Magnetic Moments of Mirror Nuclei*

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The sum of the magnetic dipole moments for pairs of mirror nuclei, as calculated with intermediate-coupling functions in the $1p$ shell, is found to be very insensitive to the degree of spin-orbit coupling. This property can be understood as being due to the rotational nature of the wave functions, and can also be interpreted in terms of wave functions from the Nilsson model. Magnetic dipole moments are predicted for those nuclei which have not been measured.

I. INTRODUCTION

THE magnetic dipole moments of mirror nuclei offer an especially favorable possibility for investigating nuclear structure. It has been pointed out by Sachs[1] that in the sum of the moments of a mirror pair the most likely forms of meson-exchange contributions are cancelled out. Therefore one can separate the effects due to nuclear structure from the uncertainty of exchange-current contributions which becloud most interpretations of magnetic moments.

The sum of the moments depends only on the $T=0$ part of the magnetic dipole moment operator, and for a state of angular momentum I, the sum Σ of the expectation values is

$$\Sigma = I + (\mu_n + \mu_p - \tfrac{1}{2})\langle\sigma_z\rangle_{I_z=I}. \tag{1}$$

Here μ_n and μ_p are the neutron and proton magnetic moments in nuclear magnetons, and the expectation value of the sum of the Pauli spin components of the individual nucleons, $\sigma_z = \sum_i \sigma_z^i$, is evaluated in the state with angular momentum component $I_z = I$. All the effects of nuclear structure are contained in the expectation value of σ_z, which is multiplied by a rather small coefficient, $\mu_n + \mu_p - \frac{1}{2} = 0.37954$. However, if the sum is measured to an accuracy of a few tenths of a nuclear magneton, useful information can be obtained. Since such measurements are in progress,[2] it seems worth while to make some theoretical estimates.

II. CALCULATION AND RESULTS

It is only for light nuclei, in which the numbers of neutrons and protons are equal that one finds pairs of mirror nuclei with one of the partners stable and the radioactive partner sufficiently long-lived to allow measurement of the magnetic moment. Most of those accessible to present experimental techniques lie in the $1p$ shell where, fortunately, there is also a fair amount of theoretical understanding of nuclear structure. The sum of magnetic moments was calculated for odd-A nuclei in the $1p$ shell by use of wave functions[3] with intermediate strength of spin-orbit coupling. The results of the calculation are given in Table I as functions of the parameter (a/K) which indicates the strength of spin-orbit coupling. For $A=7$ and $A=9$ one expects weak spin-orbit coupling with (a/K) between 1.5 and 3.0, while for $A=11$ and $A=13$, (a/K) is probably between 4.5 and 6.0. It is evident from Table I that the sum is very insensitive to the parameter (a/K). Thus the expectation value of σ_z, listed in the second half of Table I, gives a very poor indication of the changes in nuclear structure brought about by varying the degree of spin-orbit coupling. This behavior is somewhat surprising in that, aside from the LS limit $(a/K=0)$ where the wave functions are $S=\frac{1}{2}$ eigenfunctions, there are admixtures of $S=\frac{3}{2}$ functions in the ground state which become appreciable for large a/K.

However, the result has the advantage of permitting reasonably accurate predictions of the magnetic moments of the radioactive partners. Since the moments of the stable partners are all measured,[4] one can sub-

TABLE I. Sum of the magnetic dipole moments for mirror nuclei of mass number A and angular momentum I, computed as a function of the relative strength of spin-orbit coupling, a/K.

		a/K					
A	I	0	1.5	3.0	4.5	6.0	∞
7	$\frac{3}{2}$	1.880	1.881	1.886			1.880
9	$\frac{3}{2}$	1.880	1.799	1.756			1.880
11	$\frac{3}{2}$	1.880		1.818	1.840	1.880	1.880
13	$\frac{1}{2}$	0.373		0.376	0.372	0.365	0.373
		Expectation value of σ_z					
7	$\frac{3}{2}$	1.000	1.004	1.017			1.000
9	$\frac{3}{2}$	1.000	0.788	0.675			1.000
11	$\frac{3}{2}$	1.000		0.838	0.896	1.000	1.000
13	$\frac{1}{2}$	−0.333		−0.327	−0.337	−0.356	−0.333

TABLE II. Magnetic moments for pairs of mirror nuclei in the $1p$ shell. Values for stable partners are experimental; values for radioactive partners are predicted from the sums of Table I.

A	I	μ(stable partner) (nm)	μ(radioactive partner) (nm)
7	$\frac{3}{2}$	+3.256	−1.375 to −1.370
9	$\frac{3}{2}$	−1.177	+2.976 to +2.933
11	$\frac{3}{2}$	+2.688	−0.848 to −0.808
13	$\frac{1}{2}$	+0.702	−0.330 to −0.337

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[1] Previous references and a thorough discussion of the problem are found in R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Reading, Massachusetts, 1953), Chap. 9.

[2] J. L. Snider, M. Posner, A. M. Bernstein, and D. R. Hamilton, Bull. Am. Phys. Soc. **6**, 224 (1961).

