Measurement of Electron Scattering in Carbon to Compare with a Muon Experiment*

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A measurement of the inelastic scattering of high-energy electrons in carbon has been carried out with the express purpose of obtaining an experimental cross section to compare with a recently-completed measurement of muon scattering. A scaling law which connects the electron result with the (higher-energy) muon result is described. Results for the summed (elastic plus all inelastic) cross section at two values of the muon scattering angle are presented. At the smaller angle (momentum transfer ~200 Mev/c) agreement with the theoretical (Drell-Schwartz) sum rule is good; at the larger one (~280 Mev/c) it is poorer but within the experimental and theoretical uncertainties. Comparison with the muon results does not alter the previous conclusion: That no muon scattering anomaly is seen.

INTRODUCTION

RECENTLY the absolute cross section for nuclear scattering of 2-Bev/c muons was measured[1] to see whether the elementary interaction contained any contribution in addition to the predicted electromagnetic one. For practical reasons, carbon rather than hydrogen was used as the scatterer, and the results were compared with the summed (elastic plus inelastic) carbon cross section as calculated by Drell and Schwartz,[2] making use of the size parameters of the C^{12} nucleus as determined by Hofstadter and collaborators from elastic electron scattering.[3,4] The uncertainties of the theoretical evaluation are discussed in reference 1 and are expected to be no worse than 10–15%. A measurement of electron-deuteron scattering by Friedman[5] checked the Drell-Schwartz "constant-q" sum (which is closely related to the constant-angle sum used in the muon experiment) to 5%. Existing experimental studies of inelastic electron scattering in carbon[6,7] are not directly helpful, since they are at different energies and are addressed to other questions. It was felt that an experimental comparison of electron scattering in carbon with the muon scattering would be useful in so far as it would answer, without reference to nuclear theory, the question: "Does the muon scatter just like a heavy electron?"

We have therefore carried out some measurements of electron scattering in carbon, using the Stanford Mark III Electron Linear Accelerator, programmed to be comparable with the muon scattering results at two muon angles. Two complications prevent this comparison from being direct: (a) The energies available to us were not more than 900 Mev in the incident beam and not more than 275 Mev in the analyzing magnet, as compared to an incident energy of 2 Bev in the muon experiment, and all final energies from 2 to 1.36 Bev; (b) Radiative corrections, negligible in the muon experiment, were large in the electron experiment. The first difficulty is surmounted with the use of a very general scaling law for inelastic scattering of electrons which was known to the Stanford physicists; the second proves to be one of several factors which limit the accuracy of the experiments reported here. Within the rather limited accuracy we find agreement with the conclusions already reached in reference 1; the muon is indistinguishable, in our experiment, from a heavy electron.

PRINCIPLE OF THE MEASUREMENT

The experiment was suggested to us by Panofsky on the basis of an unpublished result of Bjorken[8] on the general form of inelastic electron cross sections. Similar considerations were published, at about that time, by von Gehlen,[9] who gives the result in a slightly more general and more implicit form than that quoted below.

Consider the electromagnetic scattering of an electron (or muon) by any nuclear system and observe the scattered electron but no other products of the collision. Then, in the one-photon-exchange approximation, invariance arguments can be used to show that the differential cross section for the electron has a rather simple general form, regardless of nuclear complications such as pion production, final-state interactions, etc. If we neglect $(1-\beta^2)$ with respect to 1 (allowable in the experiments under consideration), we can write the

* Supported in part by the Office of Naval Research and the National Science Foundation.

[1] G. E. Masek, L. D. Heggie, Y. B. Kim, and R. W. Williams, Phys. Rev. **122**, 937 (1961).
[2] S. D. Drell and C. L. Schwartz, Phys. Rev. **112**, 568 (1958).
[3] H. Ehrenberg, R. Hofstadter, U. Meyer-Berkhout, D. Ravenhall, and S. Sobottka, Phys. Rev. **113**, 666 (1959).
[4] U. Meyer-Berkhout, K. W. Ford, and A. E. S. Green, Ann. Phys. **8**, 119 (1959).
[5] J. I. Friedman, Phys. Rev. **116**, 1257 (1959).
[6] J. H. Fregeau, Phys. Rev. **104**, 225 (1956).
[7] H. F. Ehrenberg and R. Hofstadter, Phys. Rev. **110**, 544 (1958).
[8] J. D. Bjorken and D. H. Coward (private communication).
[9] G. von Gehlen, Phys. Rev. **118**, 1455 (1960).

