Bipionic Decay of K'

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The branching ratio of the bipionic decay of the K' particle, $w(K'\rightarrow K+2\pi)/w(K'\rightarrow K+\pi)$, has been calculated in Fermi's statistical theory. It turns out that this ratio is a few percent with a reasonable choice of the interaction range.

It has been pointed out[1] that the radiative and bipionic decay channels of the K' particle[2] ($K-\pi$ resonant state of mass 885 Mev and width 16 Mev) are forbidden for a scalar K', but allowed for a vector K'. Thus the experimental study of these rare decay modes is crucial for the spin determination. The expected partial width of the radiative decay mode was given elsewhere.[1,3] In this note we estimate the branching ratio of the bipionic decay mode $K'\rightarrow K+2\pi$.

Since there is no reliable dynamics for strong interactions at present, we shall avoid the details of the reaction mechanism and represent the "black box" by a small number of phenomenological parameters. The simplest approach on these lines is Fermi's statistical theory[4,5] in which the matrix elements connecting the initial and various final states are essentially constant.

The branching ratio of the bipionic to the main decay mode is

$$\frac{\Gamma(K'\rightarrow K+2\pi)}{\Gamma(K'\rightarrow K+\pi)}=\frac{\Omega}{(2\pi)^3}\frac{\rho_{K\pi\pi}}{\rho_{K\pi}},$$

where Γ is the partial width, Ω is the interaction volume, $\rho_{K\pi\pi}$ and $\rho_{K\pi}$ are the corresponding phase space volume.[6] The statistical weight factors, which come from charge independence and the indistinguishability of pions, become unity when one adds up all possible charge states belonging to the same mode. A numerical integration gives

$$\rho_{K\pi\pi}=\frac{2\pi^2}{3}\int_0^{p_{\max}}p^2\left[1-\frac{4m_\pi^2}{(m_{K'}-\epsilon)^2-p^2}\right]^{\frac{1}{2}}\times\left[3(m_{K'}-\epsilon)^2-p^2\left(1-\frac{4m_\pi^2}{(m_{K'}-\epsilon)^2-p^2}\right)\right]dp$$
$$=\tfrac{2}{3}\pi^2 m_{K'}{}^5(1.184)10^{-3},$$

$$\rho_{K\pi}=\frac{\pi}{2}\left[m_{K'}{}^2-\frac{(m_K{}^2-m_\pi{}^2)^2}{m_{K'}{}^2}\right]\times\left[1-\frac{2(m_K{}^2+m_\pi{}^2)}{m_{K'}{}^2}+\frac{(m_K{}^2-m_\pi{}^2)^2}{m_{K'}{}^4}\right]^{\frac{1}{2}}$$
$$=\tfrac{1}{2}\pi m_{K'}{}^2(5.877)10^{-1},$$

where p and ϵ are the magnitude of the three-momentum and the energy of the final kaon, and $p_{\max}$ is given by

$$p_{\max}=(2m_{K'})^{-1}\{[m_{K'}{}^2-(m_K+2m_\pi)^2]\times[m_{K'}{}^2-(m_K-2m_\pi)^2]\}^{\frac{1}{2}}.$$

The only parameter in the theory is Ω, which we shall choose as the volume of a sphere whose radius equals the pion Compton wavelength. Then the branching ratio turns out to be 3.9%. The spin of the K' might also suppress the bipionic mode through the angular momentum barrier, besides the phase space volume given here. It is worth noting that charge independence predicts the probability of finding a $(\pi^+\pi^-)$ pair in the final state to be one half of the total bipionic process.

We may apply the statistical theory to the radiative decay mode as well, if we single out the electromagnetic interaction, as a perturbation, from the rest of strong interactions. We have

$$\frac{\Gamma(K'\rightarrow K+\gamma)}{\Gamma(K'\rightarrow K+\pi)}=\frac{e^2}{4\pi}\cdot\frac{\rho_{K\gamma}}{\rho_{K\pi}}=\frac{1.06}{137}=0.8\%,$$

which is consistent with the previous estimate.[1,3]

Note added in proof. The choice of the interaction volume certainly leaves some arbitrariness. The physical picture behind the text may be described as follows. The decay of the K' particle is a spontaneous "explosion," releasing its mass energy into space. When the system arrives in thermodynamic equilibrium, the "expanded" K' is found in fragments, which are identified either as a kaon or a pion. Then the fragments separate from each other keeping their individuality. The radius of the "cloud" after the formation of the decay products is thus of the order of $(\hbar/m_K C)+(\hbar/m_\pi C)+\cdots$. So far as the order of magnitude is concerned, we may take the interaction radius to be equal to the Compton wavelength of the *lightest* decay product.

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