Electromagnetic Potentials in Quantum Mechanics*

HENRI E. MITLER†
Physics Department, Brandeis University, Waltham, Massachusetts
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Electromagnetic potentials are found to be significant in quantum theory, even when the fluctuations of the vacuum are taken into account. Conditions are found under which the fluctuations will, nevertheless wash out an interference pattern.

IN a paper by Aharonov and Bohm[1] it is demonstrated that in quantum mechanics, electromagnetic potentials have physical significance, contrary to the case in classical mechanics, where they are merely mathematical fictions which facilitate the computation of fields.

In doing this, Aharonov and Bohm assumed that quantum-mechanical electrons interact with classical electromagnetic fields. The question naturally arises, to what extent this effect—the measurability of the electromagnetic potentials—is washed out by the zero-point fluctuations of the fields. That is, we would like to take into account the *dynamical* character of the vacuum by quantizing the electromagnetic field.

Aharonov and Bohm show that the additional phase difference in the wave functions between two electron beams which have been subjected to two different potentials is just

$$\Delta\varphi=(e/\hbar)\oint A^{\mu}dx_{\mu},$$

i.e., just the change of phase of one electron resulting from traveling around a closed path which follows one beam from source to screen, and the other beam back to the source. This difference then determines the shift in the observed interference pattern between the two beams. (This shift is independent of position, i.e., the whole pattern will shift.)

Now consider their first experiment: It is a two-slit diffraction experiment, where the packets pass through two long metal pipes before they have a chance to interfere with each other—this they do only upon emergence from these Faraday cages. The top packet (or beam, if we ignore fringing fields) has its phase changed relative to the bottom one. One need therefore only consider this one packet (beam), and the extent to which its phase difference with any reference phase —say, that at the origin of its path—becomes blurred. Classically, the action is

$$S(t)=-\int_0^t H_{\mathrm{int}}(t')dt',$$

where the interaction is between the electron and the electromagnetic field:

$$H_{\mathrm{int}}(t)=\int \mathcal{H}_{\mathrm{int}}(x)d^3x,$$

and

$$\mathcal{H}_{\mathrm{int}}(x)=j_\mu(x)A_\mu(x),$$

where the summation convention over $\mu=1, 2, 3, 4$ is used, and

$$x_4=ix_0=ict.$$

(Boldface quantities are three-vectors, italics are four-vectors.)

The current density for a smeared-out charge distribution is

$$j_\mu(x)=-e\rho(x)v_\mu(x),$$

where $-e$ is the (negative) charge of the electron, $\rho(x)$ the density, and v_μ the μth component of its four-velocity. For a strongly localized distribution

$$\rho(x)\simeq\delta(\mathbf{x}-\mathbf{x}_0),$$

where

$$\mathbf{x}_0=\mathbf{x}_0(t)$$

is the centroid of the particle at time t.

In making the transition to quantum mechanics, $A_\mu(x)$ becomes the operator $\hat{A}_\mu{}^{\mathrm{S}}(x,0)$ in the Schrödinger picture (indicated by the superscript S), at time $t=0$. If we do not second-quantize the electron-positron field, j_μ becomes

$$-es_\mu=-(e\hbar/2mi)[\bar{\psi}\partial_\mu\psi-\psi\partial_\mu\bar{\psi}],\tag{1}$$

and $e^{iS/\hbar}\Phi_{\mathrm{original}}$ becomes a WKB approximation to the solution of the Schrödinger equation

$$H\Phi=i\hbar(\partial\Phi/\partial t),$$

where H includes the interaction, and Φ_{original} is the (exact) solution of the equation without the interaction. The uncertainty in the action then is defined by the second moment:

$$(\Delta S)^2\equiv\langle S^2\rangle-\langle S\rangle^2,$$

where the brackets indicate taking the expectation value in a state of the system which will be defined by Eq. (2).

Assuming the vacuum is unique, this expression is gauge invariant. From this follows the first result:

The potential has no significance as far as the *fluctuations* are concerned, and the Aharonov-Bohm effects is not "washed out." That is, in the experiments they

* Work supported by the National Science Foundation.
† Present address: Smithsonian Astrophysical Observatory, Cambridge, Massachusetts.
[1] Y. Aharonov and D. Bohm, Phys. Rev. 115, 485 (1959).

propose, the potential will be manifest by a shift in the interference fringes from the no-potential case, and this effect will be masked by the vacuum fluctuations only to the same extent as the ordinary (no-potential) interference pattern is thus masked. It is still interesting to see what effect is predicted, if any.