result, in the laboratory system, as

$$\frac{\partial^2\sigma}{\partial\Omega\partial p'}=\frac{Z^2e^2\cos^2(\theta/2)}{4E^2\sin^4(\theta/2)}\times\left\{Q(q^2,\Delta E)+\tan^2(\theta/2)\left[\frac{q^2}{2M^2}R(q^2,\Delta E)\right]\right\}. \quad (1)$$

Here $\mathbf{p}$ and E are the momentum and total energy of the incident electron (or muon), p' is the magnitude of its final momentum, $\Delta E=E-E'$ is the energy lost by the electron in the collision, and $q^2=|\mathbf{p}-\mathbf{p}'|^2-\Delta E^2$ is the magnitude of the four-momentum transfer squared. We put $\hbar=c=1$. In the relativistic approximation, the relation is

$$q^2=4E(E-\Delta E)\sin^2(\theta/2). \quad (2)$$

The functions Q and R, which contain all the nuclear effects, depend only on q^2 and ΔE. (The factor $q^2/2M^2$ in front of R is of course unnecessary, but is suggested by the derivation; M is the nucleon mass.) For a fixed value of q^2 and ΔE, Q and R can be determined by measuring $\partial^2\sigma/\partial\Omega\partial p'$ at two (E,θ) values. One can then predict the cross section at any energy, for this value of q^2 and ΔE. The kinematic effects of the muon's larger rest mass are unimportant for 2-Bev muons.

Insight into the general nature of the major contributions to Q and R can be obtained from the sum rule of Drell and Schwartz, which is the integral over p' of the differential cross section, neglecting pion production and related effects. The major terms, for a light nucleus, are

$$\int Qdp'\approx\frac{1}{Z}+\frac{Z-1}{Z}\langle e^{i\mathbf{q}\cdot(\mathbf{r}_1-\mathbf{r}_2)}\rangle_0+\frac{q_0^2}{4M^2Z^2}(Z\mu_p{}^2+N\mu_N{}^2),$$

$$\int Rdp'\approx\frac{1}{Z^2}(Z\mu_p{}^2+N\mu_N{}^2).$$

Here $\langle\ \ \rangle_0$, the form factor for two-body correlations, is completely determined by the ground-state nuclear wave function; μ_p and μ_N are the nucleon magnetic moments in nuclear magnetons; and a common factor, the square of the nucleon form factor, has been omitted from all terms.

Our program is to determine the integrated cross section at fixed angle and high energy, including both elastic and inelastic processes, by computing the differential cross section at various values of ΔE and the corresponding values of q^2, from Eq. (1), and then integrating over ΔE. To do this we obtain Q and R from pairs of measurements with the lower-energy electrons. In this way we have constructed the differential cross-section curves, $\partial^2\sigma/\partial\Omega\partial p'$, to be expected for 2-Bev/c muon scattering at two scattering angles: 5.73° and 8.03°, corresponding to momentum transfer, for elastic collisions, of 200 Mev/c and 280 Mev/c. These are integrated over p' to give $\partial\sigma/\partial\Omega$. The range of ΔE is of course more limited than in the muon experiment; the effect of this on our result is thought to be small, and is discussed below.

PROCEDURE

The electron beam from the Mark III Linear Accelerator was collimated to $\frac{1}{4}$-in. diameter and momentum-selected for 1% full width at half maximum; it was used over a variety of energies as required by the program. Beam intensity was measured with a large Faraday cup; all beam energies were below the maximum (600 Mev) for which the cup had been calibrated. The scattered electrons were analyzed with the double-focusing zero-dispersion spectrometer described by Alvarez *et al.*,[10] with a Lucite Čerenkov counter as detector. The entrance window of the spectrometer was masked to $\frac{1}{2}$ in. in the horizontal plane, corresponding to a scattering-angle resolution of 0.74°. The momentum window was set for $\Delta p/p=0.0067$.

