# Intermediate Meson Contributions to Hyperon Decays*

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Dispersion relations have been used to estimate the contributions to $\Lambda$ and $\Sigma$ hyperon decays from one- and two-meson intermediate states for both even and odd $\Lambda-\Sigma$ relative parity. The results indicate that these contributions will have a negligible effect on the electron modes and give perhaps a 10% correction to the rates for the muon modes; the lepton decay modes are apparently dominated by the $V$ and $A$ form factors at zero momentum transfer which appears as subtraction constants in the calculations. For even $\Lambda-\Sigma$ parity the two-meson contributions to the pion mode amplitudes are roughly 20% of the experimental values. The contributions to the $\Lambda$-decay amplitudes are essentially unchanged when one assumes odd $\Lambda-\Sigma$ parity, but the contributions to the $\Sigma$ amplitudes are reduced to about 6% of the experimental values.

## 1. INTRODUCTION

TWO of the many questions which might be asked concerning the weak decays of hyperons are: (1) How important are the effects "induced" by strong interactions in the decays $Y \rightarrow N+l+\nu$; could these effects possibly explain the "reduced" rate of these decays? (2) Is there a connection between the symmetries among the $s$- and $p$-wave amplitudes for the decays $Y \rightarrow N+\pi$ and the symmetries of strong interactions?

Goldberger and Treiman[1] have considered the analog of question (1) in neutron beta decay and $\mu$ capture. They found only small corrections to the direct interaction for $\beta$ decay but an appreciable "induced pseudoscalar" term in $\mu$ capture. Because of the large momentum transfer, and because the total rate is reduced, we might also expect important "induced" effects in the lepton decay modes of hyperons.

Several authors have suggested answers to question (2). Treiman[2] and Pais[3] have sought to explain the decay amplitude symmetries by postulating approximate symmetries in the strong couplings. Other authors[4–6] have suggested that the symmetries among the amplitudes are due to the dominance of the contributions of certain simple diagrams to these decays in addition to symmetries in the strong couplings.

In this paper we make what must be regarded as at best a crude preliminary attempt to answer these questions by using dispersion relations to estimate the contributions of one- and two-meson intermediate states to hyperon decays. These contributions correspond to the diagrams which Wolfenstein[6] has suggested might be important in the pion decay modes. Other authors[7,8] have previously calculated the single-meson contribution to the decay $\Lambda \rightarrow p+l+\nu$, while Flamm[9] has made a perturbation theory calculation of the two-meson contribution to this decay. These authors conclude that the one- and two-meson contribution, by themselves, are too small to account for the observed decay rate.

The results of our calculations depend upon the relative parities of the particles involved and the values of the renormalized strong coupling constants. At present, only the pion and nucleon parities and the $\pi-N$ coupling constant are well known. For the remaining parities and coupling constants we consider two cases which seem most probable at the present time:

(1) The "even" case—the $\Lambda-\Sigma$ parity is even, the $N-\Lambda-K$ parity odd, all pion-baryon coupling are approximately equal, and all $K$-meson–baryon couplings are approximately equal.[10]

(2) The "odd" case—the $\Lambda-\Sigma$ parity is odd,[4,11] the $N-\Lambda-K$ parity odd, and the coupling constants are approximately as suggested by Ferrari and Fonda[12] based upon hypernuclei binding energies.

As Wolfenstein[6] has pointed out, if one assumes the one- and two-meson intermediate states are dominant in the pion decay modes and takes the "even" case, many of the relations among the $Y \rightarrow N+\pi$ amplitudes are automatically explained provided the $\Lambda-\Sigma$ mass difference is ignored. In this paper we estimate the magnitude of these contributions for both "even" and "odd" cases without neglecting the mass differences.

In Sec. 2 we define the form factors and amplitudes and review the experimental information which applies to them. The dispersion relations which we use are introduced in Sec. 3. This section also contains a dis-

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[1] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 354 (1958).
[2] S. B. Treiman, Nuovo cimento **15**, 916 (1960).
[3] A. Pais, Phys. Rev. **122**, 317 (1961); Nuovo cimento **18**, 1003 (1960).
[4] S. Barshay and M. Schwartz, Phys. Rev. Letters **4**, 618 (1960).
[5] G. Feldman, P. Matthews, and A. Salam, Phys. Rev. **121**, 302 (1961).
[6] L. Wolfenstein, Phys. Rev. **121**, 1245 (1961).
[7] E. M. Ferreira, Nuovo cimento **8**, 359 (1958).
[8] N. Cabibbo and R. Gatto, Nuovo cimento **13**, 1086 (1959).
[9] D. Flamm, Nuovo cimento **16**, 194 (1960).
[10] This is an extension of the global symmetry suggested by M. Gell-Mann, Phys. Rev. **106**, 1296 (1957). As shown by A. Pais, Phys. Rev. **110**, 574 (1958), the symmetry among the couplings cannot be exact.
[11] Y. Nambu and J. J. Sakurai, Phys. Rev. Letters **6**, 377 (1961).
[12] F. Ferrari and L. Fonda, Nuovo cimento **9**, 843 (1959).

cussion of the necessity for subtractions and the anomalous thresholds which occur in the two-meson contribution. By approximating the $K\pi l\nu$ and $K\pi\pi$ amplitudes with constants and using only the one-baryon intermediate state contributions to the $YN\pi K$ amplitudes we are able to obtain an estimate of the two-meson contributions. This calculation, along with the results for the one-meson contribution, is contained in Sec. 4. The qualitative features of our results are summarized and discussed in Sec. 5.

