Two-Body Potential in Nuclear Matter

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An effective two-nucleon central potential inside nuclear matter is proposed on the basis of Barker's recent analysis of doublet splittings for $A\approx 16$. This potential has less singlet even and more singlet odd attraction than for free nucleons. It is inferred that nonlocal potentials are unavoidable for shell model calculations.

I. INTRODUCTION

MOST calculations on complex nuclei are still made by assuming the internucleon forces to be two-body, static, and central in character. Even under this simplification four independent types of exchange potential are possible, and it is customary to assign them identical spatial dependences, such as the Yukawa form. Then

$$V(r)=[W+BP^{\sigma}-HP^{\tau}-MP^{\sigma\tau}]V_0(e^{-\kappa r}/\kappa r), \quad (1)$$

where P^{σ}, P^{τ} are exchange operators. If the well depth V_0 is taken as an independent parameter, there is one arbitrary normalization of the other coefficients, which is conventionally $W+B+H+M=1$. The exchange parameters can also be specified in terms of the total isotopic spin T and real spin S of the two nucleons by the quantities A_{TS}, where

$$\begin{aligned} A_{01}&=W+B+H+M=1, & A_{00}&=W-B+H-M,\\ A_{10}&=W-B-H+M, & A_{11}&=W+B-H-M. \end{aligned} \quad (2)$$

Determination of these parameters has proved difficult and lengthy. Early proposals[1] were based on the notion that exchange should account for nuclear saturation, which means that $W+\frac{1}{2}(B-H)-\frac{1}{4}M=\frac{1}{16}(3A_{01}+3A_{10}+A_{00}+9A_{11})\approx 0$. Recent studies of nuclear saturation[2] have made this unnecessary and in fact untenable. Meanwhile, shell model studies of light nuclei have suggested other exchange mixtures.[3,4] These mixtures are supposed to represent *effective* two-body forces in nuclear matter under a static, central approximation and therefore do not have to reproduce exactly the scattering properties of free nucleons. Nevertheless, there should be some general correspondence—for example, the potential of Eq. (1) should be more strongly attractive in even than in odd states. One might also hope to explain major discrepancies between two-body forces for free and bound nucleons in terms of the action of nuclear matter.

The present note attempts to extract a form of Eq. (1) for nucleons embedded in nuclear matter that is compatible with most recent analysis, and to discuss its qualitative features.

II. DETERMINATION OF PARAMETERS

Selected doublet splittings in light nuclei provide the basis for one of the most careful and extensive analyses of exchange parameters to date.[5] The resulting two-body force laws of Yukawa shape are not unique but form a one-parameter family approximated by

$$\begin{aligned} &A_{10}=0.20w, \quad A_{00}=0.03+0.31w,\\ &A_{11}=-0.80+0.47w,\\ &V_0=46+(1/7)(7w-2.5)^2\text{ Mev}, \quad 1\lesssim w\lesssim 3.5. \end{aligned} \quad (3)$$

The best fits obtained in reference 5 do not really distinguish between values of w over the range indicated in Eq. (3), which allows a considerable variation in the potential. Since Eq. (3) is a one-parameter family, however, it should be possible to determine w by comparison with various simple nuclear quantities. We shall use three such comparisons, with (a) the exchange quantity y deduced for the $E1$ giant resonance[6]; (b) the average value of the potential in nuclear matter[7]; (c) the free n-p system at low energies. According to the remarks above, item (c) can be taken only as providing general guidance for the effective forces inside nuclear matter, although a number of earlier exchange proposals have included the low-energy free-nucleon value $A_{10}/A_{01}\approx 0.7$ as a basic assumption.

(a) The relevant exchange quantity here is $y=2(3A_{01}+A_{10}-A_{00}-3A_{11})/(3A_{01}+3A_{10}+A_{00}+9A_{11})$, for which an empirical value $y\sim 1$ (error of a factor 2) has been suggested.[6] In spite of this large uncertainty, y is sufficiently sensitive to limit w to the corresponding values

$$w=2.0\pm 0.5. \quad (4a)$$

(b) Analysis of nuclear saturation yields a relation[7] for the effective two-body force law in nuclear matter $\int -\bar{V}d^3r=(4\pm 1)\times 10^2$ Mev f^3. Using the form of Eqs.

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[7] R. Karplus and K. M. Watson, Am. J. Phys. **25**, 641 (1957); L. C. Gomes, J. D. Walecka, and V. F. Weisskopf, Ann. Phys. **3**, 241 (1958); K. Kumar and R. K. Bhaduri, Phys. Rev. **122**, 1926 (1961).

