Stability of Nuclear Octupole Deformation

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The effect of octupole deformation on the single-particle energy levels of Nilsson has been investigated by an exact diagonalization method, and the total single-particle energies for a number of rare-earth and actinide nuclei are calculated. It has been found that while in the rare-earth region a stable octupole deformation is possible energetically, the favored shape for the actinide nuclei is the familiar prolate spheroid.

THE possibility of an octupole type of nuclear deformation was suggested by the systematic occurrence of odd-parity rotational levels in even nuclei in the neighborhood of radium.[1] Strutinski[2] pointed out that the mixing of single-nucleon states of opposite parity in a pear-shaped nucleus tends to stabilize such a deformation. One way of examining the stability of octupole deformation of a particular nucleus is to calculate the single-particle energies in a model potential whose equipotential surfaces run parallel to the nuclear surface, and to add the single-particle energies for the particular configuration. One of course assumes that the total energy is first minimized for the spheroidal potential, in the manner used by Mottelson and Nilsson.[3] The validity of this method has been discussed recently by Lee and Inglis.[4] Although the neglect of residual two-particle interactions is not by any means justified in such calculations, it is hoped that the essential conclusion will remain unaffected even after such detailed calculations. Lee and Inglis[5] calculated the single-particle energies in a pear-shaped potential superimposed on a spheroidal harmonic oscillator potential. The total energy for a pear-shaped nucleus was found to be slightly greater than that of a spheroidal one, thus making it unstable. This calculation was improved by us[6] by introducing $\mathbf{l}\cdot\mathbf{s}$ and $\mathbf{l}^2$ terms into the Hamiltonian, and it was seen that the stable octupole shape is not favored energetically in the radium region.

In both these works the additional energy due to octupole deformation was calculated by the second order perturbation method. In our work we used Nilsson's wave function for the perturbation calculations and neglected the effect of $N\pm2$ mixing. Thus, the above-mentioned works cannot claim a great deal of accuracy and the problem of the stability of the octupole type of deformation has still remained open.

[1] F. Stephens, F. Asaro, and I. Perlman, Phys. Rev. **100**, 1543 (1955).
[2] V. M. Strutinski, J. Nuclear Energy **4**, 523 (1957).
[3] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 16 (1955); B. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Skrifter **1**, No. 8 (1959).
[4] K. Lee and D. R. Inglis, Phys. Rev. **120**, 1298 (1960).
[5] K. Lee and D. R. Inglis, Phys. Rev. **108**, 774 (1957).
[6] I. Dutt and P. Mukherjee, Progr. Theoret. Phys. (Kyoto) **22**, 814 (1959).

The recent discovery[7] of $E3$ Coulomb excitation of the 3− state in Th^{232} by O^{16} nuclei indicates further the possibility of such octupole deformations. The need for an accurate calculation is thus essential, and here we shall present the result of the exact diagonalization of the single-particle Hamiltonian in a pear-shaped potential.

The Hamiltonian we have used is[6]

$$\begin{aligned}H_{\mathrm{pear}}=&-\tfrac{1}{2}\hbar\omega_0(\nabla^2-r^2)\\&+\chi\hbar\omega_0{}^0r^2\{[-\tfrac{4}{3}(\pi/5)^{\frac{1}{2}}\eta+(5/64\pi)(a_3{}^2a_2/\chi)]\\&\times[9(3/140\pi)^{\frac{1}{2}}a_3Y_{10}(\theta\varphi)+Y_{20}(\theta\varphi)\\&+(a_3/a_2)Y_{30}(\theta\varphi)]-2\mathbf{l}\cdot\mathbf{s}-\mu\mathbf{l}^2\},\quad(1)\end{aligned}$$

where the parameters μ and χ are taken from Nilsson.[3]

In diagonalizing (1) we have chosen the following representation:

$$|j_z\rangle=\sum_{Nll_zs_z}A_{Nll_zs_z}|Nll_zs_z\rangle,\qquad(2)$$

where $|Nll_zs_z\rangle$ are the eigenkets corresponding to the spherical limit ($a_2=a_3=0$).

The Hamiltonian (1) is solved for $a_2=-0.62$, which roughly corresponds to most of the stable spheroidal deformations of the nuclei under consideration, and for different values of a_3 ranging from 0 to 0.1. In Eq. (2) the summation over N, the principal quantum number, includes states up to $N=6$, which is sufficient for most of the nuclei under review. The highest order matrix is then 28×28 for $j_z=\frac{1}{2}$. These matrices are diagonalized by the URAL electronic computer of the Indian Statistical Institute, Calcutta.