## 2. FORM FACTORS AND AMPLITUDES

### A. $Y \to N+l+\nu$ Form Factors

Assuming a current-current structure of the weak interaction, we find for the $Y\to N+l+\nu$ decay amplitude:

$$(G/\sqrt{2})\langle N|J_\mu(0)|Y\rangle \times[(m_l/E_l)(m_\nu/E_\nu)]^{\frac{1}{2}}\bar{u}_l\gamma^\mu(1+\gamma_5)u_\nu. \quad (1)$$

In (1), $J_\mu(x)$ is the weak current constructed from the fields of strongly interacting particles. It increases strangeness by one unit and transforms as a linear combination of a vector and a pseudovector. For convenience we have factored out the "universal" weak coupling constant $G\approx m_N^{-2}10^{-5}$.

Because of the transformation properties we assume for $J_\mu(x)$, we can always write[1]

$$\langle N|J_\mu(0)|Y\rangle = \left(\frac{m_N}{E_N}\frac{m_Y}{E_Y}\right)^{\frac{1}{2}}\bar{u}_N\Big[f_1(s)\gamma_\mu+f_2(s)\sigma_{\mu\nu}\frac{k^\nu}{m_Y}+f_3(s)\frac{k_\mu}{m_Y} +g_1(s)\gamma_\mu\gamma_5+g_2(s)\sigma_{\mu\nu}\gamma_5\frac{k^\nu}{m_Y}+g_3(s)\gamma_5\frac{k_\mu}{m_Y}\Big]u_Y. \quad (2)$$

In (2), and below,

$$k=p_Y-p_N,$$

and

$$s=k^2. \quad (3)$$

The six form factors $f_1(s)$, $\cdots$, $g_3(s)$ are dimensionless functions of the invariant $s$ and can all be chosen real if we assume time-reversal invariance. An unrenormalized universal $V-A$ theory[13] would correspond to $f_1=g_1=1$ and $f_2=f_3=g_2=g_3=0$.

Up to the present time only a few of these decays have been seen; the rates are apparently reduced by a factor of 10 or more from the rates predicted by a universal $V-A$ theory.[14] For example, in the decay $\Lambda\to p+e+\nu$, if we neglect the possible $s$ dependence of the form factors we find[15]

$$\omega_{\Lambda\to p+e+\nu}=1.43\times10^7\ \text{sec}^{-1}[|f_1|^2+2.96|g_1|^2 +0.014|f_2|^2+0.043|g_2|^2 +0.023\ \text{Re}f_1f_2^*-0.062\ \text{Re}g_1g_2^*], \quad (4)$$

while the experimental rate is roughly $0.5\times10^7$ sec$^{-1}$.[14] If, as our results below indicate, the $f_2$ and $g_2$ corrections are negligible, then $|f_1|^2+2.96|g_1|^2\approx0.3$.

### B. $Y\to N+\pi$ Amplitudes

The amplitude for the decays $Y\to N+\pi$ can be written

$$\langle N\pi|H_w(0)|Y\rangle=\left(\frac{1}{2E_\pi}\frac{m_N}{E_N}\frac{m_Y}{E_Y}\right)^{\frac{1}{2}}\bar{u}_N\left[A_s+\gamma_5\frac{A_p}{R}\right]u_Y. \quad (5)$$

In (5), $H_w(x)$ is the weak interaction density, $A_s$ and $A_p$ are the $s$- and $p$-wave amplitudes, respectively, and

$$R=\left[\frac{(m_Y-m_N)^2-m_\pi^2}{(m_Y+m_N)^2-m_\pi^2}\right]^{\frac{1}{2}}=\frac{|\mathbf{p}|}{E_N+m_N}. \quad (6)$$

In general the constants $A_s$ and $A_p$ may be complex, but if we assume time-reversal invariance then their phases are the small $\pi-N$, $J=\frac{1}{2}$ phase shifts.

The experimental decay rates[16] determine $|A_s|^2+|A_p|^2$ for each decay, while measurements of the asymmetry of the decays[16] determine the ratio of $|A_s|$ to $|A_p|$ (except that we cannot distinguish $|A_s|:|A_p|$ from $|A_p|:|A_s|$). The resulting values for the $|A_s|$ and $|A_p|$ are given in Table I. This table is somewhat schematic since the ratios of $s$- to $p$-wave amplitudes are not determined accurately by the experiments. Thus $|A_s|=|A_p|$ in the table really means $\frac{1}{2}\lesssim|A_s|:|A_p|\lesssim2$, while $|A_s|=0$ really means $|A_s|:|A_p|\lesssim0.05$.

TABLE I. Experimental values for the $s$- and $p$-wave amplitudes in the decays $Y\to N+\pi$. $A_0\approx2\times10^{-7}$.

| Amplitude | $\Lambda\to p+\pi^-$ | $\Lambda\to n+\pi^0$ | $\Sigma^+\to p+\pi^0$ | $\Sigma^+\to n+\pi^+$ | $\Sigma^-\to n+\pi^-$ |
|---|---|---|---|---|---|
| $\lvert A_s\rvert$ | $\sqrt{2}A_0$ | $A_0$ | $\sqrt{2}A_0$ | $2A_0$ (0) | 0 $(2A_0)$ |
| $\lvert A_p\rvert$ | $\sqrt{2}A_0$ | $A_0$ | $\sqrt{2}A_0$ | 0 $(2A_0)$ | $2A_0$ (0) |

The amplitudes as given in Table I are consistent with the requirements of the $|\Delta I|=\frac{1}{2}$ rule:

$$A(\Lambda\to p+\pi^-)=\sqrt{2}A(\Lambda\to n+\pi^0), \quad (7)$$

and

$$A(\Sigma^+\to n+\pi^+)+\sqrt{2}A(\Sigma^+\to p+\pi^0)=A(\Sigma^-\to n+\pi^-).$$

[13] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[14] W. E. Humphrey *et al.*, Phys. Rev. Letters **6**, 478 (1961).

