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Left-Right Asymmetry in Compton Scattering by Transversally Polarized Electrons

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A left-right asymmetry in the Compton scattering cross section is found for the case when only the target electron is polarized. The incident photon is unpolarized and the polarization of the outgoing photon and electron are assumed not to be measured. This asymmetry results from a radiative correction. It is found that the maximum asymmetry is about one part in a thousand.

I. INTRODUCTION

SCATTERING asymmetries for polarized particles are well known in Mott scattering[1] and nuclear scattering. Recently similar asymmetries have been predicted for the leptonic scatterings[2,3] $\mu-e$, e^--e^-, and e^--e^+. In this paper we find a similar spin-dependent left-right asymmetry effect for Compton scattering. We consider the case where unpolarized incident radiation strikes a free polarized electron, the polarizations of the recoil particles not being observed. The possible interaction of the electron with a nucleus to which it may be bound is not taken into account.

II. CALCULATION OF CROSS SECTION

The spin-sensitive correction to the Klein-Nishina formula was calculated by the standard Feynman method[4] described in many books on quantum electrodynamics.[5] Essentially, the calculation is a modification of that of Brown and Feynman[6] (hereafter called BF), who found the lowest order radiation correction to the Klein-Nishina formula. We adopt their notation by choosing q_1, q_2, p_1, and p_2 as the initial and final four-momenta of the photon and electron, respectively. In the rest frame of the target electron the necessary modification of the unpolarized case consists of inserting into the Dirac traces, which occur in that case, the spin projection operator, $\frac{1}{2}(1+\boldsymbol{\sigma}\cdot\mathbf{s})$. Here $\mathbf{s}$ is a unit vector in the direction of the electron spin, and $\boldsymbol{\sigma}$ has as its three components the four-by-four Pauli spin matrices. In terms of Dirac γ matrices,

$$\boldsymbol{\sigma}\cdot\mathbf{s}=\tfrac{1}{2}i\epsilon_{lmn}s\,\gamma_m\gamma_n, \tag{1}$$

where ϵ_{lmn} is the antisymmetric three-index symbol and repeated Latin indices are to be summed from 1 to 3.[7] In the lowest order calculations the only imaginary part is the pure imaginary $\boldsymbol{\sigma}\cdot\mathbf{s}$. Therefore, the traces involving this factor either vanish or cancel each

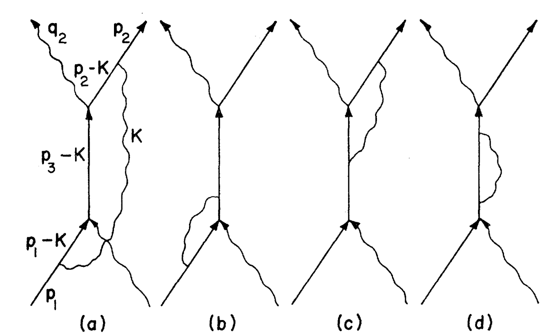

FIG. 1. Diagrams contributing to spin-dependent asymmetry.

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[1] N. F. Mott, Proc. Roy. Soc. (London) **A124**, 425 (1929); **A135**, 429 (1932).
[2] A. O. Barut and C. Fronsdal, Phys. Rev. **120**, 1871 (1960).
[3] C. Fronsdal and B. Jaksic, Phys. Rev. **121**, 916 (1961).
[4] R. P. Feynman, Phys. Rev. **76**, 749, 769 (1949).
[5] See, e.g., S. Schweber, H. Bethe, and F. de Hoffmann, *Mesons and Fields* (Row, Peterson and Company, Evanston, Illinois, 1956), Vol. I.
[6] L. M. Brown and R. P. Feynman, Phys. Rev. **85**, 231 (1952).
[7] The metric used is $g_{00}=1$, $g_{11}=g_{22}=g_{33}=-1$. Also units are chosen such that $\hbar=c=1$.

other, since otherwise they would contribute a purely imaginary part to the cross section. Thus, the lowest order in which $\boldsymbol{\sigma}\cdot\mathbf{s}$ gives a contribution results from the interference between the matrix elements proportional to e^2 and e^4. An argument similar to that above shows that in the resulting e^6 order trace, only the imaginary parts of the radiative correction integrals give non-vanishing terms. This is in contrast to the unpolarized case treated in BF where only the real parts of the integrals contribute. These integrals were calculated in general in BF and we have abstracted the necessary imaginary parts from them. The sign of the imaginary part is determined by the requirement that for purposes of integration the electron and photon masses be given a negative imaginary part. Fortunately the imaginary parts involve no divergences. As a check, we have also calculated the imaginary parts by the method of residues used for $e^- - e^+$ scattering.[3] As emphasized in that paper, the only diagrams which can give rise to an imaginary part for the integrals are those which can be cut by a horizontal line into two parts such that each part corresponds to a real process. The relevant radiative correction diagrams for this process shown in Fig. 1 are examples of this.

