Pressure Dependence of the Mössbauer Effect

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The pressure dependence of the Mössbauer effect is examined and expressions are given (in the Debye approximation) for the probability of recoiless emission (absorption) as a function of the specific volume. It is concluded that the effect of pressure is slight for metals such as Zn, Sn, etc., but that more compressible elements should exhibit observable pressure dependencies at moderate pressures. A calculation is given which predicts that the recoiless emission of the 81-kev line in Cs^{133} should become observable at a pressure of about 5000 atm. Some additional experiments involving the above pressure dependence are also discussed.

I. INTRODUCTION

THE study of recoiless radiation (Mössbauer effect) and possible applications involving it have, in the short time since Mössbauer's original work,[1] become major topics of research in several areas of physics.[2] In particular, the occurrence of recoiless radiation has aroused widespread interest among solid-state physicists because of the sharpness of the resonant lines and of their sensitive dependence upon both the immediate environment of the emitting nuclei and upon the lattice characteristics of the crystals in which they are located. By now, a number of papers[2] are to be found in the literature on both of these relationships and also on the many other facets of Mössbauer's discovery. In the remainder of the present note, a new relationship, namely the dependence of recoiless radiation upon the ambient pressure of a sample, is investigated and the conditions under which an observable effect is present are discussed. Also presented are brief discussions of some experiments in which this dependence may be further investigated and possibly utilized to advantage.

II. THEORY OF PRESSURE EFFECT

Before going into our calculations, we first remark that a pressure dependence of the amount of recoiless radiation is to be expected on the same grounds as is the occurrence of this radiation in the first place. Thus we recall that recoiless radiation is due to the occurrence of nuclear transitions in which the recoil momentum of the emitted gamma ray is imparted to the lattice as a whole rather than just to the emitting nucleus. Then, speaking in a naive way, recoiless radiation would predominate in the limit of a very rigid lattice. This therefore suggests that the amount of recoiless radiation from an actual decaying system should *increase* as the rigidity of the lattice is *increased*. That this in fact will be the case when pressure is applied is predicted from the work of Mössbauer,[3] in which the theory of recoiless emission is presented using the Debye model of lattice vibrations. Here the probability of recoiless emission f is found to be given by

$$f=\exp\{-0.75E_\gamma^2[1+6.6(T/\theta)^2]/mc^2k\theta\}, \quad (2.1)$$

where E_γ is the energy of the emitted gamma ray, T and θ are the absolute and Debye temperatures, respectively, c is the velocity of light, k is Boltzmann's constant, and m is the mass of the emitting nucleus. This equation, derived for $T/\theta\ll 1$, will be used throughout our work and many of our subsequent conclusions are valid only to the extent that it describes conditions in an actual crystal. On proceeding, we have, as is well known,[4] that θ is related to the maximum vibration frequency $\nu_{\max}$ of the crystal by the relation

$$\theta=(h/k)\nu_{\max}, \quad (2.2)$$

where h is Planck's constant. Furthermore, we know qualitatively that $\nu_{\max}$ is related directly to the lattice force constants. Since the latter will depend upon the pressure on the system, we thus see the effect we are trying to describe appearing in a sensitive (exponential) manner. For example, an increase in θ by a factor of two would give an enhancement of an order of magnitude in a typical fraction of recoiless radiation. Continuing the analysis, we now recall that θ and V, the volume of the system, are related through the Grüneisen parameter γ by the relation[4]

$$d\ln\theta/d\ln V=-\gamma. \quad (2.3)$$

For the present purpose, we may neglect the volume dependence of γ and write this equation as

$$\theta_2=(V_2/V_1)^{-\gamma}\theta_1\approx(1+\gamma\Delta V/V_1)\theta_1, \quad (2.4)$$

where $\Delta V=(V_1-V_2)$ and where the subscripts denote properties of the sample at two different volumes. Substituting this expression into Eq. (2.1), we find the probabilities of gamma-ray emission at two different

[1] R. L. Mössbauer, Z. Physik **151**, 124 (1958); Naturwissenshaften **45**, 538 (1958); Z. Naturforsch. **14a**, 211 (1959).