If H_{int} were zero, then the state vector for the system electron+electromagnetic fields could be written, in the Schrödinger picture, as

$$\Phi(\mathbf{r},t)=\psi^{S}(\mathbf{r},t)\Phi^{S}, \tag{2}$$

where Φ^{S} is the state vector of the electromagnetic field. In the case where there are no fields, $\Phi=\Phi_0$, corresponding to a transverse vacuum—i.e., the state of no real transverse photons. In case $H_{\text{int}}\neq 0$, Eq. (2) is a first-order approximation to the true state vector. We shall use this approximation.

Since the electromagnetic four-potential $\hat{A}_\mu$ can be written in covariant form in the interaction picture, we shall work in the latter: The Hamiltonian of the system is

$$H=H_0+H_{\text{int}}=H_0{}^f+H_0{}^r+H_{\text{int}},$$

where $H_0{}^f$ is the Hamiltonian for the free electron, $H_0{}^r$ that for the radiation field. Then in the vacuum,

$$e^{iH_0t}\Phi(\mathbf{r},t)=\psi(\mathbf{r},0)\Phi_0, \tag{3}$$

where the omission of superscripts indicates that the quantities are in the interaction picture. The equation conjugate to (3) is

$$\Phi^\dagger(\mathbf{r},t)=\Phi_0{}^\dagger\psi^*(\mathbf{r},0)e^{iH_{0}(r)t}.$$

(We use the natural units $\hbar=c=1$ wherever no confusion can arise.) Collecting all terms, we find that

$$\langle\Phi|S(t)|\Phi\rangle=e\left\langle\Phi_0\middle|\int\psi^*(\mathbf{r},0)e^{iH_0t}\int_0^t dt'\int s_\mu{}^S(\mathbf{x},t')\right.$$
$$\left.\times A_\mu{}^S(\mathbf{x},0)d^3xe^{-iH_0t}\psi(\mathbf{r},0)d^3r\middle|\Phi_0\right\rangle.$$

Now $H_0{}^f(\mathbf{r})$ commutes with any operator function of $\mathbf{x}$, so that

$$e^{iH_0t}s_\mu{}^S(\mathbf{x},t')A_\mu{}^S(\mathbf{x},0)e^{-iH_0t}=s_\mu{}^S(\mathbf{x},t')A_\mu(x),$$

and we can integrate over $\mathbf{r}$ immediately, the integral just giving us unity, so that

$$\langle S\rangle=e\int_0^t dt'\int s_\mu{}^S(\mathbf{x},t')\langle\Phi_0|A_\mu(x)|\Phi_0\rangle d^3x.$$

In the transverse-photon vacuum, which we have here, the Gupta-Bleuler formalism[2] shows us that

$$\langle A_\mu(x)\rangle=\partial_\mu\Lambda(x),$$

where Λ is a function satisfying

$$\Box^2\Lambda(x)=0.$$

Let us work in the gauge where there are no longitudinal (and therefore scalar) photons at all, i.e., $\Lambda=0$. This is a total (photon) vacuum, i.e., $\Phi_0=|0\rangle$, and

$$\langle 0|A_\mu|0\rangle=0 \Rightarrow \langle S\rangle=0. \tag{4}$$

Similarly,

$$S^2(t)=\int_0^t H_{\text{int}}(t')dt'\int_0^t H_{\text{int}}(t'')dt''$$
$$=e^2\int_0^t dt'\int_0^t dt''\int s_\mu(\mathbf{x},t')A_\mu{}^S(\mathbf{x},0)d^3x$$
$$\times\int s_\nu(\mathbf{y},t'')A_\nu{}^S(\mathbf{y},0)d^3y$$
$$\Rightarrow\langle\Phi|S^2|\Phi\rangle$$
$$=e^2\left\langle 0\middle|\exp(iH_0{}^rt)\int_0^t dt'\cdots d^3y\exp(-iH_0{}^rt)\middle|0\right\rangle$$
$$=e^2\int_0^t dt'\int_0^t dt''\left\langle 0\middle|\int A_\mu(\mathbf{x},t)s_\mu(\mathbf{x},t')d^3x\right.$$
$$\left.\times\int A_\nu(\mathbf{y},t)s_\nu(\mathbf{y},t'')d^3y\middle|0\right\rangle.$$