The calculation was greatly simplified by finding the traces before doing the integrations. Thus, the worst integral encountered was

$$J_\sigma = 2i\pi^{-2}\int k_\sigma d^4k/D, \tag{2}$$

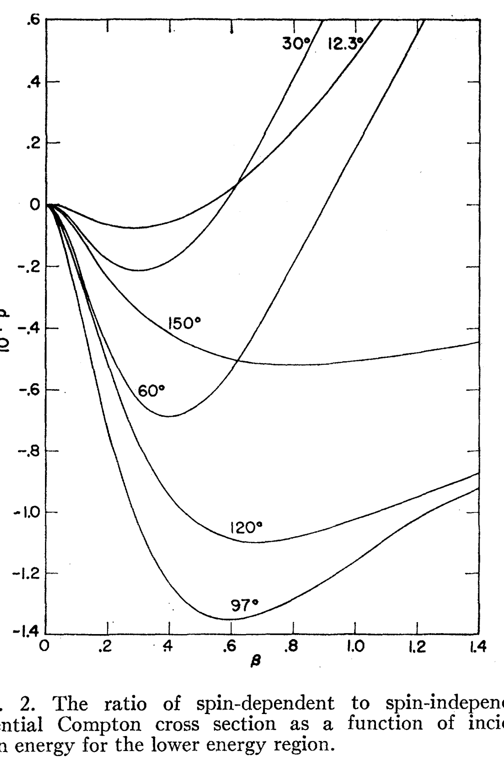

FIG. 2. The ratio of spin-dependent to spin-independent differential Compton cross section as a function of incident photon energy for the lower energy region.

with

$$D=(k^2-2p_1\cdot k)(k^2-2p_2\cdot k)(k^2-2p_3\cdot k+2p_1\cdot q_1)k^2, \tag{3}$$
$$p_3=p_1+q_1,$$

instead of the integral

$$J_{\sigma\tau\lambda}=2i\pi^{-2}\int k_\sigma k_\tau k_\lambda d^4k/D$$

which would have been necessary if the integration were performed first. The very great simplification involved may be appreciated by consulting the Appendix of BF. As indicated previously, another simplification was obtained by taking the target electron at rest. The eight relevant Dirac traces were easily calculated by machine using a computer program recently developed by the authors to obtain general Dirac traces.[8] The output gave the traces, combined over the common denominator $(p_1\cdot q_1)^3(p_1\cdot q_2)D$, in terms of symbols representing the following independent quantities: m^2, $p_1\cdot q_1$, $p_1\cdot q_2$, $\mathbf{s}\cdot(\mathbf{u}_1\times\mathbf{u}_2)$, $\mathbf{s}\cdot(\mathbf{u}_1\times\mathbf{k})$, $\mathbf{s}\cdot(\mathbf{u}_2\times\mathbf{k})$, and the factors of D. Here $\mathbf{u}_1$ and $\mathbf{u}_2$ are unit vectors in the directions of $\mathbf{q}_1$ and $\mathbf{q}_2$, respectively. After the integrals over k and the proper factors[5] are supplied, the spin-dependent part of the differential cross section is found to be[9]

$$d\sigma_s/d\Omega=-\tfrac{1}{8}\alpha r_0^2\gamma^2\beta^{-4}X\mathbf{s}\cdot(\mathbf{u}_1\times\mathbf{u}_2), \tag{4}$$

where

$$\begin{aligned}X=&-d^{-1}\theta\operatorname{csch}\theta[12\beta^5+28\beta^4+12\beta^3-(12\beta^4+16\beta^3\\&+12\beta^2)\gamma+4\beta^2\gamma^2]+d^{-1}\ln(1+2\beta)[12\beta^4+16\beta^3+3\beta^2\\&-(2\beta^2+4\beta)\gamma+(2\beta+1)\gamma^2]+(1+2\beta)^{-2}[24\beta^4+22\beta^3\\&+6\beta^2+(4\beta^3+6\beta^2+2\beta)\gamma],\end{aligned} \tag{5}$$

with

$$\begin{aligned}\beta&=(p_1\cdot q_1)/m^2, & d&=\beta-\gamma-2\beta\gamma,\\ \gamma&=(p_1\cdot q_2)/m^2, & \theta&=\cosh^{-1}(1+\beta-\gamma).\end{aligned} \tag{6}$$