Combining this value of the momentum window with the geometrical measurements of the entrance window we obtain $\Delta\Omega(\Delta p/p)=7.7\times10^{-6}$ sr, which of course provides the basis for an absolute cross-section measurement. We also calibrated the system by using the known cross section of the proton. The targets used were polyethylene, 0.423 g cm^{-2} and carbon, 0.370 g cm^{-2}. The calibration was done at 275 Mev incident energy, and $q=200$ Mev/c. The measured carbon scattering was subtracted from the CH_2 data and the resulting curve was integrated from well above the peak to a point 5.5 Mev below the peak. It is desirable to integrate over as much of the "elastic" curve as possible in order to eliminate the effect of the spread in incident-beam momentum and the details of the spectrometer resolution function. By placing the lower limit of the integral well into the radiative tail one can be sure that the radiative corrections can be applied unambiguously. The "ΔE" of the radiative-correction formulas should then be taken fron the half-maximum point on the high-energy side of the peak to the cut-off point on the low-energy side. We find the following numerical results: $\Delta E=6.6$ Mev; Schwinger correction, $\delta_S=0.134$; and bremsstrahlung correction, $\delta_B=0.055$. Using the proton form factor given by Herman and Hofstadter[11] to calculate the absolute scattering cross section, we find from this calibration that $\Delta\Omega(\Delta p/p)=7.3\times10^{-6}$ sr. This is reasonably good agreement with the nominal value quoted above; we have used the value based on the proton in our subsequent calculations.

At the time of our experiment the maximum momentum which could be analyzed by the spectrometer was 275 Mev/c. The Čerenkov counter began to lose

[10] R. A. Alvarez, K. L. Brown, W. K. H. Panofsky, and C. T. Rockhold, Rev. Sci. Instr. **31**, 556 (1960).

[11] R. Herman and R. Hofstadter, *High-Energy Electron Scattering Tables* (Stanford University Press, Stanford, California, 1960).

efficiency at low energies; it was down to ~ 0.9 at 125 Mev/c and dropped rapidly below that. We therefore had to plan our program to fit this rather narrow range of final energy. So far as possible we kept the incident energy fixed, varying the energy loss and the angle to give the q^2 appropriate to the corresponding fixed-angle, 2-Bev curves. To solve for the two unknown functions we then ran through the same range of q^2 and ΔE at lower incident energy and larger angle. Although the second term of Eq. (1) did not contribute appreciably to the 2-Bev curves, where $\tan^2(\theta/2)$ is very small, it had to be subtracted from the electron-scattering curves to find Q. However, its contribution to the small-angle electron curves was small (and identically zero in the elastic peak, for a spin-zero nucleus), so the large-angle data serves essentially as a correction. The data were smoothed before making the subtractions to find Q.

RESULTS AND ANALYSIS

Figures 1 and 2 show the small-angle data ($E=275$ Mev) for a q^2, ΔE relation corresponding to $E=2$ Bev, $\theta=5.73°$ and $8.03°$, respectively. The yield of scattered electrons has been corrected for spectrometer dispersion but not for radiative effects. There is a general similarity to the data of Fregeau.[6] A few spot checks were made at very large values of ΔE (250 Mev, 350 Mev) to see whether the cross section rises again in the pion-production region, as it does, for example, in the large-q, large-angle data of Ehrenberg and Hofstadter.[7] The yield at these points was small and it appeared that after radiative corrections had been applied a small but uncertain contribution would be left. These points have not been included in our analysis, since they have poor statistics and very large corrections. The contribution of this to the integral is discussed below.

