$$
\begin{aligned}
(I_0/I_\infty)_{\text{fcc}} \approx [& (1.0+3.8805092u+7.8281044u^2+13.419386u^3+14.181302u^4-1.7565205u^5-38.657407u^6 \\
& -60.306173u^7-93.189067u^8-77.874280u^9-75.149353u^{10}+27.847042u^{11}-48.093601u^{12})/ \\
& (1.0+3.8805092u+7.8281044u^2+13.419386u^3+14.181302u^4-1.7565205u^5-31.990740u^6 \\
& -34.436111u^7-41.001704u^8+11.5882963u^9+19.392664u^{10}+96.136905u^{11}-53.146689u^{12})]^{3/10},
\end{aligned}
$$

$$
\begin{aligned}
(I_0/I_\infty)_{\text{bcc}} \approx [& (1.0+3.5481591u+7.8806661u^2-1.1714755u^3-19.310113u^4-46.487253u^5-37.851754u^6 \\
& -26.364680u^7-39.358346u^8)/(1.0+3.5481591u+7.8806661u^2-1.1714755u^3-12.643446u^4 \\
& -22.832859u^5+14.686020u^6+19.158817u^7-9.9683914u^8)]^{3/10},
\end{aligned}
$$

$$
\begin{aligned}
(I_0/I_\infty)_{\text{sc}} \approx [& (1.0+3.5495749u-5.7826623u^2-24.453557u^3-8.2315745u^4+15.570039u^5+49.840321u^6 \\
& +13.621297u^7)/(1.0+3.5495749u-5.7826623u^2-17.786890u^3+15.432258u^4+17.018957u^5 \\
& +11.021825u^6-35.665913u^7)]^{3/10}.
\end{aligned}
$$

PHYSICAL REVIEW VOLUME 124, NUMBER 3 NOVEMBER 1, 1961

Measurement of the Refractive Index of Lucite by Recoilless Resonance Absorption*

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(Received June 20, 1961)

A method of frequency-modulating a monochromatic electromagnetic wave by varying the optical path length between the source and detector is described. The method has been applied to the measurement of the refractive index of Lucite for the 0.86 A radiation emitted from Co^{57}; the small frequency shift was detected by recoilless resonance absorption. The refractive index was found to be $1-n=(1.29\pm0.03)\times10^{-6}$, in agreement with classical theory.

THIS paper describes a method of frequency-modulating a monochromatic electromagnetic wave by varying the optical path length between the source and detector. The method has been applied to, and is described in terms of, the measurement of the refractive index of Lucite for the 14.4-kev radiation emitted from Co^{57}. The measured refractive index agrees, within the 2% experimental uncertainty, with the simple theory applicable when the radiation energy is much greater than the binding energy of the electrons in the refractive medium, as in this case. The technique is in principle applicable to the nearly monochromatic radiation emitted from optical-frequency masers.

It is instructive to consider the method from two points of view, first in terms of frequency modulation and then in terms of a Doppler shift. Consider a source S and an observer (in our case a recoilless resonance absorber) A separated by a distance x [Fig. 1(a)]. A wave of angular frequency ω emitted by S will have the form $e^{i\omega(t-x/c)}$ at A. If a length L of material with refractive index n is placed in the optical path, the wave becomes $e^{i\omega(t-x/c)+i\phi}$, where the phase advance

$$\phi=(1-n)\omega L/c. \tag{1}$$

If ϕ changes with time, the instantaneous frequency seen by A will be $(\omega+d\phi/dt)$. This is done by moving a wedge-shaped piece of material to produce a frequency shift

$$\frac{1}{2\pi}\frac{d\phi}{dt}=\Delta\nu=\nu\frac{(1-n)}{c}\frac{dL}{dt}. \tag{2}$$

An equivalent point of view considers the radiation as being Doppler-shifted during the refraction by the moving wedge [Fig. 1(b)]. As it leaves the wedge the radiation is deflected (toward the normal, since $n<1$) by an angle

$$\Delta\theta=(1-n)\tan\alpha.$$

The change in momentum of the photon is $\Delta p=p\Delta\theta$, and since the wedge is moving at a speed V it does work on the photon, increasing its energy by

$$\Delta E=V\Delta p=Vp(1-n)\tan\alpha=E[(1-n)/c]V\tan\alpha,$$

which is equivalent to Eq. (2) above.