[15] This, and other expressions for observables in terms of the six form factors, can be obtained from the papers of C. H. Albright, Phys. Rev. **115**, 750 (1959); V. P. Belov, B. S. Mingalev, and V. M. Shekhter, Zhur. Eksptl. i Teoret. Fiz. **38**, 541 (1960) [translation: Soviet Phys.—JETP **11**, 392 (1960)]; and D. R. Harrington, Phys. Rev. **120**, 1482 (1960).

[16] Recent experimental data concerning the decays $Y\to N+\pi$ can be found in the *Proceedings of the 1960 Annual Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), especially in the report by J. W. Cronin, p. 540, and the table on p. 878.

They show additional symmetry, however, satisfying, for example, the relation suggested by Treiman[2]:

$$\sqrt{2}A(\Lambda\to p+\pi^-)=A(\Sigma^+\to n+\pi^+)+A(\Sigma^-\to n+\pi^-). \quad (8)$$

It should also be noted that the decays $\Sigma^\pm - n+\pi^\pm$ are parity conserving (or at least very nearly so).

### 3. DISPERSION RELATIONS

We assume that the form factors and amplitudes introduced in Sec. 2 satisfy the following dispersion relations in the variable $s=(p_Y-p_n)^2$:

$$f_1(s)=f_1(0)+\frac{s}{\pi}\int_c \frac{\mathrm{Im}f_1(s')}{s'(s'-s)}ds',$$

$$g_1(s)=g_1(0)+\frac{s}{\pi}\int_c \frac{\mathrm{Im}g_1(s')}{s'(s'-s)}ds',$$

$$f_2(s)=\frac{1}{\pi}\int_c \frac{\mathrm{Im}f_2(s')}{s'-s}ds',\quad g_2(s)=\frac{1}{\pi}\int_c \frac{\mathrm{Im}g_2(s')}{s'-s}ds', \quad (9)$$

$$f_3(s)=\frac{1}{\pi}\int_c \frac{\mathrm{Im}f_3(s')}{s'-s}ds',\quad g_3(s)=\frac{1}{\pi}\int_c \frac{\mathrm{Im}f_3(s')}{s'-s}ds',$$

$$A_s=\frac{1}{\pi}\int_c \frac{\rho_s(s')}{s'-m_\pi^2}ds',\qquad A_p=\frac{1}{\pi}\int_c \frac{\rho_p(s')}{s'-m_\pi^2}ds'.$$

The spectral functions $\mathrm{Im}f_i$ and $\mathrm{Im}g_i$ are obtained by projecting out the appropriate terms from

$$A_\mu(s)=\frac{1}{2}\left(\frac{E_Y}{m_Y}\right)^{\frac{1}{2}}\bar{u}_N\sum_n(2\pi)^4\delta^4(p_Y-p_N-p_n)\times\langle 0|J_\mu(0)|n\rangle\langle n|J_N(0)|Y\rangle, \quad (10)$$

while $\rho_s$ and $\rho_p$ are given by

$$\bar{u}_N\left(\rho_s+\gamma_5\frac{\rho_p}{R}\right)u_Y=\frac{1}{2}\left(\frac{E_Y}{m_Y}\right)^{\frac{1}{2}}\bar{u}_N\sum_n(2\pi)^4\delta^4(p_Y-p_N-p_n)\times\{\langle 0|\,\mathcal{O}_\pi|n\rangle\langle n|J_N(0)|Y\rangle+\langle 0|J_\pi(0)|n\rangle\langle n|\,\mathcal{O}_N|Y\rangle\}. \quad (11)$$

In (11),

$$\mathcal{O}_\alpha=-i\int d^4x\, e^{ip_\alpha\cdot x}\theta(x)[J_\alpha(x),H_w(0)], \quad (12)$$

and $J_\pi$ and $J_N$ are the pion and nucleon currents, respectively.

Normally we expect the contribution to the dispersion integrals from a given intermediate state to begin at a value of $s'$ corresponding to the physical threshold. In this problem, however, there are anomalous thresholds below the physical thresholds, and these must be treated as described in the papers of Mandelstam[17] and of Blankenbecler and Cook.[18] The anomalous thresholds for the processes considered here are only very slightly below the normal thresholds; their main effect is to allow the spectral function to be nonzero at the normal threshold.

We have made subtractions in the dispersion relations for $f_1(s)$ and $g_1(s)$ for several reasons. There may be terms of the form $\bar{\psi}_N\gamma_\mu\psi_Y$ and $\bar{\psi}_N\gamma_\mu\gamma_5\psi_Y$ in $J_\mu$; these would give "equal-time" contributions to $f_1(s)$ and $g_1(s)$ which would not be included in unsubtracted dispersion relations. Also, in our approximate calculation, an unsubtracted dispersion relation for $f_1(s)$ would not converge [there is no contribution to $g_1(s)$ from the intermediate states we consider]. Finally, if we do use an unsubtracted dispersion relation for $f_1$ and make a reasonable cutoff, our approximate results give a $\Lambda\to p+e+\nu$ rate which is too small by several orders of magnitude (of course this neglects the contributions of the more complicated intermediate states). Because there is no reason to expect equal-time contributions, and because the integrals, with our approximate spectral functions, converge, we make no subtractions in the dispersion relations for the remaining form factors and amplitudes.