We want the absolute value of $\partial^2\sigma/\partial\Omega\partial p'$ with all radiative effects removed.[12] Radiative corrections in inelastic (continuum) scattering were treated by

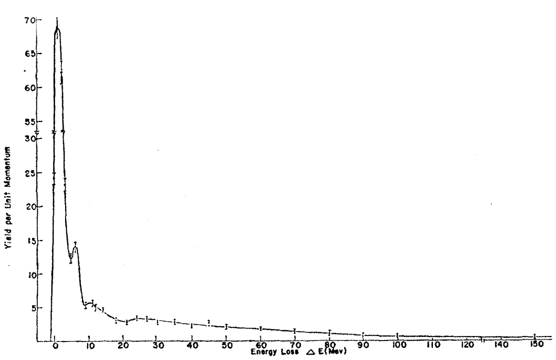

FIG. 1. Yield of scattered electrons, corrected for spectrometer dispersion, in arbitrary units. Incident energy 275 Mev, final energy $(275-\Delta E)$ Mev, angles chosen to give same momentum transfer as 5.73° at 2 Bev. Note break in scale.

[12] Radiative corrections to muon scattering are negligible under the conditions of the muon experiment.

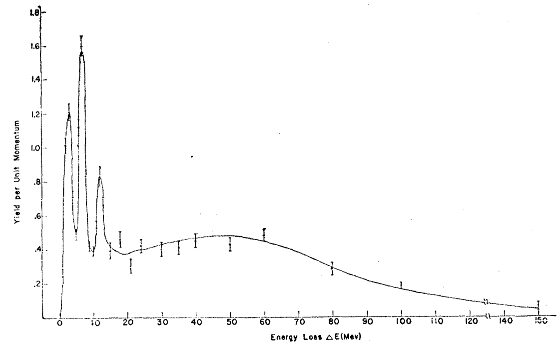

FIG. 2. Same as Fig. 1, but for 8.03° at 2 Bev.

Panofsky and Allton[13]; we have followed their method, as discussed in more detail by Friedman.[5] The effects to be considered are: (1) Reduction of the area under the elastic and inelastic-level scattering peaks due to real and virtual soft-photon effects (the Schwinger correction). These areas must be restored. (2) Bremsstrahlung in the target. This degradation reduces the elastic peak and distorts the rest of the curve. It is thickness-dependent and the target thickness (0.367 g cm^{-2} of graphite) was chosen so that it was less than the other corrections. (3) Bremsstrahlung during the scattering process. The approximate cross section for large-angle bremsstrahlung quoted by Panofsky and Allton was used. In this approximation this correction has the same form (though with an artificial "thickness") as (2) and we can discuss them together, treating both as though the processes of radiation and scattering were separate.

As a first step the elastic peak was corrected for effects (1) and (2) and resolved inelastic levels were similarly corrected. As a result of (2) and (3), each level contributes scattered, degraded electrons which appear with an energy ΔE_R below that which they would have without radiation. In these processes the nonradiative energy loss $\Delta E'$, which can be recoil or nuclear excitation, is discrete, with a cross section $\partial\sigma/\partial\Omega(E)$. If an electron of initial energy E has a probability $P(E,\Delta E_R)dE'$ to radiate energy ΔE_R into dE' then the differential cross section for the following process—radiate then scatter, with total energy loss $\Delta E=\Delta E_R+\Delta E'$—is clearly $\partial^2\sigma/\partial\Omega\partial p'=P(E,\Delta E_R)\partial\sigma/\partial\Omega(E-\Delta E_R)$. The process with the opposite order leads to $\partial\sigma/\partial\Omega(E)P(E-\Delta E',\ \Delta E_R)$, which in general is different. We use the approximate expressions given by Friedman[5] for the correct combinations of these terms, and for the corresponding expression for large-angle bremsstrahlung, effect (3). With a knowledge of the appropriate cross sections we can then remove the unwanted contributions of any given level to the levels of higher excitation and to the continuum. This was done using some published data[6] and extrapolations from the

[13] W. K. H. Panofsky and E. A. Allton, Phys. Rev. **110**, 1155 (1958). They use the basic result of L. I. Schiff, Phys. Rev. **87**, 750 (1952).