Now

$$\langle 0|A_\mu(\mathbf{x},t)A_\nu(\mathbf{y},t)|0\rangle=i\delta_{\mu\nu}D^+(x-y)$$
$$=\frac{\delta_{\mu\nu}}{2(2\pi)^3}\int\frac{d^3k}{k}e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})},$$

and summing over ν, we find

$$\langle S^2\rangle=\frac{e^2}{16\pi^3}\int_0^t dt'\int_0^t dt''\int\int\int s_\mu(\mathbf{x},t')s_\mu(\mathbf{y},t'')$$
$$\times e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\frac{d^3k}{k}d^3xd^3y. \tag{5}$$

In the presence of an electromagnetic field consisting of just n transverse photons, Eq. (4) still holds, since A_μ is linear in creation and annihilation operators. Similarly, Eq. (5) still holds, too, except for a factor $(n+1)$ on the right-hand side. In order to obtain an explicit result, we must calculate the electronic current. Thus, consider the electron's wave packet:

$$\psi(\mathbf{r},t)=(2\pi)^{-\frac{3}{2}}\int g(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}}e^{-i\omega_kt}d^3k,$$

where $\hbar\omega_k=E_k=\hbar^2k^2/2m$, nonrelativistically.[3] If the packet, with momentum $\hbar k_0$, has its centroid moving in the z direction, and is centered at the origin when $t=0$, we can take the momentum-space packet to be

$$g(\mathbf{k})=N\exp\{-\tfrac{1}{2}[k_x{}^2\sigma_x{}^2+k_y{}^2\sigma_y{}^2+(k_z-k_0)^2\sigma_z{}^2]\},$$

with $N=(\sigma_x\sigma_y\sigma_z/\pi^{\frac{3}{2}})^{\frac{1}{2}}$, where σ_x, σ_y, σ_z are the half-widths

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[2] K. Bleuler, Helv. Phys. Acta **23**, 567 (1950), S. N. Gupta, Proc. Phys. Soc. (London) **A63**, 681 (1950).

[3] In using the nonrelativistic expression for the energy, Eq. (1) will give us an s_4 which will no longer keep $s_\mu s^\mu$ invariant.

of the packet. Then with $\tau \equiv \hbar t/2m$,

$$\psi(\mathbf{r},t)=\frac{N\exp\left[izk_0-i\tau k_0^2-\dfrac{x^2}{2(\sigma_x^2+2i\tau)}-\dfrac{y^2}{2(\sigma_y^2+2i\tau)}-\dfrac{(z-2k_0\tau)^2}{2(\sigma_z^2+2i\tau)}\right]}{(\sigma_x^2+2i\tau)^{\frac{1}{2}}(\sigma_y^2+2i\tau)^{\frac{1}{2}}(\sigma_z^2+2i\tau)^{\frac{1}{2}}}.$$

It follows that the density is

$$\rho(\mathbf{r},t)=\frac{N^2}{(\Sigma_x\Sigma_y\Sigma_z)^2}\exp-\left\{\frac{x^2\sigma_x^2}{\Sigma_x^4}+\frac{y^2\sigma_y^2}{\Sigma_y^4}+\frac{(z-2k_0\tau)^2\sigma_z^2}{\Sigma_z^4}\right\}, \quad (6)$$

where

$$\Sigma_i^4\equiv\sigma_i^4+4\tau^2.$$

Since

$$2\tau=\hbar t/m\leq(\hbar/mv)L=L/k_0,$$

where L is the distance from source to screen, the second term will be much smaller than the first whenever

$$k_0\sigma_i\gg L/\sigma_i.$$

Thus for $L=10$-cm and 50-ev electrons, $k_0\sim 4\times 10^8$ cm^{-1}, and the condition is easily satisfied for $\sigma_i\geq 10^{-3}$ cm. This corresponds to no appreciable spreading of the packet, i.e. the geometrical optics limit.

