECTIONS

a. Rotation Picture

In the rotation picture the zero-order approximation eigenstates of the problem are the eigenstates of $H_{\text{stat}}+H_{\text{free}}$ according to (12)–(14). The zero-order approximation eigenvalues to order $1/g^2$ in the coupling constant are given by

$$(H_{\text{stat}}+H_{\text{free}})\Theta(I',\mathbf{k}_{01},\rho_1\cdots\mathbf{k}_{0n},\rho_n) = \left[\frac{g^2}{2a}\begin{pmatrix}\frac{3}{2}\\-\frac{1}{2}\end{pmatrix}+\frac{I'^2-\frac{1}{4}}{2V}+\sum_n \epsilon_n\right] \times\Theta(I',\mathbf{k}_{01},\rho_1\cdots\mathbf{k}_{0n},\rho_n) \quad (29)$$

the τ_3 term giving only $1/g^4$ corrections and the zero-point energy being omitted. The orthogonal system of these eigenstates $\Theta(I',\mathbf{k}_{01},\rho_1\cdots\mathbf{k}_{0n},\rho_n)$ with the total isospin I', containing n mesons with wave numbers $\mathbf{k}_{01}\cdots\mathbf{k}_{0n}$ and isospin polarizations $\rho_1\cdots\rho_n$, can be constructed by means of the production and destruction operators $\tilde{a}_{\rho\mathbf{k}\text{o}}^*$, $\tilde{a}_{\rho\mathbf{k}\text{o}}$ introduced by (9). Since these operators $\tilde{a}_{\rho\mathbf{k}\text{o}}^*$, $\tilde{a}_{\rho\mathbf{k}\text{o}}$ commute with the total isospin operator I, they don't change the total isospin quantum number I' if they act on $\Theta(I',\mathbf{k}_{01},\rho_1\cdots\mathbf{k}_{0n},\rho_n)$. So if the meson charge is increased or decreased by applying $\tilde{a}_{\pm\mathbf{k}\text{o}}^*=(1/\sqrt{2})(\tilde{a}_{1\mathbf{k}\text{o}}^*\pm i\tilde{a}_{2\mathbf{k}\text{o}}^*)$ on $\Theta(I',\mathbf{k}_{01},\rho_1\cdots\mathbf{k}_{0n},\rho_n)$, the compound nucleon charge is decreased or increased automatically. But this cannot be seen explicitly in the zero-order approximation of the rotation picture because meson charge and compound nucleon charge are not good quantum numbers there. To investigate this question in the rotation picture the first order eigenvalue corrections have to be considered. In this connection the most important term of the $\tilde{H}^i$ expression of Pais and Serber and Nickle and Serber is

$$\tilde{H}_0{}^i=-2I\Sigma^{02}/2V, \quad (30)$$

which also is of order $1/g^2$ as well as the I'^2 term of the zero-order eigenvalue (29). Σ^0 is the part of Σ belonging only to the $\delta(\mathbf{k}-\mathbf{k}_0)$ part of $v_{\mathbf{k}\mathbf{k}\text{o}}$ (10),

$$\Sigma^0=I^f. \quad (30')$$

Taking into account the first-order eigenvalue corrections of (30), we obtain the expectation value between the zero-order eigenstates:

$$\langle I',\mathbf{k}_{01},\rho_1\cdots\mathbf{k}_{0n},\rho_n|H_{\text{stat}}+H_{\text{free}}+\tilde{H}_0{}^i|I',\mathbf{k}_{01},\rho_1\cdots\mathbf{k}_{0n},\rho_n\rangle =\frac{g^2}{2a}\begin{pmatrix}\frac{3}{2}\\-\frac{1}{2}\end{pmatrix}+\frac{T'^2-\frac{1}{4}}{2V}+\sum_n \epsilon_n. \quad (31)$$

The main difference between (31) and (30) is that the expectation value $\langle\Sigma^0\rangle$, being the meson charge, has been subtracted from I' so that I' is replaced by the compound nucleon charge

$$T'=I'-\langle\Sigma^0\rangle. \quad (32)$$

It is characteristic for the rotation picture that the dependence of the eigenvalues on the compound nucleon isospin or on the free meson charge initially appears in the first-order perturbation approximation, not in the zero-order approximation. For meson vacuum states $|0I'\rangle$ the first-order contribution of (30), and so the difference between (29) and (31), vanishes because of $\langle\Sigma^0\rangle=0$. But if mesons are present, so that $\langle\Sigma^0\rangle\neq0$, then (30) plays an important role as can be seen by the calculation of Pais and Serber of scattering including the thresholds for isobaric excitation.