For 14.4-kev radiation, the refractive index of Lucite is (see below)

$$(1-n)=1.29\times10^{-6},$$

so that

$$(\Delta\nu/\nu)_{14\text{ kev}}=4\times10^{-17}dL/dt.$$

The frequency shift thus obtained for reasonable values of dL/dt can be detected by recoilless resonance scattering.[1]

A schematic drawing of the experimental arrangement is shown in Fig. 2(a). The recoilless resonance

* This work is supported in part through a U. S. Atomic Energy Commission contract, by funds provided by the U. S. Atomic Energy Commission, the Office of Naval Research, and the Air Force Office of Scientific Research.

[1] R. L. Mössbauer, Z. Physik **151**, 124 (1958); R. V. Pound and G. A. Rebka, Jr., Phys. Rev. Letters **4**, 337 (1960).

apparatus has been described previously.[2] The 14.4-kev gamma rays from a Co^{57} source diffused into Armco iron passed through a rotating wheel, shown in profile in Fig. 2(b), then through a movable 0.5-mil Armco Fe absorber to the Be-window NaI(Tl) detector.

To construct the wheel, 12-in. diam. pieces of $\frac{1}{16}$-in. brass and $\frac{1}{4}$-in. Lucite were clamped together and 120 radial slots $\frac{1}{16}$ in. wide were cut through both at an angle of 60°. Since the gamma rays are stopped by the brass, they are allowed to pass through only one side of each Lucite tooth. As the wheel rotates, every gamma ray which passes through it does so when the thickness of Lucite in the tooth is changing in the same direction. Thus all the gamma rays detected undergo a frequency shift of the same direction and magnitude.

The absorption line profiles for four wheel speeds are shown in Fig. 3. The shift in the line position is evident. The high-speed runs, 1500 rpm clockwise and counterclockwise, show a broader line which we attribute to vibration transmitted through the air from the rotating wheel to the source and absorber. This effect diminished rapidly with decrease of angular speed and no attempt was made to alleviate it. At speeds below 1000 rpm, line broadening resulted in an error of less than 2% in the determination of the line shift.

The shift in the position of an accurately known line profile is most efficiently determined by measuring the change in counting rate at the maximum slope points of the absorption line profile, 0.013 cm/sec in this case. The counting rates for absorber speeds towards and away from the source were separately recorded for each of a set of speeds of the Lucite wheel; the resulting line shift as a function of wheel speed is shown in Fig. 4. A least-squares fit of the data (from 4 to 12×10^6 counts per point) between 900 rpm clockwise and 900 rpm counterclockwise yields a slope

$$\frac{\Delta v}{\Delta\omega_{\text{wheel}}}=c\frac{\Delta\nu}{\nu}\frac{1}{\Delta\omega_{\text{wheel}}}=(3.32\pm0.05)\times10^{-6}\frac{\text{cm/sec}}{\text{rpm}}, \quad (3)$$

where $\Delta\nu/\nu$ is the relative frequency shift.

The effective radius from the axis of the wheel to the gamma ray path was 14.2 cm so that

$$\frac{(dL/dt)}{\Delta\omega_{\text{wheel}}}=2.58\frac{\text{cm/sec}}{\text{rpm}}. \quad (4)$$

A combination of Eqs. (2)–(4), yields the refractive index:

$$(1-n)=\frac{c(\Delta\nu/\nu)}{(dL/dt)}=\frac{3.32\times10^{-6}}{2.58}=(1.29\pm0.04)\times10^{-6}.$$

The stated error includes uncertainties in the effective radius, the absorption line depth, and the absorber velocity.