We then find, for the flux,

$$\mathbf{s}=\left[\mathbf{v}+\frac{\hbar^2 t}{m^2}\left(\frac{\hat{\imath}x}{\Sigma_x^4}+\frac{\hat{\jmath}y}{\Sigma_y^4}+\frac{\hat{k}(z-vt)}{\Sigma_z^4}\right)\right]\rho(\mathbf{r},t).$$

As above, however, we find that $\Sigma_i\simeq\sigma_i$, and the term in parentheses is very much smaller than the first term. Hence,

$$s_i\simeq v_i\rho(\mathbf{r},t). \quad (7)$$

Within exactly the same kind of approximation, we find that

$$s_4\simeq(iv^2/2c)\rho(\mathbf{r},t), \quad (8)$$

and thus calling

$$iv^2/2c\equiv V_4,\quad v_i\equiv V_i, \quad (8a)$$

we have

$$s_\mu\simeq V_\mu\rho. \quad (8b)$$

For heuristic purposes, consider the particle approximation

$$\rho(\mathbf{r},t)\simeq\delta(\mathbf{r}-\mathbf{v}t) \quad (9)$$

to (8); then the space integrals in Eq. (5) can be done immediately:

$$\int s_\mu(\mathbf{x},t')e^{i\mathbf{k}\cdot\mathbf{x}}d^3x\simeq V_\mu\int\rho(\mathbf{x},t')e^{i\mathbf{k}\cdot\mathbf{x}}d^3x\simeq V_\mu e^{i\mathbf{k}\cdot\mathbf{v}t'},$$

and similarly

$$\int s_\mu(\mathbf{y},t'')e^{-i\mathbf{k}\cdot\mathbf{y}}d^3y\simeq V_\mu e^{-i\mathbf{k}\cdot\mathbf{v}t''},$$

so that

$$\langle S^2\rangle\simeq\frac{e^2V_\mu V_\mu}{16\pi^3}\int_0^t dt'\int_0^t dt''\int e^{i\mathbf{k}\cdot\mathbf{v}(t'-t'')}\frac{d^3k}{k}.$$

In integrating over $\mathbf{k}$, we must use a high-momentum cutoff, k_m. Then calling $vtk_m\equiv\xi$, we find

$$\langle S^2\rangle\simeq\frac{e^2}{2\pi^2}\frac{V_\mu V_\mu}{v^2}[\mathrm{Ci}\xi-\ln\gamma\xi+\xi\,\mathrm{Si}\xi+\cos\xi-1],$$

where γ is the Euler-Mascheroni constant. If $k_m\rightarrow\infty$, the bracket approaches $\pi\xi/2$, and writing $vt=L$, the distance traveled to the screen (since it is at that point that we are really interested in the phase uncertainty),

$$\langle S^2\rangle\simeq\frac{e^2}{4\pi}\frac{V_\mu V_\mu}{v^2}(Lk_m).$$

Finally,

$$V_\mu V_\mu=v^2+(i\beta v/2)^2=v^2[1-\tfrac{1}{4}\beta^2]\simeq v^2,$$

with the approximations made above, so that the phase uncertainty is

$$\delta\varphi=\langle S^2\rangle^{\frac{1}{2}}/\hbar\simeq\tfrac{1}{2}\alpha^{\frac{1}{2}}(Lk_m/\pi)^{\frac{1}{2}}. \quad (10)$$

On the other hand, if $k_m\rightarrow 0$, the bracket approaches $\frac{1}{2}\xi^2$, and

$$\langle S^2\rangle\simeq\frac{e^2}{4\pi^2}\frac{V_\mu V_\mu}{v^2}(Lk_m)^2\simeq\frac{e^2}{4\pi^2}(Lk_m)^2\Rightarrow\delta\varphi\simeq\tfrac{1}{2}\alpha^{\frac{1}{2}}\frac{Lk_m}{\pi}. \quad (11)$$

k_m must be the inverse of a length appearing in the problem, probably the width of the wave packet. To see this explicitly, we must do the problem with a particular packet.

Thus, instead of using the particle approximation (9), we go back to the packet (6). Then in (5), we have the integrals

$$\int_0^t dt'\int s_\mu(\mathbf{x},t')e^{i\mathbf{k}\cdot\mathbf{x}}d^3x\simeq\int_0^t dt'\int V_\mu\rho(\mathbf{x},t')e^{i\mathbf{k}\cdot\mathbf{x}}d^3x \qquad [\text{by Eq.}(7)],$$

$$=V_\mu\int_0^t dt'e^{2ik_0\tau'k_z}\exp\left\{-\frac{1}{4}\left[\frac{k_x^2\Sigma_x^4}{\sigma_x^2}+\frac{k_y^2\Sigma_y^4}{\sigma_y^2}+\frac{k_z^2\Sigma_z^4}{\sigma_z^2}\right]\right\}$$

$$=V_\mu\int_0^t dt'e^{ivk_zt'}\exp\{-\tfrac{1}{4}[k_x^2\sigma_x^2+\cdots]\}\exp\left\{-\left(\frac{\hbar t'}{2m}\right)^2\left[\frac{k_x^2}{\sigma_x^2}+\cdots\right]\right\}.$$