To see the contributions of (30) to the higher order isobaric energy corrections, we begin with the $1/g^4$ corrections which are composed, according to Nickle and Serber[8] [see (43)–(46) in their paper], of the terms

$$\tilde{H}_{g-1}{}^i=(g/4\pi)qxy^2, \quad (33)$$

and

$$\tilde{H}_{g-3}{}^i=-[(I^2-\tfrac{1}{4})/V^2]\int u_{\mathbf{k}}\tilde{q}_{1\mathbf{k}}d\mathbf{k}, \quad (34)$$

where

$$x=\frac{1}{q}\int v(k)\tilde{q}_{1\mathbf{k}}d\mathbf{k};\quad y=\frac{1}{q}\int v(k)\tilde{q}_{2\mathbf{k}}d\mathbf{k};\qquad q=\int u_{\mathbf{k}}v(k)d\mathbf{k}. \quad (35)$$

By inserting these terms into the second-order perturbation correction expression,

$$\Delta^{(2)}\epsilon_{I'}=-\sum_\rho\int\frac{|\langle0,I'|\tilde{H}^i|\mathbf{k},\rho,I'\rangle|^2}{\epsilon}d\mathbf{k}, \quad (36)$$

Nickle and Serber[8] obtained, besides the I'-independent contributions coming from (33), only the I'-dependent $1/g^4$ contribution

$$\frac{g}{4\pi}\frac{(I'^2-\frac{1}{4})}{V^2}\langle y^2\rangle\int\frac{u_{\mathbf{k}}v(k)}{\epsilon^2}d\mathbf{k}=\frac{1}{2V}(I'^2-\tfrac{1}{4})\langle y^2\rangle, \quad (37)$$

which is mixed by (33) and (34). According to (36), this is a correction to the meson vacuum state with the one-meson state as intermediate state. Because of the commutativity of the production and destruction operators $\tilde{a}_{\rho\mathbf{k}}^*$, $\tilde{a}_{\rho\mathbf{k}}$ of the rotation picture with the total isospin operator I, the transition from the meson vacuum to the intermediate one-meson state in (36) does not change the isobaric total isospin quantum number I' of the zero-order states so that only the energy ϵ of the produced meson but no isobaric energy difference appears in the denominator of (36).

It can furthermore be shown that (30) does not give any contribution to the $1/g^4$ isobaric energy correction. Formally, this contribution could only come from the

third-order perturbation theoretical expression,

$$\Delta^{(3)}\epsilon_{I'}=\sum_{\rho,\rho'}\int\int\frac{\langle 0,I'|\tilde{H}_{g-1}{}^{i}|\mathbf{k},\rho,I'\rangle\langle I',\mathbf{k},\rho|\tilde{H}_{0}{}^{i}|\mathbf{k}',\rho',I'\rangle\langle I',\mathbf{k}',\rho'|\tilde{H}_{g-1}{}^{i}|0,I'\rangle}{\epsilon\epsilon'}d\mathbf{k}d\mathbf{k}',\tag{38}$$

to which, besides (30), only (33) contributes. But the matrix elements of (33) occurring in (38) can only lead to intermediate states in (38) with $\rho=1$ having a resultant meson charge equal to zero: also

$$\langle 1|\Sigma|1\rangle=0.\tag{39}$$

Therefore (38) does not give any I'-dependent $1/g^4$ contribution.

But to higher order than $1/g^4$ in the reciprocal coupling constant, Eq. (30) gives I'-dependent contributions from higher order perturbation theoretical expressions; e.g., to $1/g^6$, from

$$\Delta^{(4)}\epsilon_{I'}=-\int\int\int\frac{\langle 0,I'|\tilde{H}_{g-1}{}^{i}|\mathbf{k},1,I'\rangle\langle\mathbf{k},1,I'|\tilde{H}_{0}{}^{i}|\mathbf{k}',2,I'\rangle\langle I',\mathbf{k}',2|\tilde{H}_{0}{}^{i}|\mathbf{k}'',1,I'\rangle\langle\mathbf{k}'',1,I'|\tilde{H}_{g-1}{}^{i}|0I'\rangle}{\epsilon\epsilon'\epsilon''}d\mathbf{k}d\mathbf{k}'d\mathbf{k}'',\tag{40}$$

because then

$$\langle 1|\Sigma|2\rangle\neq 0$$

is the essential contribution and (39) does not occur. This is also the case for all even-order perturbation theoretical expressions, which are of the type of (40). If summed they give a geometrical series in powers of I^2/V^2, which is the total contribution of (30) to the isobaric energy correction. The meaning of this result will be seen in the following by considering the splitting picture.

