Effects of Parity-Nonconserving Internucleon Potential on Li^{7*} (0.478 Mev)$\rightarrow Li^7$ Gamma Transition

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A calculation of the radiation anisotropy is made for the electromagnetic transition of Li^7 from the first excited state to the ground state on the assumption that a weak parity-nonconserving internucleon potential admixes into the two levels small amounts of wave function with the "wrong" parity and permits a small $E1$ transition to interfere with the normal $M1$ transition. The asymmetry is of the form $1+\alpha\cos\theta$ and our results give $|\alpha|\approx 10^{-8}$ assuming that the potential stems from the self-interacting current theory of weak interactions.

I. INTRODUCTION

PARITY nonconservation, though well established in the weak interactions, has yet to be found in the strong interactions. On the assumption that electromagnetic interactions conserve parity, Blin-Stoyle[1] has shown that there will arise, as a result of a parity nonconserving internucleon potential, small asymmetries in the radiation pattern of certain nuclei due to interference between multipoles of the same order. The asymmetry has the form $1+\alpha\cos\theta$, where

$$\alpha\propto\frac{m_L{}^*e_L}{|m_L|^2+|e_L|^2},$$

with e_L and m_L, the reduced matrix elements of multipole order L suitably defined so that they are relatively real. The most advantageous case to choose, experimentally, is one in which the normally occurring transition is $M1$ and the irregular transition (due to small admixtures of states of the "wrong" parity) is $E1$ since the e_1 matrix element is usually larger than the m_1.

Li^7 is a nucleus which has been used experimentally[2] for the investigation of this effect and which is also amenable to detailed theoretical calculation. The first-excited state of Li^7 (0.478 Mev) decays by $M1$ radiation to the ground state with $\Delta J=+1$ and no parity change ($\frac{1}{2}^-\rightarrow\frac{3}{2}^-$). However, small amounts of $\frac{1}{2}^+$ or $\frac{3}{2}^+$ states mixed into the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states, respectively, will cause a small $E1$ transition and we may write $|\alpha|\approx|e_1/m_1|$. If the weak interactions are interpreted in terms of self-interacting charged currents (with or without an intermediate neutral boson), then the lowest order parity-nonconserving internucleon potential gives approximately no effect in LS or jj coupling[1] although an effect is obtained in intermediate coupling. On the other hand, if neutral currents are allowed as suggested, for example, by Lee and Yang,[3] then an effect is obtained in the LS extreme. Therefore, in the following, appropriate intermediate coupled wave functions are used to calculate $|e_1/m_1|$ for two cases: (a) with only charged currents, and (b) with neutral currents allowed.

[1] R. J. Blin-Stoyle, Phys. Rev. **120**, 181 (1960).
[2] D. H. Wilkinson, Phys. Rev. **109**, 1614 (1958).
[3] T. D. Lee and C. N. Yang, Phys. Rev. **119**, 1410 (1960).

II. CALCULATION OF α

We use the static parity nonconserving potential[4] derived on the assumption of a conserved self-interacting current for weak interactions

$$V_{12}=-\frac{Gf^2}{2\pi\hbar c}\left(\frac{1}{r^4}+\frac{2}{\mu r^5}+\frac{1}{\mu^2r^6}\right)e^{-2\mu r}\mathbf{r}\cdot(\boldsymbol{\sigma}^1\times\boldsymbol{\sigma}^2)(\boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2-\tau_z{}^1\tau_z{}^2)=\boldsymbol{\chi}_{12}\cdot\mathbf{S}_{12}T_{12},$$

with $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ and (1)

$$\boldsymbol{\chi}_{12}=-\frac{Gf^2}{2\pi\hbar c}\left(\frac{1}{r^4}+\frac{2}{\mu r^5}+\frac{1}{\mu^2r^6}\right)e^{-2\mu r}\mathbf{r},$$

$$\mathbf{S}_{12}=\boldsymbol{\sigma}^1\times\boldsymbol{\sigma}^2,\quad T_{12}=(\boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2-\tau_z{}^1\tau_z{}^2),$$

where G is the weak four-fermion coupling constant ($G\approx 1.4\times10^{-49}$ erg-cm^3), f is the renormalized pseudovector coupling constant ($f^2\approx 0.08$ in units of $\hbar c$), and $1/\mu$ is the pion Compton wavelength. If the existence of a neutral current is assumed, the only change occurs in the isotopic spin part[5] which becomes simply $T_{12}=\boldsymbol{\tau}^1\cdot\boldsymbol{\tau}^2$.