Now, from the middle exponential term, the "ranges" in $\mathbf{k}$ space are $2/\sigma_x$, $2/\sigma_y$, $2/\sigma_z$. At these "maximum" values, the exponent in the last term takes the value

$$-\tau^2[4/\sigma_x^4+4/\sigma_y^4+4/\sigma_z^4].$$

Recalling that $\sigma_i^2 \gg \tau_{\max}$, the exponent is close to zero, and the integral becomes

$$\simeq \exp\{-\tfrac{1}{4}[k_x^2\sigma_x^2+\cdots]\}V_\mu \frac{e^{ivtk_z/2}}{(vk_z/2)}\sin\left(\frac{vtk_z}{2}\right).$$

The integral over $s_\mu(\mathbf{y},t'')$ is exactly the same, except that $-\mathbf{k}$ appears in place of $\mathbf{k}$. Thus we come to

$$\langle S^2\rangle \simeq \frac{e^2}{16\pi^3}\frac{4V_\mu V_\mu}{v^2}\int \exp\{-\tfrac{1}{2}[k_x^2\sigma_x^2+\cdots]\} \times \frac{\sin^2(vtk_z/2)}{k_z^2}\frac{d^3k}{k}.$$

Here we consider two cases:

Case I. The packet represents a short pulse, i.e. $\sigma_z \ll L$. Now, again because of the exponential, the range in k_z is $\simeq \sigma_z^{-1}$. The argument of the trigonometric term, at that range, is (at the screen)

$$vtk_z/2 \simeq L/2\sigma_z \gg 1.$$

Hence we may use the approximation

$$\frac{\sin(\tfrac{1}{2}Lk_z)}{k_z} \simeq \pi\delta(k_z).$$

Thus integrating, we have

$$\begin{aligned}\langle S^2\rangle &\simeq \frac{e^2}{16\pi^3}\frac{V_\mu V_\mu}{v^2}4\pi \int \exp[-\tfrac{1}{2}(k_x^2\sigma_x^2)+k_y^2\sigma_y^2)] \\ &\qquad \times \exp(-\tfrac{1}{2}k_z^2\sigma_z^2)\frac{\sin(\tfrac{1}{2}Lk_z)}{k_z}\delta(k_z)\frac{dk^3}{k} \\ &= \frac{e^2}{4\pi^2}\frac{V_\mu V_\mu}{v^2}\frac{1}{2}L\int \exp[-\tfrac{1}{2}(k_x^2\sigma_x^2+k_y^2\sigma_y^2)] \\ &\qquad \times dk_y dk_x/(k_x^2+k_y^2)^{\frac{1}{2}}.\end{aligned}$$

The calculation becomes much simpler if we assume

$$\sigma_x = \sigma_y = \sigma,$$

for then we can use cylindrical coordinates, and find

$$(\delta\varphi)^2 = \frac{1}{\hbar^2}\langle S^2\rangle \simeq \frac{e^2}{\hbar c}\left(\frac{V_\mu V_\mu}{v^2}\right)\frac{L}{\sigma}\frac{1}{4(2\pi)^{\frac{1}{2}}} \simeq \frac{\alpha}{10}\frac{L}{\sigma}.$$

This says that the concept of phase, for this situation, is meaningless for $L/\sigma \geq 5\times 10^4$, as the phase uncertainty then becomes 2π.

Case II. The packet represents a *beam*, i.e., $\sigma_z \gg L$. This time therefore, we can use[4]

$$\exp(-k_z^2\sigma_z^2/2) \simeq [(2\pi)^{\frac{1}{2}}/\sigma_z]\delta(k_z), \qquad (12)$$

and once again using cylindrical coordinates, we find

$$\langle S^2\rangle \simeq \frac{e^2 V_\mu V_\mu}{8\pi\sigma\sigma_z}t^2 \simeq \frac{e^2(vt)^2}{8\pi\sigma\sigma_z},$$

and at the screen, this is $(e^2/8\pi)(L/\sigma)(L/\sigma_z)$, i.e.,

$$\delta\varphi \simeq \left(\frac{\alpha}{8\pi}\right)^{\frac{1}{2}}\left(\frac{L}{\sigma}\right)^{\frac{1}{2}}\left(\frac{L}{\sigma_z}\right)^{\frac{1}{2}}.$$

Thus, as long as the beam is steady, then aside from some transient end effects, σ_z is effectively infinite, and the dynamical vacuum does *not* wash out the interference effect at all.