The intermediate states which contribute to $\mathrm{Im}f_1$ and $\mathrm{Im}g_1$ must have negative charge, negative strangeness, and angular momentum either zero or one. Those which contribute to $\rho_s$ and $\rho_p$ must have the same charge as the final pion, zero angular momentum, and a strangeness of either zero or minus one. We have estimated the contributions from the simplest of this infinity of contributing states: the single $K$ meson and the $K$ meson plus pion states. Thus among the states we have neglected are three-pion states (because of the difficulty of the calculation) and states containing hyperon-antinucleon pairs (because the results of Goldberger and Treiman[1] indicate that these contributions are unimportant).

### 4. ESTIMATE OF ONE- AND TWO-MESON CONTRIBUTIONS

For completeness and consistency of notation we review here the results which other authors[7,8] have obtained for the single $K$-meson contributions. Of the six $Y\to N+l+\nu$ form factors, only $f_3$ or $g_3$ (depending upon the relative $YNK$ parity) receives a contribution from one-meson intermediate states:

$$\left.\begin{matrix} f_3^{(1)} \\ g_3^{(1)}\end{matrix}\right\}=\left\{\begin{matrix} g_{YNK}^{(s)} \\ g_{YNK}^{(ps)}\end{matrix}\right\}\frac{am_Y}{m_K^2-s}, \quad (13)$$

[17] S. Mandelstam, Phys. Rev. Letters **4**, 84 (1960).

[18] R. Blankenbecler and L. F. Cook, Phys. Rev. **119**, 1745 (1960).

where

$$\langle 0|J^\mu(0)|K^-\rangle=(2E_K)^{-\frac{1}{2}}a p_K^\mu \tag{14}$$

and

$$\bar{u}_N\langle K^-|J_N(0)|Y\rangle$$
$$=\left(\frac{1}{2E_K}\frac{m_Y}{E_Y}\right)^{\frac{1}{2}}\bar{u}_N\left\{\begin{matrix}1g_{YNK}{}^{(s)}\\ \gamma_5 g_{YNK}{}^{(ps)}\end{matrix}\right\}u_Y. \tag{15}$$

Using $\omega(K\to\mu+\nu)=48\times10^6/\mathrm{sec}$,[19] we obtain $|a|=3.7\times10^{-2}m_N$. Then, taking[12] $(g_{NK\Lambda}{}^{(ps)})^2/4\pi=(g_{\Sigma NK}{}^{(ps)})^2/4\pi=2.2$ and $(g_{\Sigma NK}{}^{(s)})^2/4\pi=0.8$, we obtain for $\Lambda$ decay:

$$g_3{}^{(1)}(s)=\pm0.83\left(\frac{1}{1-s/m_K{}^2}\right), \tag{16}$$

and for $\Sigma$ decay either

$$g_3{}^{(1)}(s)=\pm1.18\left(\frac{1}{1-s/m_K{}^2}\right) \tag{17}$$

or

$$f_3{}^{(1)}(s)=\pm0.72\left(\frac{1}{1-s/m_K{}^2}\right), \tag{18}$$

for the even and odd cases, respectively. Except for uncertainties in the constants $a$ and $g_{YNK}$ these one-meson contributions are exact.

The form factors $f_3$ and $g_3$ always appear along with a factor $(m_l/m_Y)^2$ in expressions for rates.[15] Thus the effects of the above contributions will be completely negligible for the electron modes and probably small even in the muon modes.

The one-meson contributions to the decays $Y\to N+\pi$ have a similar form[5]:

$$\left.\begin{matrix}A_s{}^{(1)}\\ A_p{}^{(1)}\end{matrix}\right\}=\left\{\begin{matrix}g_{YNK}{}^{(5)}\\ Rg_{YNK}{}^{(ps)}\end{matrix}\right\}C, \tag{19}$$

where

$$\langle 0|\mathcal{O}_\pi|K\rangle=(2E_K)^{-\frac{1}{2}}C(m_K{}^2-m_\pi{}^2). \tag{20}$$

In this case, however, we cannot use experimental data to evaluate the constant $C$. The value required is not completely unreasonable, however; taking

$$\langle 0|\mathcal{O}_\pi|K\rangle=(2E_\pi)^{\frac{1}{2}}\langle\pi|J_\mu{}'(0)|0\rangle\langle 0|J_\mu(0)|K\rangle,$$

where $J_\mu(0)$ and $J_\mu{}'(0)$ are the strangeness-changing and conserving weak currents, respectively, gives a value of $C$ roughly an order of magnitude less than that needed to fit the amplitudes of Table I.

We turn now to the two-meson contributions to the spectral functions:

$$(A^\mu)^{(2)}=\frac{1}{2}\int\frac{d^4p_\pi}{(2\pi)^3}\delta(p_\pi{}^2-m_\pi{}^2)\frac{d^4p_K}{(2\pi)^3}\delta(p_K{}^2-m_K{}^2)$$
$$\times(2\pi)^4\delta^4(p_Y-p_N-p_\pi-p_K)$$
$$\times[a_1(s)p_K{}^\mu+a_2(s)(p_K+p_\pi)^\mu]$$
$$\times\bar{u}_N[A(s,t_K,t_\pi)+B(s,t_K,t_\pi)Q_\mu\gamma^\mu]\Gamma u_Y, \tag{21}$$

[19] M. Gell-Mann and A. H. Rosenfeld, *Ann. Rev. Nuclear Sci.* **7**, 407 (1957).