This result is in agreement with the theoretical value obtained for the refractive index of a gas in which the binding energy of the electrons is much less than the

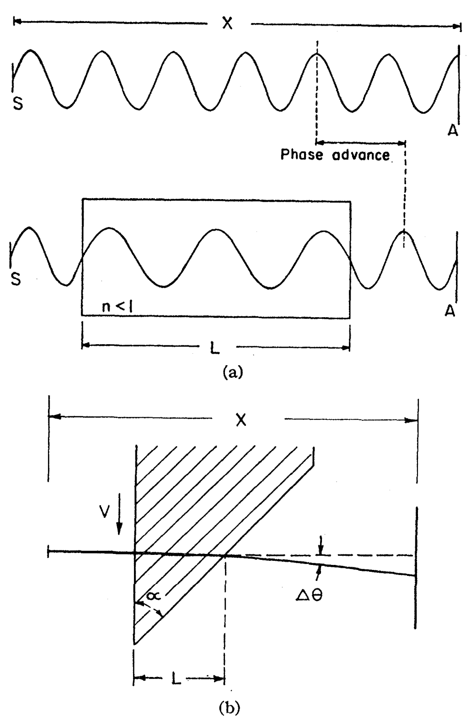

FIG. 1. (a) The phase advance produced by interposition of a length L of refractive material between source S and observer A. (b) The deflection $\Delta\theta$ of the beam when the refractive material of Fig. 1(a) is wedge-shaped.

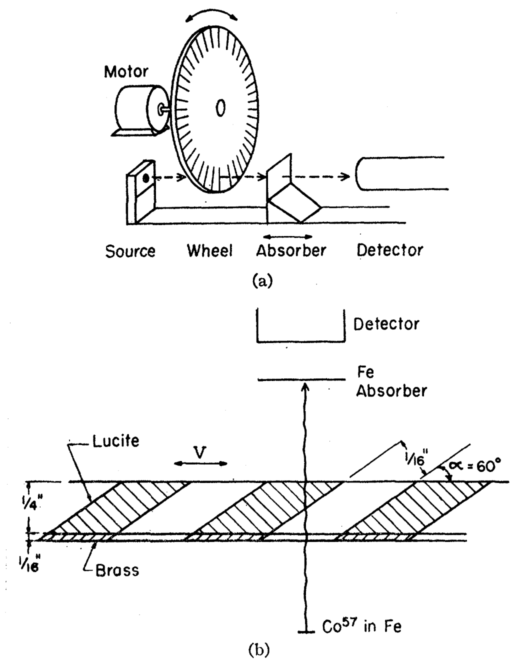

FIG. 2. (a) Schematic diagram of experimental arrangement. (b) Detail of the slotted wheel; not to scale.

[2] L. Grodzins and F. Genovese, Phys. Rev. 121, 228 (1961).

energy of the radiation. This condition holds for 14.4-kev radiation on Lucite ($C_5H_8O_2$) since E_K(oxygen) $=0.53$ kev. The result is[3]

$$(1-n)=N_0\frac{\rho Z}{2A}\frac{e^2}{m\omega^2\epsilon_0},$$

where the symbols have their conventional meaning; i.e., N_0 is Avogadro's number, ρ is the density, etc. For the case of Lucite and 14.4-kev radiation, Z/A