We note the following points:

(1) For short pulses, the length of the pulse does not appear, in first approximation.

(2) k_m is indeed σ^{-1}, the *transverse* width of the packet.

(3) The smallness of the effect is greatly enhanced by the smallness of e, which is quite reasonable.

(4) The effect, in case I, is proportional to the *root* of the distance traveled, as in Eq. (10), whereas in case II, the effect is linear in that distance, as in Eq. (11). This indicates that the use of a single cutoff, k_m, is misleading, as there are two lengths involved, σ and σ_z. One might think that this effect is due to the spreading of the wave-packet giving rise to an uncertainty in its phase. That this is not the case is seen through the dependence on $\sqrt{\alpha}$, which makes this a dynamical, rather than a kinematical, effect. Moreover, remember that in the approximation used here, the packet only spreads very little. It may be understood as follows: In the case of the packet, what we do is follow the centroid. The spreading of the packet merely makes its position more uncertain. Its centroid, by Ehrenfest's theorem, travels on undisturbed. When the interaction with the electromagnetic field is turned on, however, the fluctuations of the latter tend to "shake" the

[4] We can do better: Since the approximation (12) improves as σ_z increases, it is natural to try to improve it by adding a power series in σ_z^{-1}. This is naturally done by integrating

$$S(\sigma) \equiv \int_{-\infty}^{\infty} f(x)\exp(-\sigma^2x^2/2)dx$$

by parts, successively. Since the odd part of $f(x)$ must integrate to zero, we need consider only *even* functions f, in the interval $(0,\infty)$. So long as $f(x)$ does not increase as fast as $\exp(\sigma^2x^2/2)$, and has derivatives of all orders at the origin (and at infinity), this will yield a series which is at least asymptotically convergent:

$$S(\sigma) = \frac{(2\pi)^{\frac{1}{2}}}{\sigma}\sum_{n=0}^{\infty}\frac{f^{(2n)}(0)}{n!(2\sigma^2)^n}.$$

We may write this symbolically as

$$\exp(-\sigma^2x^2/2) \equiv \frac{(2\pi)^{\frac{1}{2}}}{\sigma}\sum_{n=0}^{\infty}\frac{\delta^{(2n)}(x)}{n!(2\sigma^2)^n},$$

when it occurs in such integrals.

electron, and do so the more violently the smaller the packet is. When the packet turns into a beam, it becomes increasingly improbable to find a fluctuation large enough to move the whole beam.

One can understand the dependence on the various factors as follows: The vacuum fluctuations can be thought of as resulting in the uncertainty in the measurement of the fields. This uncertainty will depend (as is well known from Bohr and Rosenfeld's work[5]) on the details of the measurement process: inversely on the size of the space-time region in which the field is measured. We can think of the electron as being the testing body, and its size will then be roughly the width of its packet. Thus the phase should be inversely dependent on σ (or some power thereof). Moreover, remembering that the fluctuations are random, we would have to sum the squares of the microscopic fluctuations, and hence have a dependence on $L^{\frac{1}{2}}$. Finally, the argument above indicates that we should expect an inverse dependence on the length of the packet.

An interesting question which was not looked into is the comparison of the size of this effect to the kinematical effect. That is, for a short pulse, there will be an appreciable frequency distribution, and the purely classical diffraction minima will occur at different places on the screen, leading to a blurring of the interference pattern. This effect, however, can be made as small as we like by making $k_0\sigma_z$ large enough, while still keeping $\sigma_z \ll L$.[6]

[5] N. Bohr and L. Rosenfeld, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **12**, No. 8 (1933).

ACKNOWLEDGMENTS

It is a pleasure to thank Professor K. Ford for suggesting this problem, and Dr. R. Drachman and Dr. A. Grossmann for helpful discussions.

[6] Since the writing of this article, another has appeared, bearing in part on this problem: Y. Aharonov and D. Bohm, Phys. Rev. **123**, 1511 (1961). The remark is made in Sec. 4 of that paper that ". . . in first approximation, all effects [on the electron] of the zero-point fluctuations of the field average out to zero. . . ." The results above indicate that this assertion is not always true.