(1) and (2) for V, one has $\bar{V}=(V_0/16)(9A_{11}+3A_{10}+3A_{01}+A_{00})e^{-\kappa r}/(\kappa r)$. This condition places quite narrow limits on w, which appears quadratically in V_0. Thus when $\kappa^{-1}=1.4$ f,

$$w=1.5\pm0.3. \tag{4b}$$

(c) The free n-p scattering lengths are approximately fitted by $w\approx1.1$ for the ^3S state and $w\approx2.4$ for the ^1S state. Both these values lie within the range indicated in Eq. (3), and are represented by

$$w=1.9\pm0.7. \tag{4c}$$

A weighted average of the values in Eqs. (4a)–(4c) is

$$w=1.7\pm0.2. \tag{5}$$

The limits of uncertainty in Eq. (5) are probably optimistic, but there can be no doubt that the acceptable range of w is considerably reduced from that in Eq. (3). The potential parameters corresponding to Eq. (5) are approximately

$$\begin{aligned} &A_{10}=0.34\pm0.04, \quad A_{00}=0.56\pm0.06,\\ &A_{11}=0\pm0.10, \qquad V_0=60\pm5 \text{ Mev}. \end{aligned} \tag{6}$$

The values obtained in Eq. (6) can be fed back into the previous considerations for a rough indication of the most appropriate range parameter in the effective force. The parameters in Eq. (3) were determined for $\kappa^{-1}=1.41$ f, but the results are insensitive to κ, provided that V_0 is multiplied by a scaling factor according to $V_0/\kappa^2=V_0'/\kappa'^2=\text{const}$.[5] The same is true of (4a) and (4c), but not of (4b): under the scaling condition,

$$\int \bar{V}d^3r=(\kappa/\kappa')\int \bar{V}'d^3r, \tag{7}$$

so that this quantity can be used to determine κ'. With the parameters of Eq. (6) and $\kappa^{-1}=1.41$ f, $\int -\bar{V}d^3r=(6\pm2)\times10^2$ Mev f^3. Although this overlaps the empirical value of $(4\pm1)\times10^2$ Mev f^3 quite satisfactorily, the agreement would be improved if κ^{-1} were reduced by a factor of 0.7. The indicated range parameter is then

$$\kappa^{-1}\approx(1.0\pm0.3) \text{ f}. \tag{8}$$

III. DISCUSSION

The present effective two-nucleon potential in nuclear matter compares with that for free nucleons as follows: the triplet potentials are about the same, being strongly attractive for even states and weak for odd ones. The singlet potentials are altered—the even states are less strongly attractive in nuclear matter, the odd states more so. One may perhaps make this plausible by the following arguments. In analyses of free nucleon-nucleon scattering the hard core always appears most prominently in the ^1S state[8]; it may have an appreciable effect even at laboratory energies as low as 30 Mev, while the action of tensor forces makes it difficult to be so definite about triplet states. The average momentum in nuclear matter corresponds to nucleon-nucleon collisions at about 80 Mev in the lab system. Thus it is possible that the hard core is particularly effective for the ^1S potential in nuclear matter and accounts for the reduction of A_{10}/A_{01} by about a factor of 2 from the value characteristic of free nucleons at laboratory energies $\lesssim10$ Mev.

The singlet odd potential shows the converse behavior: with increasing relative momentum it becomes more attractive, indicating the presence of a "soft core," while at sufficiently large distances it should reflect the one-pion exchange term and be the most strongly repulsive of any potential.

These qualitative features accord remarkably well with calculations of the Taketani school.[9] Figures 6–1 and 6–2 of reference 9 show a stronger repulsive core for the singlet even than for the triplet even potential, and for the singlet odd potential a repulsive exterior region with a strongly attractive core.[10] Empirical fits to the nucleon-nucleon scattering data up to about 300 Mev do not generally show these features[11]: The central part of the triplet even potential looks more repulsive than the singlet even potential; and the singlet odd potential is repulsive at all distances. Of course the effective triplet potential in nuclear matter may have substantial tensor and spin-orbit contributions, so that direct comparison is difficult. For the singlet odd potential, however, one may argue that doublet splitting calculations are possibly more informative than nucleon-nucleon scattering. For free nucleon scattering the relative statistical weights of triplet odd, triplet even, singlet even, and singlet odd states are roughly the usual 9:3:3:1, while in doublet splitting all the potentials have about equal weights on the average.[5]