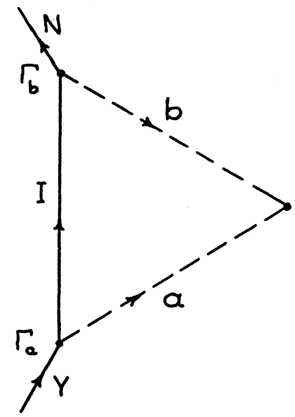


FIG. 1. Diagram corresponding to the calculation of the two-meson contribution. $\Gamma_a$ and $\Gamma_b$ are either 1 or $\gamma_5$. The particle assignments are given in Table II.

and

$$\bar{u}_N\left[\rho_s+\frac{\rho_p}{R}\gamma_5\right]u_\Lambda$$
$$=(1/2)\int\frac{d^4p_\pi}{(2\pi)^3}\delta(p_\pi{}^2-m_\pi{}^2)\frac{d^4p_K}{(2\pi)^3}(p_K{}^2-m_K{}^2)$$
$$\times(2\pi)^4\delta^4(p_Y-p_N-p_\pi-p_K)b(s)$$
$$\times\bar{u}_N[A(s,t_K,t_\pi)+B(s,t_K,t_\pi)\gamma_\mu Q^\mu]\Gamma u_Y, \tag{22}$$

where

$$\langle 0|J^\mu(0)|\pi K\rangle=(2E_K 2E_\pi)^{-\frac{1}{2}}[a_1((p_\pi+p_K)^2)p_K{}^\mu$$
$$+a_2((p_\pi+p_K)^2)(p_K+p_\pi)^\mu], \tag{23}$$

$$\langle 0|\mathcal{O}_\pi|\pi K\rangle=(2E_K 2E_\pi)^{-\frac{1}{2}}b((p_\pi+p_K)^2), \tag{24}$$

and

$$\langle\pi K|J_N(0)|Y\rangle=\left(\frac{1}{2E_K}\frac{1}{2E_\pi}\frac{m_Y}{E_Y}\right)^{\frac{1}{2}}\bar{u}_N[A(s,t_K,t_\pi)$$
$$+B(s,t_K,t_\pi)\gamma_\mu Q^\mu]\Gamma u_Y. \tag{25}$$

In the above equations $Q=\frac{1}{2}(p_K-p_\pi)$ and $\Gamma$ is 1 or $\gamma_5$, depending upon the relative parities. The three variables $s=(p_K+p_\pi)^2=(p_Y-p_N)^2$, $t_\pi=(p_Y-p_\pi)^2$, and $t_K=(p_Y-p_K)^2$ correspond to the usual invariants for $\pi KNY$ scattering and satisfy $s+t_K+t_\pi=m_\pi{}^2+m_K{}^2+m_N{}^2+m_Y{}^2$.

It is clearly impossible to evaluate the functions $a_1$, $a_2$, $b$, $A$, and $B$ exactly; here we use the simplest approximations. We replace the functions $a_1(s)$, $a_2(s)$ and $b(s)$ by constants determined from the $K\to\pi+l+\nu$ and $K\to\pi+\pi$ decay rates and approximate the functions $A$ and $B$ by the baryon pole contributions to the $\pi KNY$ matrix elements. Our calculation then includes only the contributions corresponding to diagrams of the kind shown in Fig. 1. The $\Gamma_b$ and $\Gamma_a$ are either 1 or $\gamma_5$, depending upon the parity we choose for the $\Sigma$. If we neglect mass differences within charge multiplets and factor out coupling constants, we can express our results in terms of the five diagrams, $D_i$, described in Table II.

TABLE II. Particle assignments for the five diagrams $D_i$ represented by Fig. 1.

| Line in Fig. 1 | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ |
|---|---|---|---|---|---|
| $Y$ | $\Lambda$ | $\Lambda$ | $\Sigma$ | $\Sigma$ | $\Sigma$ |
| $I$ | $N$ | $\Sigma$ | $N$ | $\Lambda$ | $\Sigma$ |
| $a$ | $K$ | $\pi$ | $K$ | $\pi$ | $\pi$ |
| $b$ | $\pi$ | $K$ | $\pi$ | $K$ | $K$ |

Each diagram will, in general, contribute to each of $f_1$, $f_2$, and $f_3$ (or $g_1$, $g_2$, and $g_3$). A detailed calculation for the $\Lambda\rightarrow p+l+\nu$ form factors with even $\Lambda-\Sigma$ parity shows, however, that the contributions to $f_1$ and $f_2$ are completely negligible. It is unlikely that this will change when we go to the other cases (i.e., $\Sigma$ decays, and odd $\Lambda-\Sigma$ parity). In any case, when we write the $\langle 0|J_\mu(0)|\pi K\rangle$ amplitude as $a_1Q_\mu+(\frac{1}{2}a_1+a_2)k_\mu$, the contributions from the $a_1Q_\mu$ term are rather complicated to calculate and, at least in the $\Lambda\rightarrow p+l+\nu$ case, quite small. We consider, therefore, only the $(\frac{1}{2}a_1+a_2)k_\mu$ term which contributes only to $f_3$ (or $g_3$). Since we are approximating $a_1$, $a_2$, and $b$ by constants, these contributions are determined by exactly the same integrals which determine the contributions to $Y\rightarrow N+\pi$. If we make use of charge symmetry and assume the $|\Delta T|=\frac{1}{2}$ rule, we find