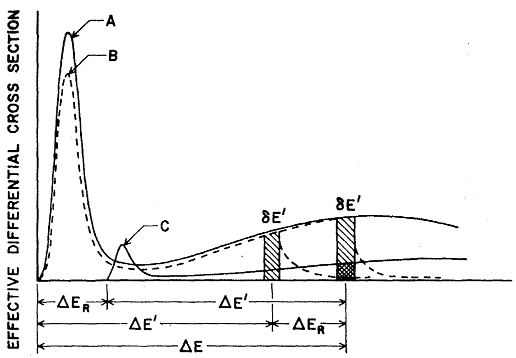

FIG. 3. Schematic illustration of some radiative corrections in inelastic scattering. The desired curve is A. At the point ΔE it is distorted by bremsstrahlung after scattering (curve B), bremsstrahlung before scattering (C), and degradation of the desired-energy electrons (tail below ΔE). The ordinate is $\partial^2\sigma/\partial\Omega\partial p'$ for Curves (A) and (B), but for curve (C) the cross section has already been multiplied by the probability to radiate energy ΔE_R into the interval $\delta E'$.

published data to provide the inelastic level cross sections in regions where we had not measured them.

The radiative degradation of the continuum can be treated in an analogous way; now however, $\Delta E'$ is no longer discrete. We fix ΔE, let $\Delta E'$ range over a small interval $\delta E'$, and replace the cross section $\partial\sigma/\partial\Omega$ by $\delta E'\partial^2\sigma/\partial\Omega\partial p'(E,\Delta E)$. We then sum over all intervals $\delta E'$, and restore whatever may have been lost from the interval around ΔE. The process is illustrated schematically in Fig. 3. Curve A is the desired corrected cross-section curve and we consider the corrections at a spectrometer setting ΔE. Curve B is the cross-section curve appropriate to the angle θ at which we are set (recall that each ΔE value has a different θ!). B is the curve we must use for the sequence: scatter, then radiate, and one such process is illustrated. The process: radiate, then scatter, introduces an entirely different set of cross-section curves, one of which, labeled C, is illustrated. These cross sections are not readily available; there is no theoretical expression for them, and they are in general different from the cross sections we

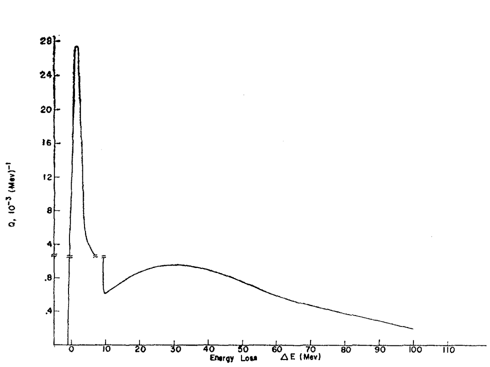

FIG. 4. The function Q of Eq. (1), for momentum transfer corresponding to 5.73° at 2 Bev, which is 200 Mev/c for elastic scattering and given by Eq. (2) for inelastic scattering. Note break in scale.

are measuring in this experiment. Equation (1) is useful in extending the measurements in certain ways, but we need approximate values of Q and R in Eq. (1) over a large continuum of q^2, ΔE values in order to make the radiative corrections to our measurements of the differential cross section, and therefore of Q and R, along a single line. These approximate values of Q and R were obtained by linear extrapolation and interpolation from inelastic cross sections measured during the experiment. Checks on the extrapolation procedure were provided by additional measured cross sections.

These corrections can change the measured cross section at the largest values of ΔE which contribute significantly ($\sim$100 Mev) by as much as a factor of two. Over most of the curve, however, they are typically 10–30%, and since our interest is in the integral of this curve we have made the corrections in the following approximate way: using the known elastic cross section, we removed the elastic peak contribution as described above. The otherwise uncorrected values of Q and R

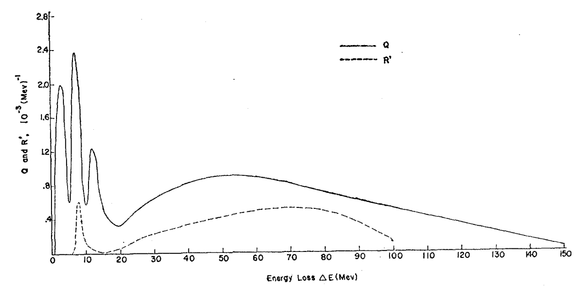

FIG. 5. The functions Q and $R'=(q^2/2M^2)R$ of Eq. (1) for momentum transfer corresponding to 8.03° at 2 Bev, which is 280 Mev/c for elastic scattering, and given by Eq. (2) for inelastic scattering.