[3] D. Kurath, Phys. Rev. **101**, 216 (1956).

[4] Theoretical values in intermediate coupling are in quite good agreement with the measurements.

tract these values from the sums to obtain the predictions in Table II. The predictions of Table II place unwarranted confidence in the intermediate-coupling functions if one believes the quoted accuracy. However, in view of the alternative estimates to be made in the following sections, the predictions are believed to be reliable to within 0.1 nm.

III. ROTATIONAL INTERPRETATION

A. Generator Method

The wave functions of the low-lying $1p$-shell functions contain correlations appropriate to particles in a rotating spheroidal well. This has been shown[5] by generating wave functions starting with a many-particle function of spheroidal single-nucleon functions. The generated functions are very nearly identical to the intermediate-coupling functions used in the first part of this paper. However, there is a real difference shown by the fact that the generated functions for the $A=7$ and $A=13$ ground states lead to expectation values of σ_z which are even more nearly constant than those of Table I. For $A=7$ the generated function leads to $\sigma_z \leqslant 1$ and lying between 0.996 and unity. For $A=13$ one gets $\sigma_z \geqslant (-\frac{1}{3})$ and lying between -0.324 and $-\frac{1}{3}$. These values differ from those listed in Table I, but only affect the third decimal place in the sum of magnetic moments. The generated functions would lead to predicted moments of $\mu=-1.378$ nm for Be^7 and $\mu=-0.325$ nm for N^{13}. Since one would not expect either the results of this approach or the interaction results of Table II to be accurate to three decimal places, the differences are mainly of academic interest.

B. Nilsson-Model Interpretation

A simplified way of looking at the rotational properties is to use the model of Bohr and Mottelson,[6] namely, a rotating deformed core strongly coupled to a single odd nucleon. The latter's wave function, as calculated by Nilsson,[7] is that of a particle in a potential well with the shape of the deformed core. Such an approach can be considered as an approximation to the generator method, because the matrix elements calculated with Bohr-Mottelson wave functions can be derived from the integrals of the generator method by assuming that the only important contributions to the integrals come when the nuclear axes in the two wave functions are aligned.

The advantage of using the model consisting of a single odd nucleon plus a core is that calculations are much simpler, and one can explicitly demonstrate the essential features. One can also easily extend the results to more complicated regions such as the $(2s$-$1d)$ shell. The price one pays in the case of magnetic moments is the introduction of a new parameter g_R, the g factor for the contribution from the rotating core. As we shall see, it plays an important role in the expression for the sum Σ of the magnetic moments of a pair of mirror nuclei.

The sum Σ can be expressed[7] in terms of matrix elements of the single-nucleon Nilsson functions identified by K, the component of angular momentum on the nuclear symmetry axis. The expression is

$$\Sigma=I+(\mu_p+\mu_n-\tfrac{1}{2})\left(\frac{K}{I+1}\right)[\langle K|\sigma_z|K\rangle+\delta_{K,\frac{1}{2}}(-1)^{I+l-\frac{1}{2}}(I+\tfrac{1}{2})\langle-\tfrac{1}{2}|\sigma^-|\tfrac{1}{2}\rangle]$$
$$+(2g_R-1)\left(\frac{1}{I+1}\right)\{I(I+1)-K[K+\delta_{K,\frac{1}{2}}(-1)^{I+l-\frac{1}{2}}(I+\tfrac{1}{2})\langle-\tfrac{1}{2}|J^-|\tfrac{1}{2}\rangle]\}. \quad (2)$$

Here g_R is the average rotational g factor for the core in the odd-neutron and odd-proton cases. The matrix elements can be evaluated in terms of the coefficients $a_\Lambda{}^l$ for expanding the Nilsson functions in terms of functions of orbital angular momentum l with projection Λ on the nuclear symmetry axis. The expression for Σ then becomes

$$\Sigma=I+(\mu_p+\mu_n-\tfrac{1}{2})\left(\frac{K}{I+1}\right)[2B_K{}^2-1+\delta_{K,\frac{1}{2}}(-1)^{I+l-\frac{1}{2}}(2I+1)B_K{}^2]$$
$$+(2g_R-1)\left(\frac{1}{I+1}\right)\{I(I+1)-K[K+\delta_{K,\frac{1}{2}}(-1)^{I-\frac{1}{2}}(I+\tfrac{1}{2})\mathcal{A}]\}, \quad (3)$$

where $B_K{}^2=\sum_l |a_{\Lambda=K-\frac{1}{2}}{}^l|^2$, l indicates the parity of the Nilsson function, and $\mathcal{A}$ is the usual decoupling parameter for a $K=\frac{1}{2}$ band, namely,

$$\mathcal{A}=(-1)^l\langle-\tfrac{1}{2}|J^-|\tfrac{1}{2}\rangle. \quad (4)$$

The decoupling parameter and $B_K{}^2$ contain the dependence on nuclear structure as determined by the Nilsson parameter η which measures the importance of deformation relative to spin-orbit coupling.