[2] See for example, Proceedings of the Mössbauer Effect Conference, University of Illinois, Urbana, Illinois, June, 1960 [University of Illinois Report AFOSR TN 60-698 (unpublished)].

[3] R. L. Mössbauer, Z. Physik **151**, 124 (1958); R. V. Pound and G. A. Rebka, Jr., Phys. Rev. Letters **3**, 440 (1959).

[4] M. Born and K. Huang, *Dynamical Theory of Crystal Lattices* (Oxford University Press, New York, 1954), Chap. II. See also *Solid-State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1958), Vol. 6, pp. 1–60, and references cited therein.

volumes to be related by the equation

$$f(V_2)=\exp\{-0.75E_\gamma^2[1+6.6(T/\theta_2)^2]/mc^2k\theta_2\} \quad (2.5)$$

$$=\exp\{-0.75E_\gamma^2[1-\gamma\Delta V/V_1+6.6(T/\theta_1)^2 \times(1-2\gamma\Delta V/V_1)]/mc^2k\theta_1\}, \quad (2.6)$$

or, when T/θ_1 is negligibly small,

$$f(V_2)=\exp\{-0.75E_\gamma^2[1-\gamma\Delta V/V_1]/mc^2k\theta_1\}. \quad (2.7)$$

We thus see that (isotropic) changes in sample volumes reflect themselves simply and importantly in the probability of occurrence of recoiless radiation. Now volume compressibilities $K[\equiv(1/V)\Delta V/\Delta P]$ range from about 5×10^{-5} atm $^{-1}$(Cs) to 2×10^{-6} atm $^{-1}$Zn) and γ has the average value two for many crystals. Using the first of these values and the corresponding value of f given in reference 2, we find that a pressure of 5000 atm will bring f to an (observable) value of 4×10^{-3}, i.e., will cause sufficient change in the crystal binding so as to change f by two orders of magnitude. We thus conclude from the relatively low values of the required pressure that it should be feasible to bring about recoiless emission of the 81-kev line in Cs this way. For the less compressible Zn however, a pressure of about 30 000 atm will bring about a change in f by a factor of 2. We then conclude from these and other similar calculations that an observable shift in the fraction of recoiless radiation should occur at commonly encountered pressures, but that, generally speaking, rather high pressures are required to produce large changes.

III. DISCUSSION

The foregoing results require little comment in themselves as they are elementary and have, for the most part, only order-of-magnitude significance. In fact, the only point we have made thus far is that a measurable pressure dependence of the amount of recoiless radiation is to be expected on the basis of theory. With this in mind, we now conclude our note by commenting upon several possible experiments where the effect might be of use.

The first point we wish to mention is that it may be possible to use pressure to bring about recoiless radiation from transitions which normally give rise to off-resonant radiation. It is clear from the foregoing that this would most likely happen with more compressible substances unless rather high pressures are involved. Also we note that the probability of recoiless transitions involving gamma rays of *higher* energy than those usually encountered should increase as the pressure is increased. In fact, even though our simple theory might not be valid under such conditions, there seems to be no reason why very high pressures could not be used to bring about rather high-energy recoiless radiation. However, it is to be expected that this latter situation would present a number of experimental difficulties and our conclusion here should be considered as tentative.

Finally we note that resonant absorption within an emitting sample can give rise to a measurable change in the number of gamma rays emitted per unit time by the decaying sample. It is then to be expected that samples at two different pressures will give rise to different apparent rates of decay. This fact might be of some use in investigating the effects of extreme pressure on the amount of recoiless radiation since it is not then necessary to make measurements on the system while it is subjected to pressure. It might also be feasible to adopt the technique of Frauenfelder *et al.*,[5] and use the x rays arising from internal conversion of the re-emitted gamma rays in studying pressure effects. Here it might be expected that the change with pressure of the number of reabsorbed gamma rays could be followed by observing the number of x rays emitted from the sample.

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[5] H. Frauenfelder, D. R. F. Cochran, D. E. Nagle, and R. D. Taylor, Nuovo cimento **19**, 183 (1961).