b. Splitting Picture

In the splitting picture the zero-order-approximation eigenstates of the problem are the eigenstates of $H^{0c}+H^{0f}$ according to (26)–(28). The zero-order approximation eigenvalues to $1/g^2$ are given by

$$(H^{0c}+H^{0f})\Theta(T',\mathbf{k}_{01},\rho_{01}\cdots\mathbf{k}_{0n},\rho_{0n})=\left[\frac{g^2}{2a}\begin{pmatrix}\frac{3}{2}\\-\frac{1}{2}\end{pmatrix}+\frac{T'^2-\frac{1}{4}}{2V}+\sum_n\epsilon_n\right]\times\Theta(T',\mathbf{k}_{01},\rho_{01}\cdots\mathbf{k}_{0n},\rho_{0n}).\tag{41}$$

(41) differs from (29) by having the compound-nucleon isospin quantum number T' instead of total isospin quantum number I', and the n mesons with wave numbers $\mathbf{k}_{01}\cdots\mathbf{k}_{0n}$ and isospin polarization $\rho_{01}\cdots\rho_{0n}$ are created and destroyed by the creation and destruction operators $a_{\rho_0\mathbf{k}_0}{}^*$, $a_{\rho_0\mathbf{k}_0}$ instead by $\tilde{a}_{\rho\mathbf{k}_0}{}^*$, $\tilde{a}_{\rho\mathbf{k}_0}$ [see (18)]. Since, contrary to the $\tilde{a}_{\rho\mathbf{k}_0}{}^*$, $\tilde{a}_{\rho\mathbf{k}_0}$, the $a_{\rho_0\mathbf{k}_0}{}^*$, $a_{\rho_0\mathbf{k}_0}$ commute with the compound-nucleon isospin operator $\mathcal{T}$, instead with the total isospin operator I, they do not change the compound-nucleon isospin quantum number T' but instead the total isospin quantum number I' if acting on the eigenstates of (41). This shows the independence of the free-meson system and the compound-nucleon system in the splitting picture.

Now the eigenvalues in (41) are identical with the right-hand side of (31). That means that the first-order perturbation theoretical contributions of (30) of the rotation picture are included in the zero-order approximation of the splitting picture. The reason for that is that the corresponding parts of (30) are no longer contained in the interaction part H^i of the splitting picture [see (26)] but are instead included in the zero-order compound-nucleon part H^{0c} [see (27)] of the splitting picture by means of the subtraction (19) according to

$$\mathcal{T}=I-I^f,\quad[\text{see }(30')].\tag{42}$$

This has the following consequence for the calculations of the isobaric energy corrections:

In the expressions (35) one has to consider in the splitting picture, instead of the $\tilde{a}_{\rho\mathbf{k}_0}{}^*$, $\tilde{a}_{\rho\mathbf{k}_0}$, the right-hand side of (18). Then the change of the compound-nucleon isospin quantum number T', necessary for total isospin conservation, is done by the compound-nucleon operators $s_{\rho'\rho}$. Therefore, in the second order perturbation theoretical expression for the isobaric energy corrections formed in the representation relative to zero-order eigenstates (41) of the splitting picture, we have a change in T' between the meson vacuum state and the one-meson intermediate state. But according to (41) this has the consequence that, contrary to (36), isobaric energy differences must appear in the perturbation-theoretical denominators of the splitting picture so that, instead of (36), we have

$$\Delta\epsilon_{I'}=-\sum_{\rho_0,T'}\int\frac{|\langle 0,I'|H^i|\mathbf{k}_0,\rho_0,T'\rangle|^2}{\epsilon_0-B_{I'T'}}d\mathbf{k}_0,\tag{43}$$

with

$$B_{I'T'}=(I'^2-T'^2)/2V.\tag{44}$$

To compare (43) with the results of the rotating picture, we expand

$$\frac{1}{\epsilon_0-B_{I'T'}}=\frac{1}{\epsilon_0}+\frac{B_{I'T'}}{\epsilon_0{}^2}+\frac{(B_{I'T'})^2}{\epsilon_0{}^3}+\cdots.\tag{45}$$

Then if the expansion (45) is introduced into (43), the first term gives the I'-dependent $1/g^4$ contributions (36) and (37). From the second term the $1/g^4$ contribution is identical with (38), of which the I'-dependent part had vanished because of (39). In the splitting picture, this vanishing of the I'-dependent $1/g^4$ part of the second term can be shown by considering the summation over T' which reduces to

$$\begin{aligned}\sum_{T'}(I'|s_{1\rho 0}|T')(T'^2-I'^2)(T'|s_{1\rho 0}|I') &= |(I|s_{1\rho 0}|I+1)|^2(2I+1)\\ &\quad +|(I|s_{1\rho 0}|I-1)|^2(-2I+1). \quad (46)\end{aligned}$$