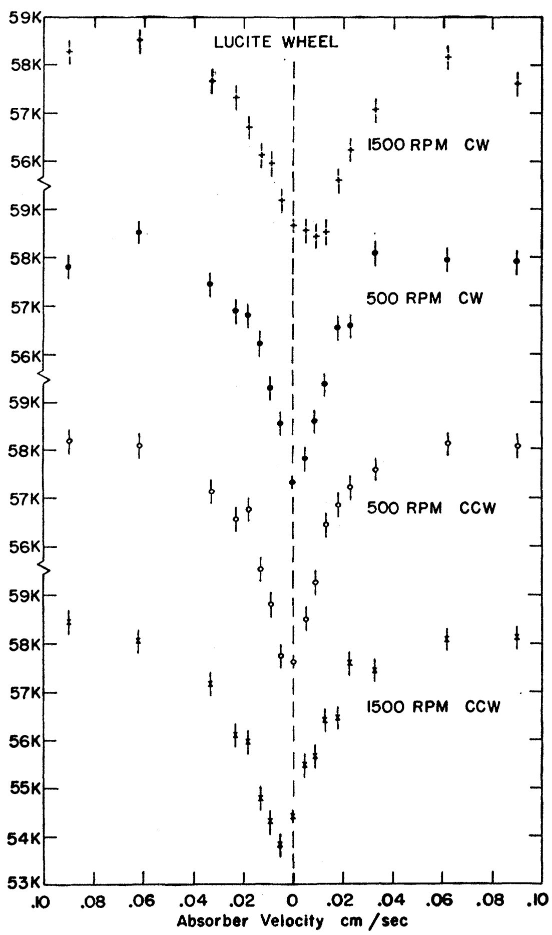

FIG. 3. Absorption line profile versus wheel speed.

[3] F. K. Richtmeyer and E. H. Kennard, *Introduction to Modern Physics* (McGraw-Hill Book Company, New York, 1947), 4th ed., pp. 522–527.

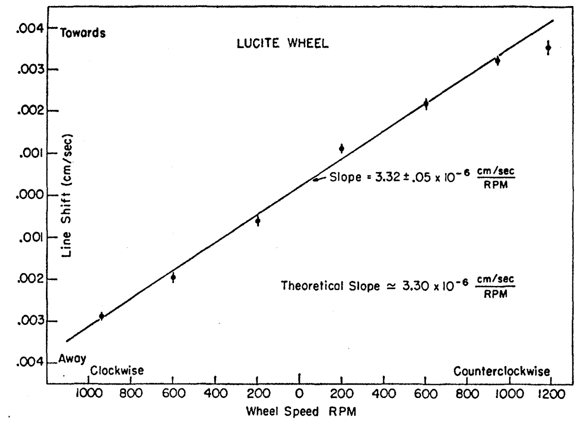

FIG. 4. Reduced data: Line shift versus wheel speed; the theoretical slope contains an uncertainty in the wheel radius. (See text.)

$=0.54$, $\rho=1.185$, and $\omega=2.185\times10^{19}$. Then

$$1-n=1.285\times10^{-6}.$$

The exact theory of the index of refraction, which takes into account the binding energies of the electrons,[4,5] yields a result differing from the above number by about 0.1%.

The measurement of the index of refraction at x-ray wavelengths is, of course, not new.[3] Indeed, Bearden[4] measured the refractive index of diamond at 1.39 A to an accuracy of 1 part in 10^4. The phase modulation technique can, if desirable, be made as accurate for those wavelengths observed by recoilless nuclear gamma emission.

We have shown that the frequency of a nearly monochromatic electromagnetic wave may be shifted by modulating the optical path between source and observer. The application to an optical-frequency maser where $\Delta\nu/\nu \lesssim 10^{-9}$ is evident. Since $(1-n)$ is $\sim10^6$ times as large for optical frequencies as for x rays, frequency modulation may be observed by varying either n or L. For example, L may be varied by vibrating a mirror [$1-n=2$ in Eq. (2)] from which the light is reflected. The corresponding experiment for recoilless gamma radiation has been reported by Ruby and Bolef,[6] who acoustically vibrated the source.

[4] J. A. Bearden, Phys. Rev. **54**, 698 (1938).
[5] J. A. Prins, A. Physik **47**, 479 (1928).
[6] S. L. Ruby and D. I. Bolef, Phys. Rev. Letters **5**, 5 (1960).