The following wave functions, neglecting the four-particle core of Li^7 ,were used for the calculation[6]:

$$\psi(J=\tfrac{3}{2},M_J=\tfrac{1}{2})=0.966\,{}^{22}P[3]-0.0114\,{}^{22}P[21]+0.221\,{}^{24}P[21]+0.095\,{}^{22}D[21]-0.094\,{}^{24}D[21],\quad(2)$$

$$\psi(J=\tfrac{1}{2},M_J=\tfrac{1}{2})=0.940\,{}^{22}P[3]+0.309\,{}^{22}P[21]+0.127\,{}^{24}P[21]-0.053\,{}^{24}D[21]+0.047\,{}^{22}S[111],\quad(3)$$

where the notation is ${}^{2T+1,2S+1}L[\lambda]$ and λ designates the spatial symmetry properties of the wave function. The various states were then expressed with the aid of frictional parentage coefficients in such a way as to separate out the third particle in the usual manner. The potential was then allowed to act between particles

[4] R. J. Blin-Stoyle, Phys. Rev. **118**, 1605 (1960). Higher order contributions to this potential would not radically alter its general form.
[5] G. Barton, Nuovo cimento **19**, 512 (1961).
[6] J. M. Soper (private communication). These values were derived with $a/k=1.4$ and $L/k=4.62$.

1 and 2. The single-particle wave functions were taken to be properly normalized P-shell oscillator functions of the form $r\exp(-r^2/2b^2)$.

Equation (39) of reference 1, derived using a completeness approximation, was used as a basis for the calculation:

$$\langle\psi_{\frac{3}{2}}{}^{M_J}|\,\mathbf{E1}\,|\psi_{\frac{1}{2}}{}^{M_J'}\rangle = -\frac{e}{2\hbar\omega}\langle\psi_{\frac{3}{2}}{}^{M_J}|\sum_{j\neq k}[(1+\tau_z{}^{(j)})\mathbf{r}_jV_{jk} + V_{jk}(1+\tau_z{}^{(j)})\mathbf{r}_j]|\psi_{\frac{1}{2}}{}^{M_J'}\rangle, \quad (4)$$

where $\hbar\omega$ is the oscillator spacing and was taken to be 20 Mev. By use of the Wigner-Eckert theorem and the symmetry properties of the potential and the wave functions involved, we may write

$$|e_1| = |\langle\psi_{\frac{3}{2}}||\mathbf{E1}||\psi_{\frac{1}{2}}\rangle| = (6e/\hbar\omega)(\tfrac{3}{2})^{\frac{1}{2}} \times|\langle\psi_{\frac{3}{2}}{}^{\frac{1}{2}}|z_2\chi_{12}\cdot\mathbf{S}_{12}\tau_z{}^1\tau_p{}^2|\psi_{\frac{1}{2}}{}^{\frac{1}{2}}\rangle|, \quad (5)$$

where $\tau_p=1+\tau_z$. In the case of no neutral boson current we have

$$|e_1| = (6e/\hbar\omega)(\tfrac{3}{2})^{\frac{1}{2}}|\langle\psi_{\frac{3}{2}}{}^{\frac{1}{2}}|z_2\chi_{12} \cdot\mathbf{S}_{12}(\tau_+{}^1\tau_-{}^2+\tau_-{}^1\tau_+{}^2)|\psi_{\frac{1}{2}}{}^{\frac{1}{2}}\rangle|, \quad (6)$$

where $\tau_\pm=(1/\sqrt{2})(\tau_x\pm i\tau_y)$.

It turns out, after making a transformation to relative and center-of-mass coordinates, that all the integrals either can be done analytically or reduced to the complementary error function.