An examination of the eigenvalues shows that in the region Z (or N)$=40$ to 86, some fifteen single-particle levels are depressed in energy as a_3 is increased. For Z (or N)>90, there are only eight such levels. So in the rare earth region both the proton and neutron core of the nucleus may show stability towards octupole deformation. In the actinide region, although the proton core may show such stability, the outer neutron core will have a tendency to make the pear shape unstable. The total single-particle energies for those nuclei, which

[7] B. Elbek, *Proceedings of the International Conference on Nuclear Structure, Kingston, 1960* (University of Toronto Press, Toronto, 1960), p. 563.

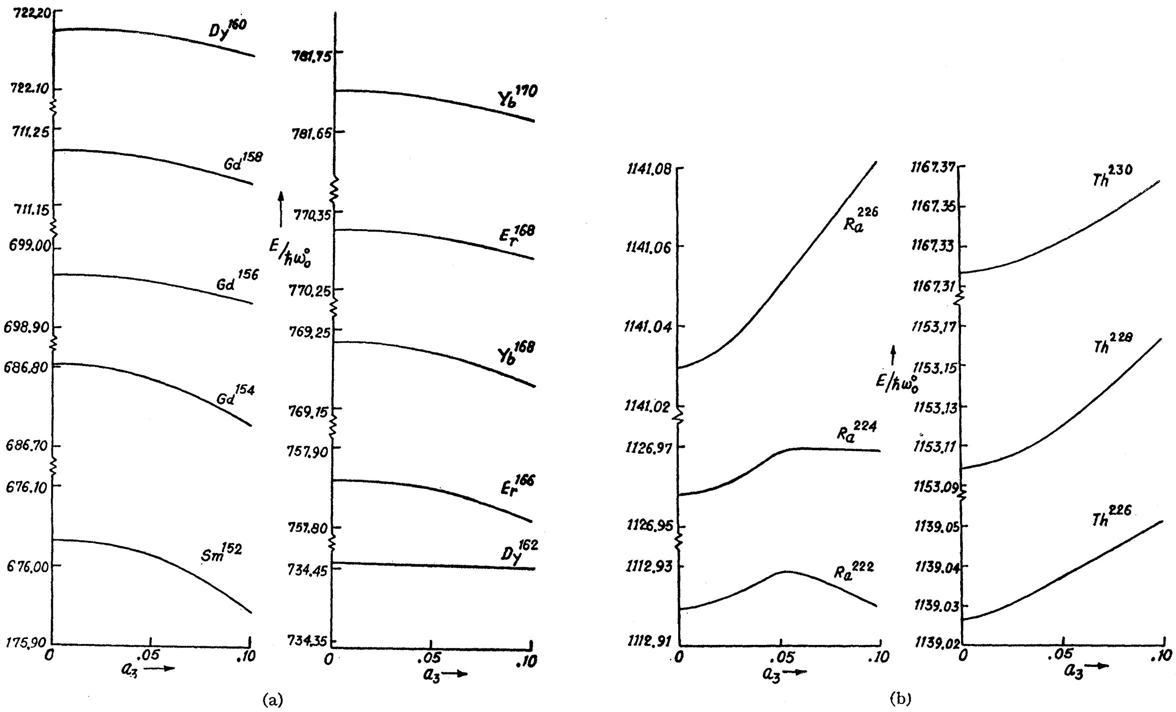

FIG. 1. (a) The total single-particle energies for various rare-earth nuclei as a function of the octupole parameter a_3. (b) The total single-particle energies for various actinide nuclei plotted against the octupole parameter a_3.

have stable spheroidal deformation a_2 around -0.62, are plotted against a_3 in Figs. 1(a) and 1(b). The combinations of single-particle levels that have been chosen to calculate such total energies give the lowest energy at each a_3. From the figures it is obvious that some of the rare-earth nuclei have equilibrium values of a_3 lying above 0.1. It is expected that at large a_3 the total energy will again increase due to the volume-preserving term in the Hamiltonian (1).

The total-energy curves in the actinide region show a different trend. The stable shape favored energetically corresponds to $a_3=0$, making the pear shape unstable. However, for these heavy nuclei, inclusion of states for $N\geqslant 7$ may profoundly alter these conclusions.