$$\begin{aligned}
&m_\Lambda^{-1}f_3^{(2)}(\Lambda\rightarrow p+l+\nu)=(\tfrac{1}{2}a_1+a_2)[3g_{\Lambda NK}g_{NN\pi}D_1(s)+3g_{\Lambda\Sigma\pi}g_{N\Sigma\pi}D_2(s)],\\
&m_\Sigma^{-1}\left\{\begin{matrix} f_3^{(2)} \\ g_3^{(2)}\end{matrix}\right\}(\Sigma^-\rightarrow n+l+\nu)=(\tfrac{1}{2}a_1+a_2)[-\sqrt{2}g_{\Sigma NK}g_{NN\pi}D_3(s)+\sqrt{2}g_{\Sigma\Lambda\pi}g_{N\Lambda K}D_4(s)-2\sqrt{2}g_{\Sigma\Sigma\pi}g_{N\Sigma K}D_5(s)],\\
&A_s(\Lambda\rightarrow p+\pi^-)=\sqrt{2}A_s(\Lambda\rightarrow n+\pi^0)=b[\sqrt{2}g_{\Lambda NK}g_{NN\pi}D_1(m_\pi^2)+\sqrt{2}g_{\Lambda\Sigma\pi}g_{N\Sigma K}D_2(m_\pi^2)],\\
&\left.\begin{matrix} A_s \\ R^{-1}A_p\end{matrix}\right\}(\Sigma^+\rightarrow p+\pi^0)=b[\sqrt{2}g_{\Sigma NK}g_{NN\pi}D_3(m_\pi^2)+\sqrt{2}g_{\Sigma\Lambda\pi}g_{N\Lambda K}D_4(m_\pi^2)],\\
&\left.\begin{matrix} A_s \\ R^{-1}A_p\end{matrix}\right\}(\Sigma^+\rightarrow n+\pi^+)=b[2g_{\Sigma NK}g_{NN\pi}D_3(m_\pi^2)+g_{\Sigma\Lambda\pi}g_{N\Lambda K}D_4(m_\pi^2)+g_{\Sigma\Sigma\pi}g_{N\Sigma K}D_5(m_\pi^2)],
\end{aligned}\tag{26}$$

and

$$\left.\begin{matrix} A_s \\ R^{-1}A_p\end{matrix}\right\}(\Sigma^-\rightarrow n+\pi^-)=b[g_{\Sigma\Lambda\pi}g_{N\Lambda K}D_4(m_\pi^2)-g_{\Sigma\Sigma\pi}g_{N\Sigma K}D_5(m_\pi^2)].$$

The $D_i(s)$ are given by

$$D(s)=\frac{-1}{(2\pi)^3}\int\frac{ds'}{s'-s}\frac{\bar{u}_N\Gamma_b[(K^\mu\gamma_\mu+m_I)I^{(0)}(s')-k^\mu\gamma_\mu I^{(k)}(s')-P^\mu\gamma_\mu I^{(P)}(s')]\Gamma_a u_Y}{\bar{u}_N\Gamma_b\Gamma_a u_Y},\tag{27}$$

where

$$\begin{aligned}
K&=\tfrac{1}{2}(p_Y+p_N),\\
k&=p_Y-p_N,\\
P&=k_\mu-2(m_Y^2-m_N^2)^{-1}k^2K_\mu.
\end{aligned}\tag{28}$$

The functions $I^{(\alpha)}(s)$ are given by

$$\begin{aligned}
I^{(0)}&=-\tfrac{1}{4}\pi p^{-1}\ln[(d+pq)/(d-pq)],\\
I^{(k)}&=[(m_a^2-m_b^2)/2s]I^{(0)},
\end{aligned}\tag{29}$$

and

$$I^{(P)}=-[(m_Y^2-m_N^2)/2sp^2][\tfrac{1}{2}\pi q+dI^{(0)}],$$

with

$$\begin{aligned}
q^2&=[s-(m_a+m_b)^2][s-(m_a-m_b)^2],\\
p^2&=[s-(m_Y+m_N)^2][s-(m_Y-m_N)^2],
\end{aligned}\tag{30}$$

and

$$d=s^2-(m_Y^2+m_N^2+m_a^2+m_b^2-2m_I^2)s+(m_a^2-m_b^2)(m_Y^2-m_N^2).$$

Normally, the principal branch of the logarithm is used in evaluating $I^{(0)}$. For $D_1$ and $D_3$, however, there is an anomalous threshold which must be treated as described in the papers of Mandelstam[17] and Blankenbecler and Cook.[18]

The integrations indicated in (27) are all convergent. Because of the complicated form of the integrands, however, and because it is extremely doubtful that our approximations are valid for large $s$,[20] we evaluate these integrals graphically, cutting off at $s'=(m_Y+m_N)^2$. Furthermore, since we need the $D_i(s)$ only where $s=m_\pi^2$ for the pion modes and only where $m_l^2\leq s\leq(m_Y-m_N)^2$

TABLE III. Numerical values of the quantities $D_i(0)$. $m_0c^2=1$ Bev.