were then obtained from the data as functions of ΔE, for $q^2 \approx (200\ \text{Mev}/c)^2$ and $\approx (280\ \text{Mev}/c)^2$. These values of Q and R were then corrected for the radiative contributions from the inelastic continuum as outlined above and in Fig. 3. The inelastic levels were corrected and their contributions removed by hand calculation, and the continuum was corrected by a digital computer program. Various numerical checks were made on the accuracy of our approximations. In all cases we made only lowest-order corrections (each effect is considered to be independent of the others), and we found that iteration of the correction procedure was in general not necessary. The uncertainty in the final integrated cross section which radiative effects introduce is thought to be about 5% and unlikely to be as much as 10%. We have assigned it 5% in our estimate of errors.

The results for the corrected values of Q and $R'\equiv(q^2/2M^2)R$ are shown in Figs. 4 and 5. At the lower momentum transfer only Q is shown, since $R'\tan^2(\theta/2)$ is small and not well determined.

The differential cross sections calculated from these Q

and R' values for comparison with the muon experiment are shown in Figs. 6 and 7.

Numerical results are given in Table I. One might expect that the elastic cross section in carbon would afford a good check on the experiment, but it turns out that the absolute value of the carbon cross section has never been determined with great accuracy. Detailed discussion is given by Meyer-Berkhout *et al.*[4] who show that the experimental numbers must be adjusted by $\pm 20\%$ or so at various points in order to give a smooth fit to a form-factor curve. If we take their smoothed form-factor curve we find it gives a cross section 20% larger than our result. However, if we take one of the "best-fit" analytic expressions for the form factor which they quote and which is used in our evaluation of the Drell-Schwartz sum rule,[1] we find it gives an elastic cross section in agreement with our result. At 280 Mev/c the situation is worse; the best-fit curve lying 35% above our experimental result and the analytic expression lying below the experimental result. It should be noted that the sum rule evaluation is much less sensitive to the parameters of the wave function than is the elastic cross section.

TABLE I. Electron scattering in carbon: Elastic cross sections, total cross sections projected to 2 Bev, comparison with the Drell-Schwartz sum rule and part of the data from the muon scattering experiment. All cross sections are in units of cm^{2}sr^{-1}. The errors quoted should be understood as estimates only; they are compounded from statistics plus our estimates of radiative-correction uncertainties and systematic errors.

	Momentum transfer for elastic scattering	
	200 Mev/c (5.73° at 2 Bev)	280 Mev/c (8.03° at 2 Bev)
Elastic scattering cross section, $E=275$ Mev	$(11.0\pm0.7)\times10^{-30}$	$(0.200\pm0.014)\times10^{-30}$
Summed cross section (elastic plus inelastic), scaled to 2 Bev by Eq. (1)	$(1.49\pm0.11)\times10^{-27}$	$(0.18\pm0.03)\times10^{-27}$
Summed cross section calculated from sum rule	1.45×10^{-27}	0.25×10^{-27}
Cross section inferred from muon-scattering data in regions indicated	$(1.7\pm0.2)\times10^{-27}$ (5.2°–7.2°)	$(0.35\pm0.10)\times10^{-27}$ (7.2°–9°)

The object of this experiment was to obtain the summed inelastic cross section at 2 Bev by scaling the electron measurements. This is simply the area under the curves of Figs. 6 and 7, and is given in the second line of Table I. The integrations were cut off where the data stops, at 100 and 150 Mev, respectively; it appears that most of the nuclear-disintegration region was included. There is neither theoretical nor (as mentioned above) experimental reason to expect pion production to add markedly to the integral at large ΔE values. Energy losses up to 640 Mev were available in the muon experiment, however, and it is somewhat unsatisfactory that neither the present measurement nor our shell-model sum rule evaluation deals with large energy losses.

The statistical uncertainty in our integrals is about 3% in the 200-Mev/c sum and 10% in the 280-Mev/c sum (the latter is influenced by the necessity of subtracting the R component). Systematic errors are clearly of at least this magnitude.