In Eq. (4) α is the fine structure constant and r_0 is the classical electron radius. Equation (4), valid in the laboratory system, will apply to a more general Lorentz frame if $\mathbf{s}\cdot(\mathbf{u}_1\times\mathbf{u}_2)$ is replaced by $(m^3\beta\gamma)^{-1}\times\det\{s,q_1,q_2,p_1\}$, the four-by-four determinant formed from the vectors s, q_1, q_2, and p_1 with s now the covariant spin vector.[10] Using the substitution symmetry of the traces under interchange of ingoing and outgoing particles, one can easily modify Eq. (4) so that it applies to the case where the target electron is unpolarized but the polarization of the recoil electron is observed.

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[8] S. C. Miller and R. M. Wilcox (to be published).

[9] *Note added in proof.* Similar calculations were reported by G. V. Frolov, JETP **12**, 1277 (1961). A number of terms given there agree with those of Eq. (5) but others disagree. His cross section does not reduce to the simple nonrelativistic form of Eq. (7) and it appears to approach $\pm\infty$ for small scattering angles.

[10] See, e.g., C. Fronsdal and H. Überall, Phys. Rev. **111**, 580 (1958), and other references given there. In our metric $s^2=-1$.

At low energies Eq. (4) reduces to the simple form,

$$d\sigma_s/d\Omega = -\tfrac{1}{3}\alpha r_0^2(\lambda_c/\lambda)^2\mathbf{s}\cdot(\mathbf{u}_1\times\mathbf{u}_2), \tag{7}$$

where λ_c is the Compton wavelength and λ is the wavelength of the incident radiation.

III. RESULTS

The ratio of the cross section in Eq. (4) to the unpolarized Compton cross section, $P=(d\sigma_s/d\Omega)/(d\sigma_0/d\Omega)$, has been plotted in Figs. 2–4. We have taken the case of maximum asymmetry by choosing s normal to the scattering plane with

$$\mathbf{s}\cdot(\mathbf{u}_1\times\mathbf{u}_2)/|\mathbf{u}_1\times\mathbf{u}_2|=1.$$

For various fixed angles Figs. (2) and (3) give P versus

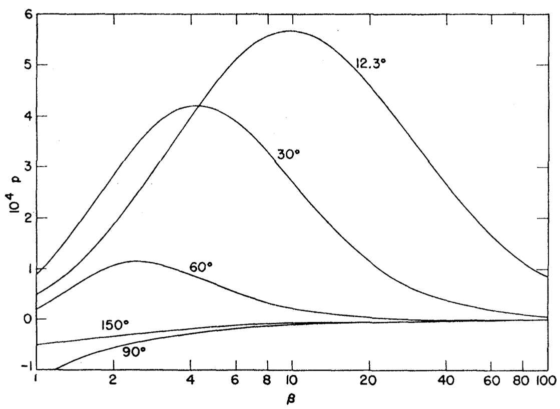

FIG. 3. The ratio of spin-dependent to spin-independent differential Compton cross section as a function of incident photon energy for the higher energy region.

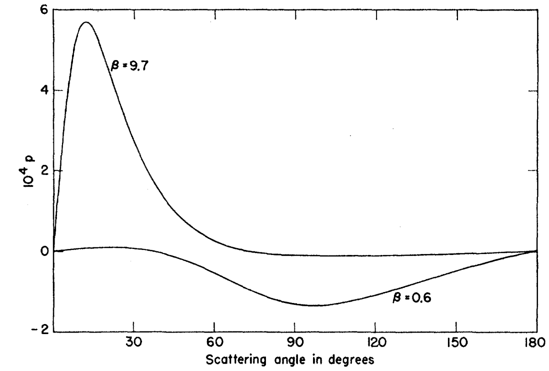

FIG. 4. The ratio of spin-dependent to spin-independent differential Compton cross section as a function of angle. The two energies used correspond to those that give the maximum effect for each sign.

β, the incident photon energy in units of the electron rest energy. Figure 4 gives this as a function of scattering angle for two energies. The maximum effect occurs for an angle of 12.3° with $\beta=9.7$. The smaller but broader maximum of opposite sign occurs at about 97° with $\beta=0.6$. The scattering asymmetry is nowhere very large as was also the case with the leptonic scatterings.[2,3]

ACKNOWLEDGMENTS

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