Since one expects that the g factor for rotation is approximately equal to the value of (Z/A) for the deformed core, these nuclei should have an average

[5] D. Kurath and L. Pičman, Nuclear Phys. **10**, 313 (1959).

[6] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab Selskab, Mat.-fys. Medd. **27**, No. 16 (1953).

[7] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys Medd. **29**, No. 16 (1955).

value $g_R \approx \frac{1}{2}$. Therefore let us first consider this approximation in Eq. (3) for the $1p$-shell cases. For these cases $l=1$ is the only l value, and $K=\frac{1}{2}$ for $A=7$ and $A=13$. For $A=7$, the ground state has $I=\frac{3}{2}$, and Eq. (3) becomes

$$\Sigma(A=7)=\tfrac{3}{2}+\tfrac{1}{5}(\mu_p+\mu_n-\tfrac{1}{2})(6B^2-1), \qquad (5)$$

where $B^2=|a_{\Lambda=0}^{l=1}|^2$ for Nilsson level number 3. Except very near the jj-coupling limit, B is quite close to unity. An estimate of the value of η appropriate to $A=7$ is found in the work of Clegg[8] who has recently treated the energy spectra and inelastic proton data for the $1p$ shell[9] with the Nilsson model. For $A=7$, Clegg chooses $\eta=+8$ which gives $B^2=0.977$. Inserting this value in Eq. (5) gives $\Sigma=1.869$, very close to the values of Table I. From the point of view of this model, the expectation value of $\sigma_z=\sum_i \sigma_z^i$ is never greater than unity because it is determined solely by the odd nucleon, and it is very close to unity because the wave function of this odd nucleon is almost entirely one with the projection of spin angular momentum equal to $+\frac{1}{2}$.

For $A=13$ and $I=\frac{1}{2}$, the approximation with $g_R=\frac{1}{2}$ gives a very simple result:

$$\Sigma(A=13)=\tfrac{1}{2}-\tfrac{1}{3}(\mu_p+\mu_n-\tfrac{1}{2})=+0.3735. \qquad (6)$$

This result is independent of η and again very close to the results of Table I.

The effect of the term including the factor $(2g_R-1)$ must be considered carefully because the coefficient of $(2g_R-1)$ is not particularly small. For $A=7$, the correction term is $(2g_R-1)(1.4+0.4\mathcal{A})$. Clegg's choice of $\eta=+8$ gives $\mathcal{A}=-1.4$ so that Eq. (3) becomes

$$\Sigma(A=7)=1.869+1.68(g_R-0.5). \qquad (7)$$

Therefore the result is very sensitive to deviations of g_R from its expected value of 0.5, so on this model the experimental determination of Σ would be interpreted as an accurate measurement of g_R. If the experimental results turn out to be close to the predictions of Table I, it will mean that g_R must be very close to 0.5.

Similarly, for $A=13$ the complete evaluation of Eq. (3) gives

$$\Sigma(A=13)=0.3735+\tfrac{2}{3}(1-\mathcal{A})(g_R-0.5). \qquad (8)$$

In this case there is unfortunately no good determination of $\mathcal{A}$, which probably lies somewhere between the values of $+\frac{2}{3}$ and zero, the values obtained for $\eta=-2$ and $\eta=-4.5$, respectively. But again, if experimental deviations of Σ from 0.3735 are small, one can interpret the information to give a fairly accurate value for g_R.

So far there has been no discussion of the rotational interpretation for the cases $A=9$ and $A=11$. This is because such interpretations are complicated by the problem of band mixing since there are two Nilsson states near each other, one with $K=\frac{1}{2}$ the other with $K=\frac{3}{2}$. Because B^9 is unstable to decay into a proton plus two alpha particles, its magnetic moment is likely to remain unknown. For $A=11$, one can probably make a rotational interpretation in terms of band mixing but, because of the uncertainty in g_R and the decoupling parameter as well, it will probably not be possible to draw any unambiguous conclusions.

IV. CONCLUSIONS

Theoretical calculations show that for the ground states of nuclei in the $1p$ shell, the sum of the magnetic moments of pairs of mirror nuclei is determined almost entirely by the rotational nature of these states. The results are very insensitive to other features of nuclear structure such as the degree of spin-orbit coupling. Therefore, one cannot expect to obtain information about spin-orbit coupling from experimental determination of magnetic moments of mirror pairs.

The simple strong-coupling picture of Bohr, Mottelson, and Nilsson also gives results comparable to those of intermediate-coupling calculations. On the strong-coupling model the interpretation of experimental results will be essentially to give an accurate determination of the rotational g factor, g_R.

If the measurements are close to predicted values, they will serve to confirm the rotational nature of these ground states.

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[8] A. B. Clegg, Clarendon Laboratory Report No. 40/61, 1961 (unpublished).

[9] Although these results represent the experimental evidence as well as they do in the $(2s\text{-}1d)$ shell, they are not nearly as good as, for example, in the rare-earth region.