| $\Lambda-\Sigma$ parity | $m_0D_1(0)$ | $m_0D_2(0)$ | $m_0D_3(0)$ | $m_0D_4(0)$ | $m_0D_5(0)$ |
|---|---|---|---|---|---|
| Even | $0.98\times10^{-3}$ | $1.35\times10^{-3}$ | $0.98\times10^{-3}$ | $1.25\times10^{-3}$ | $1.35\times10^{-3}$ |
| Odd | $0.98\times10^{-3}$ | $10.9\ \times10^{-3}$ | $8.10\times10^{-3}$ | $12.5\ \times10^{-3}$ | $4.55\times10^{-3}$ |

[20] P. Federbush, M. L. Goldberger, and S. B. Treiman, Phys. Rev. **112**, 642 (1958), have found that the one-nucleon exchange contribution exceeds the limits imposed by unitarity in the physical region for the process $N+\bar{N}\rightarrow 2\pi$.

TABLE IV. Summary of estimated one- and two-meson contributions to hyperon decays.

| | Even $\Lambda-\Sigma$ parity | | Odd $\Lambda-\Sigma$ parity | |
|---|---|---|---|---|
| Decay | One-meson contribution | Two-meson contribution | One-meson contribution | Two-meson contribution |
| $\Lambda\rightarrow p+l^-+\bar{\nu}$ | $g_3=0.83$ | $f_3=0.73$ | $g_3=0.83$ | $f_3=0.70$ |
| $\Sigma^-\rightarrow n+l^-+\bar{\nu}$ | $g_3=1.18$ | $f_3=-0.39$ | $f_3=0.72$ | $g_3=-1.30$ |
| $\Lambda\rightarrow p+\pi^-$ | $A_p=\sqrt{2}(0.28)C$ | $A_s=0.61\times10^{-7}$ | $A_p=\sqrt{2}(0.28)C$ | $A_s=0.56\times10^{-7}$ |
| $\Lambda\rightarrow n+\pi^0$ | $A_p=(0.28)C$ | $A_s=0.43\times10^{-7}$ | $A_p=(0.28)C$ | $A_s=0.40\times10^{-7}$ |
| $\Sigma^+\rightarrow p+\pi^0$ | $A_p=\sqrt{2}(0.52)C$ | $A_s=0.57\times10^{-7}$ | $A_s=\sqrt{2}(3.2)C$ | $A_p=0.18\times10^{-7}$ |
| $\Sigma^+\rightarrow n+\pi^+$ | $A_p=0$ | $A_s=0.82\times10^{-7}$ | $A_s=0$ | $A_p=0.27\times10^{-7}$ |
| $\Sigma^-\rightarrow n+\pi^-$ | $A_p=2(0.52)C$ | $A_s=-0.02\times10^{-7}$ | $A_s=2(3.2)C$ | $A_p=-0.01\times10^{-7}$ |

for the lepton modes, it seems sufficient to calculate only $D_i(0)$. This could at most produce errors of the order of

$$(m_k+m_\pi)^{-2}s_{\max}\lesssim(m_K+m_\pi)^{-2}(m_\Sigma-m_N)^2\approx16\%.$$

The results of the integrations are given in Table III.

To obtain numerical values for the two-meson contributions to the form factors and amplitudes we must have numerical values for the constants $a_i$, $b$, and $g$. Since we are ignoring the $s$ dependence of $a_1$ and $a_2$ we can obtain the magnitude of these two constants from the $K\rightarrow\pi+l+\nu$ decay rates. The constant $a_1$ is determined by the $K\rightarrow\pi+e+\nu$ rate; the experimental rate[19] of $3.4\times10^6$ sec$^{-1}$ gives $|a_1|=0.30$. The experimental value of roughly unity[19] for the ratio of the muon and electron modes requires that $a_2/a_1$ be either $-5$ or 0.5.[21] As suggested by Fujii and Kawaguchi,[21] we choose the negative value, finally obtaining $|\frac{1}{2}a_1+a_2|=1.35$. Similarly, the experimental $K_1{}^0\rightarrow\pi^++\pi^-$ decay rate[19] of $0.66\times10^{10}$ sec$^{-1}$ gives $|b|=2.57\times10^{-7}m_0$, with $m_0c^2=1$ Bev. It should be noted here that, by ignoring the $s$ dependence of the $K\pi\pi$ amplitude, we have neglected a possible contribution from a term odd in the isotopic spin indices of the two pions. This term would, for example, permit contributions from intermediate states containing charged $K$ mesons. Of course it gives no contribution to the actual decays since it must vanish when both pions are on the mass shell. The strong-coupling constants, with the exception of $g_{NN\pi}$, have not been accurately determined. In order to obtain numerical estimates we have chosen the following, hopefully "reasonable," values[12]:

$$(g_\pi{}^{(ps)})^2/4\pi=14,\quad (g_K{}^{(ps)})^2/4\pi=2.2,$$
$$(g_{\Sigma\Lambda\pi}{}^{(s)})^2/4\pi=0.5,\quad\text{and}\quad(g_{\Sigma NK}{}^{(s)})^2/4\pi=0.8.\quad(31)$$

From these values for $a_1$, $a_2$, and $b$, and the strong-coupling constants, and the values for the $D_i(0)$ given in Table III, we obtain the two-meson contribution listed in Table IV.