The effective two-body force determined by Elliott and Flowers[3] differs from Eq. (6) mainly by requiring A_{10}/A_{01} to resemble closely the value for free nucleons; this may reflect determination of the ratio[12] from low-lying states of the extra-core nucleons at $A=18$, which are not very closely bound into nuclear matter. This comparison emphasizes the likelihood of a rather strong density dependence for the effective two-nucleon potential in nuclear matter. The present potential is probably more representative of nuclear matter than other conventional exchange mixtures, although it also

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[9] Y. Nogami and H. Hasegawa, reported in S. Machida and T. Toyoda, Progr. Theoret. Phys. Suppl. **3**, 106 (1956). Also, S. Gartenhaus, Phys. Rev. **100**, 900 (1955).

[10] This last feature is maintained in more recent meson-theoretical calculations: M. Sugawara and S. Okubo, Phys. Rev. **117**, 605 (1960).

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[12] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A229**, 536 (1955).

may not correspond to fully saturated conditions, since it is based[5] on light nuclei. Another apparent example of density dependence has long been recognized in the spin-orbit potential, which seems to increase in strength as a shell fills[13]: For example, the splitting at $A=17$ is only about half as strong as the p-hole splitting at $A=15$.

Such strong density dependences suggest that shell model calculations cannot hope to find general agreement with experiment by using a fixed set of strictly local two-body potentials.

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Beta-Gamma Directional Correlation in the Decay of Eu^{154}†

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The directional correlation for the (1855 kev-beta) (123 kev-gamma) cascade in the decay of Eu^{154} has been measured. The directional correlation coefficient A_2 varies from -0.15 at a beta energy of 1100 kev to -0.18 at a beta energy of 1600 kev. In terms of a single nuclear matrix element parameter, ζ_1, the directional correlation suggests $\zeta_1=2.6$ or -0.2 while the beta spectral shape correction suggests $\zeta_1=1.3$. The results suggest that the attenuation of ordinary first-forbidden matrix elements relative to B_{ij} is less marked in Eu^{154} than in the comparable transition in Eu^{152}. Interpretation of the data in terms of a less restrictive formulation of the theory is investigated. Relations between matrix element ratios may be found but no unique set of matrix element ratios is demanded by the experimental data.

INTRODUCTION

THE principal transitions in the decay of Eu^{154} are shown in Fig. 1.[1] The 1855-kev beta transition is believed to be first forbidden with spin change 1. The $\log ft$ product for the 1855-kev beta group is about 12.4 which is unusually high for a first forbidden transition. A negative anisotropy for the (1855-kev beta) (123-kev gamma) directional correlation has been found[2] which is somewhat lower in magnitude than the directional correlation for the comparable cascade in Eu^{152}.[3] Additional measurements of the directional correlation as a function of beta energy in Eu^{154} have been reported.[4,5] The shape of the spectrum for the 1855-kev beta group has been measured and found to be intermediate between an allowed and a unique shape,[6] but somewhat closer to the allowed shape than the comparable transition in Eu^{152}.[6]

In the present report we present our final data for the directional correlation in Eu^{154}. Comparison is made with the approximate formulation of beta-decay theory in terms of a single nuclear matrix element parameter. Interpretation of the data in terms of a less restrictive formulation of the theory is also discussed.

EXPERIMENTAL PROCEDURE AND RESULTS

The Eu^{154} was produced at the Oak Ridge National Laboratory by irradiation of isotopically enriched Eu^{153} (about 0.6% Eu^{151}). Two different irradiations were

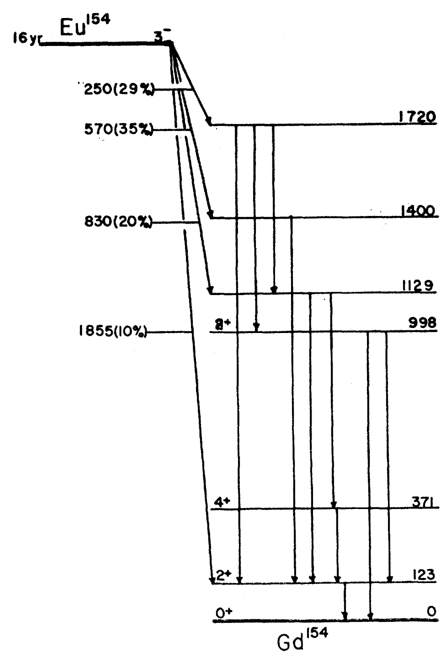

FIG. 1. Principal transitions in the decay of 16-year Eu^{154}.[1]

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