Here we have used

$$T'=I'\pm 1, \quad (47)$$

which follows as a property of the matrix elements of $s_{1\rho}$, which are

$$\left(I'\left|\begin{matrix}s_{1\rho}\\ s_{1\rho}\end{matrix}\right|T'\right)=\frac{1}{2\pi}\int\left\{\begin{matrix}\frac{1}{2}(e^{i\varphi}+e^{-i\varphi})\\ \frac{1}{2}(e^{i\varphi}-e^{-i\varphi})\end{matrix}\right\}e^{i(T'-I')\varphi}d\varphi =\frac{1}{2}\left\{\begin{matrix}\delta_{I',T'+1}+\delta_{I',T'-1}\\ \delta_{I',T'+1}-\delta_{I',T'-1}\end{matrix}\right\}. \quad (48)$$

So the $(I'|s_{1\rho}|T')$ are independent of I' and T'. For this reason the I'-dependent contributions to (46), which are linear in I', cancel, so that the vanishing of the I'-dependent $1/g^4$ term of the second term of (45) is proved. This corresponds to (39).

The third term of the expansion (45) gives the same contribution as (40). Its I'^2 dependency to order $1/g^6$ follows in the splitting picture from the fact that in the sum over T', which now reduces to

$$\begin{aligned}\sum_{T'}(I'|s_{1\rho}|T')(T'^2-I'^2)^2(T'|s_{1\rho}|I') &= |(I'|s_{1\rho}|I'+1)|^2(4I'^2+4I'+1)\\ &\quad +|(I'|s_{1\rho}|I'-1)|^2(4I'^2-4I'+1), \quad (49)\end{aligned}$$

a I'^2 dependency remains.

In this way it is evident that the perturbation series of the rotation picture produced by the interaction term (30) as shown from (33) to (40) can be considered as consisting of expansions like (45) introduced into (43). But in the splitting picture these contributions are obtained in an unexpanded form like (43), which is also valid for $B_{I'T'}$ of the order of magnitude of ϵ_0, when the expansion is no longer valid. This would be important for resonance-scattering damping calculations where, for the resonance energy correction and the resonance width, the following expression has been obtained[11]:

$$\Delta^R\epsilon_{I'}=-\sum_{\rho_0,T'}\int\frac{|(0,I'|H^W|\mathbf{k}_0,\rho_0,T')|^2}{(1+a_{T'})(\epsilon_0-\omega_{T'}-i\alpha)}d\mathbf{k}_0, \quad (50)$$

where[14] $a_{T'}$ is a certain coefficient and $\omega_{T'}$ is given by

$$\omega_{T'}=\epsilon_N-\frac{B_{T'\frac{1}{2}}+\Delta\epsilon_T-\Delta\epsilon_{\frac{1}{2}}}{1+a_{T'}}, \quad (51)$$

with the energy ϵ_N of the outgoing meson $B_{T\frac{1}{2}}$ see (44) and $\Delta\epsilon_T-\Delta\epsilon_{\frac{1}{2}}$ is a certain isobaric energy correction.[14] Now by considering (50) it is obvious that $1/(\epsilon_0-\omega_{T'}-i\alpha)$, which corresponds to $1/(\epsilon_0-B_{I'T'})$ of (43), cannot be expanded as in (46) because the expansion corresponding to (46) would be

$$\frac{1}{\epsilon_0-\omega_{T'}-i\alpha}=\frac{1}{\epsilon_0-\epsilon_N}+\frac{B_{T'\frac{1}{2}}+\Delta\epsilon_{T'}-\Delta\epsilon_{\frac{1}{2}}}{(\epsilon_0-\epsilon_N)^2}+\frac{(B_{T'\frac{1}{2}}+\Delta\epsilon_{T'}-\Delta\epsilon_{\frac{1}{2}})^2}{(\epsilon_0-\epsilon_N)^3}+\cdots,$$

in which the denominator surely will have a zero within the space of the integration.

From this it seems necessary to conclude that only the splitting picture and not the rotating picture can be used for such damping calculations to give the resonance width. For $T=\frac{1}{2}$ the denominator of (50) has a zero for $\epsilon=\epsilon_N$; the integral (50) then has to be carried through according to

$$\frac{1}{\epsilon_0=\omega_{T'}-i\alpha}=P\frac{1}{\epsilon_0-\omega_{T'}}+i\pi\delta(\epsilon_0-\omega_{T'}),$$

where the principal-value integral gives a resonance energy correction and the imaginary δ-function part gives the resonance width.[15]

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[14] a_T, $\Delta\epsilon_T-\Delta\epsilon_{\frac{1}{2}}$ are given in (A).

[15] The detailed derivation and a numerical calculation are given in (A).