III. RESULTS AND DISCUSSION

In the case (a) the calculation yields $|e_1|=1.25\times10^{-31}$ and using a value[7] of the $M1$ lifetime $\tau=1.1\times10^{-13}$ sec we get, by the usual formula, $|m_1|=1.34\times10^{-23}$ so that $|\alpha|\approx|e_1/m_1|=0.93\times10^{-8}$. In case (b) the calculation gives $|e_1|=0.58\times10^{-31}$ and $|\alpha|\approx0.42\times10^{-8}$, about the same order of magnitude. These values were computed for a value of the parameter $b=2\times10^{-13}$ cm and for a cutoff of $\epsilon=0.6\times10^{-13}$ cm in the relative coordinate. Such a cutoff is necessary to prevent the integrals from diverging and also simulates the effect of a strong repulsive core in the regular parity-conserving internuclear potential. Further investigation showed that these results were very insensitive to a shift in b ($b=1.5\times10^{-13}$ cm changed the final value by less than a factor of two) and to a shift in ϵ until ϵ became very small since ultimately in the limit of $\epsilon\rightarrow0$ some of the integrals diverge. Since the divergent terms go as $1/\epsilon$, the lowering of ϵ from $\epsilon=0.6$ fermi to, say, $\epsilon=0.4$ fermi has little effect. These terms do not dominate the other (i.e., nondivergent) terms that appear until $\epsilon\lesssim0.01$ fermi.

The reason that cases (a) and (b) do not differ more is due to the fact that there is a great deal of destructive interference in both cases. Though there are more nonvanishing terms in case (a), they tend to cancel each other leaving very little net result. For this reason, the same calculation for a different nucleus in this region might show a much bigger difference in the two cases.

The actual experiment requires a polarization of the nucleus but this could be accomplished by various means[8] with polarizations of order unity. Even with such polarizations, the experiment is, at present, outside the range of experimental feasibility. The one previous experiment done on this nucleus[2] yielded an upper limit on the amplitude of the irregular part of the wave function[9] of $\mathfrak{F}\lesssim10^{-2}$ which is a long way from the expected value for $\mathfrak{F}$.[10] If the strength of the $E1$ matrix element in this region (around Li^7) is taken to be[11] 10^{-23} then we get, in the most favorable case of $|\alpha|=0.93\times10^{-8}$, that $\mathfrak{F}=1.25\times10^{-8}$ since $\mathfrak{F}R=\alpha$, where R is the ratio of a normal $E1$ transition to a normal $M1$ transition in the P shell.[2] It should be noted that the estimate made for R in reference 2 does not hold here as this $M1$ matrix element is anomalously large for this region (see reference 11).

It should also be pointed out that it is possible to look for a partial circular polarization of the emitted γ's in such a nucleus since by Eq. (30) of reference 1 we have, for the degree of circular polarization of the emitted γ's,

$$\delta=\frac{2\sum_L m_L{}^*e_L}{\sum_L(|m_L|^2+|e_L|^2)} \quad\text{or}\quad \delta=2\times10^{-8},$$

in case (a) assuming the m_1 and e_1 transitions to be the only significant ones. This is for unoriented nuclei, which obviates the problem of polarization which may in the asymmetry effect lessen the magnitude of α by $\approx\frac{1}{2}$. Hence, experimentally, one picks up a factor of 4 over the other experiment, but at the expense of having to detect circularly polarized radiation.

[7] C. P. Swann, V. K. Rasmussen, and F. R. Metzger, Phys. Rev. **114**, 862 (1959).

[8] R. J. Blin-Stoyle and M. A. Grace, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 42, p. 555.

[9] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956). The argument given above is strictly true only in the case that just one "wrong" parity state mixes in with the regular wave function.

[10] L. Grodzins and F. Genovese, Phys. Rev. **121**, 228 (1961) have carried out an asymmetry experiment of the type described in this paper on Fe^{57} and conclude that $\mathfrak{F}\lesssim10^{-5}$.

[11] A. M. Lane, Revs. Modern Phys. **32**, 519 (1960).