### 5. DISCUSSION

The results of Sec. 4 indicate that the one- and two-meson contributions to the $Y\rightarrow N+l+\nu$ form factors will have little effect upon the observables connected with these decays; these observables should be largely determined by the $V$ and $A$ subtraction constants, $f_1(0)$ and $g_1(0)$. This should be especially true for the electron modes, where the contributions of $f_3$ and $g_3$ are reduced by a factor of $(m_e/m_Y)^2$. The relative effect of the "induced" $f_3$ and $g_3$ in the muon modes will depend upon the values of $f_1(0)$ and $g_1(0)$, but probably will be fairly small. For example, choosing $f_1(0)=g_1(0)$ to fit the present experimental estimate for the $\Lambda\rightarrow p+e+\nu$ decay rate, the values for $f_3$ and $g_3$ given in Table III give a correction to the muon energy spectrum for the decay $\Lambda\rightarrow p+\mu+\nu$ of about 10%, due mainly to the $f_1-f_3$ interference term.

Our estimates for the two-meson contributions to the $A_s(Y\rightarrow N+\pi)$, for the case in which the $\Sigma-\Lambda$ relative parity is even, are all smaller than the experimental values listed in Table I by factors of 4 or 5, but the relative magnitudes are almost exactly in the ratio suggested by experiment. The correct ratios, of course, follow directly from our use of common strong-coupling constants $g_K$ and $g_\pi$ if we ignore the $\Lambda-\Sigma$ mass difference.[6] Our results for the quantities $D_i$ show that the effects of the mass difference tend to cancel, although because of the approximations we have made it is not clear how meaningful this conclusion is. The $\Lambda-\Sigma$ mass difference does have a large effect, however, in the one-meson contributions to the $A_p$. The relative magnitudes of these amplitudes would be exactly as described in Table I were it not for the factor $R$, which is a measure of the final state momentum. For $\Sigma\rightarrow N+\pi$, $R\approx0.10$, while $R\approx0.054$ for $\Lambda\rightarrow N+\pi$, so that the $A_p(\Lambda\rightarrow N+\pi)$ are reduced by almost a factor of 2 relative to the $A_p(\Sigma\rightarrow N+\pi)$.

Our estimates for the one- and two-meson contributions to the $Y\rightarrow N+\pi$ amplitudes thus do not come particularly close to the values required to fit the experimental data, indicating that the intermediate states which we have neglected may be important. One can think of several ways in which the agreement might be improved, however. For the decays with large asymmetries we could make use of the range of the $s$- to $p$-wave ratio permitted by experiment, choosing the constant $C$ to give large values for $A_p$ and thus requiring smaller values for $A_s$. It might also be true that the value we have chosen for $g_K$ is too small. Finally there is

[21] A. Fujii and M. Kawaguchi, Phys. Rev. **113**, 1156 (1959).

the possibility that the $K \rightarrow 2\pi$ amplitude $b(s)$ is somewhat larger in the region of integration that at $s=m_\pi^2$ because, say, of an $s$-wave $\pi-K$ resonance.

If the $\Sigma-\Lambda$ parity is odd it is even less likely that the experimental amplitudes are due predominantly to one- and two-meson intermediate states. In this case the two-meson contributions to the $\Sigma \rightarrow N+\pi$ $p$-wave amplitudes are too small by roughly a factor of 15. Furthermore, with the values for the strong coupling constants we use, we find that $A_p(\Lambda \rightarrow p+\pi^-)=\sqrt{2}(0.28)C$ and $A_s(\Sigma^+ \rightarrow p+\pi^0)=\sqrt{2}(3.2)C$, while experiments indicate that these amplitudes should be almost equal. Even in this "odd" case, however, it is not absolutely impossible to obtain agreement, provided we are willing to make adjustments of the kind mentioned in the previous paragraph.

All the above results and conclusions depend, of course, upon many assumptions and approximations. An accurate calculation of the two-meson contributions should include rescattering corrections to the $\pi KNY$ amplitudes and an estimate of the $s$ dependence of the amplitudes $a_1$, $a_2$, and $b$. At the present time, however, with so little known about the strong interactions of $K$ mesons and hyperons, it appears that only extremely crude estimates of these corrections could be made. Even the results obtained in this paper depend upon coupling constants and parities which are not well known.

It would be of great interest to have experimental confirmation of the qualitative conclusions drawn from our results. For example, it should be possible to fit $Y \rightarrow N+l+\nu$ data quite well, especially when $l=e$, with a baryon current of the form $\bar{u}_N(f_1\gamma_\mu+g_1\gamma_\mu\gamma_5)u_Y$, where $f_1$ and $g_1$ are constants (the corresponding expressions for observables can be obtained from references 15). Also, our results indicate (although certainly not conclusively) that if the $\Sigma-\Lambda$ parity is even then $A_s(\Sigma^+ \rightarrow n+\pi^+) \gg A_p(\Sigma^+ \rightarrow n+\pi^+)$ and $A_p(\Sigma^- \rightarrow n+\pi^-) \gg A_s(\Sigma^- \rightarrow n+\pi^-)$, while the reverse inequalities should hold if the $\Sigma-\Lambda$ parity is odd. When the $\Sigma-\Lambda$ parity is known, these conclusions could be verified by measuring the transverse polarization of the emitted neutron.[22,23] Unfortunately, both of these experimental programs seem quite difficult.

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[22] T. D. Lee and C. N. Yang, Phys. Rev. **108**, 1645 (1957).
[23] Chou Kuang-Chao, Zhur. Eksptl. i Teoret. Fiz. **38**, 1342 (1960) [translation: Soviet Phys-JETP **11**, 966 (1960)].