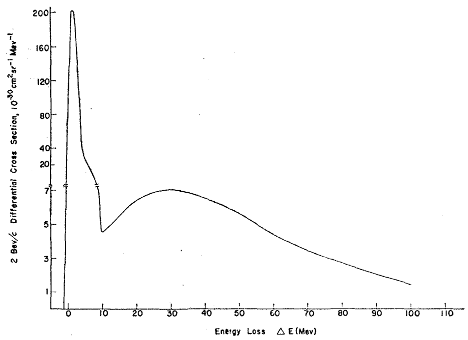

FIG. 6. Differential cross section for scattering of 2-Bev electrons (or muons with no anomalous interactions) at 5.73°, corresponding to a momentum transfer, at $\Delta E=0$, of 200 Mev/c. Note break in scale.

Evaluation of the Drell-Schwartz sum is discussed in reference 1 and some numerical results are indicated in Table I; these numbers are directly comparable with the experimental summed cross section given there. It is seen that at 5.73° ($q=200$ Mev/c elastic) the agreement is excellent, better in fact than is warranted by either the theory or the experiment. This is the highest momentum transfer for which Drell and Schwartz claim good ($\sim$5%) accuracy for their approximations, exclusive of uncertainty of the wave function, and it is also a region in which the muon experiment had an appreciable yield. The last line shows the muon cross section inferred from the number of events which fell in a range (centered with respect to number) around the angle in question. It is in agreement with the electron data and the calculation, and indicates, as stated in reference 1, that with respect to this kind of observation a muon is simply a heavy electron.

The higher momentum-transfer result (280 Mev/c) was obtained primarily because it is in a region in which the sum rule is less reliable. Unfortunately the lack of

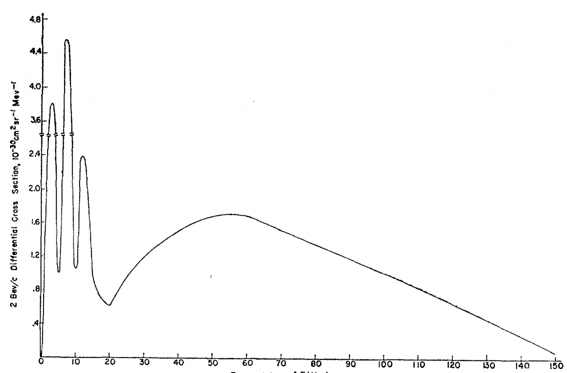

FIG. 7. Differential cross section for scattering of 2-Bev electrons (or muons with no anomalous interactions) at 8.03°, corresponding to a momentum transfer, at $\Delta E=0$, of 280 Mev/c.

good data at large ΔE makes the electron result less reliable than that at 200 Mev/c. For example, the contribution to the cross section of the region from 100 to 150 Mev (see Fig. 7) is 17% of the whole, but it rests on a point at 150 Mev which has very large and uncertain radiative corrections. The possibility that considerably more area lies beyond 150 Mev in this case, plus the much larger magnetic contribution which must be subtracted out, cause us to assign a fairly large uncertainty to this result, perhaps 20%. The electron result is some 28% below the sum rule calculation and we believe this range is a fair indication of the accuracy to which the theoretical and experimental work have determined this cross section. The muon result, based on 16 counts, is higher but not, we believe, significantly so.

CONCLUSIONS

We have used the scaling law, Eq. (1), in constructing high-energy differential inelastic cross sections from lower-energy electron scattering.[14]

[14] It would be possible to check the validity of the scaling law by analogous measurements, but this has not been done.

At moderate momentum transfers ($\leq$200 Mev/c) no essential difficulties were encountered, and a result was obtained which checked the Drell-Schwartz sum for carbon. At higher momentum transfers the importance of large energy transfers, and the prominence of the non-Coulomb term in the scaling law, make the method more difficult to apply.

The muon experimental results were compared in reference 1, over the entire range for which data was available, with the Drell-Schwartz sum. The present work has not altered the conclusion of reference 1: that no anomaly is found in the muon scattering.

ACKNOWLEDGMENTS

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