crystal sites. Using our data for the relative density of chromic ions in the crystal and in frozen solution, we require that more than 13% of the crystal used by KS be occluded solution. As this is really an underestimate of the fraction of solution, we conclude that the crystal used by KS probably had a lower density of chromium in the crystal relative to that in the solution than existed for the crystals which we have grown. (See Sec. I, Note added in proof.)

VI. CONCLUSIONS

Chromic ions and possibly many other trivalent ions can be put into two different sites of cerium double nitrate crystals which are not any of the sites usually occupied by the metal ions. Such crystals provide an excellent method for the orientation of chromium nuclei because the large negative value of D should prevent complications due to Ce–Cr interactions. The very small value of $g_{\parallel}$ for cerium ions in this compound permits the application of a field along the symmetry axis so to produce a large polarization of the chromium nuclei. If complications due to occluded solution are to be avoided only very clear crystals should be used and they should be checked for solution content by the proton resonance method described in Sec. III.

Rather large amounts of solution may be trapped in small pockets within the double nitrate crystals. If the pockets are sufficiently small, they will be cooled by contact with the crystalline material. The small value of $g_{\parallel}$ for cerium ions in the double nitrate permits one to apply a field to the crystal so to establish on orientation axis for the electron spins.

The amount of orientation which can be established by a given field will depend critically upon the distribution of nondiagonal interactions present for the ions in the solution. The nature of the distribution of nondiagonal interactions can often be inferred from the paramagnetic resonance spectrum. For ions of spin greater than $\frac{1}{2}$, the off diagonal interactions will usually be most severe. If the spin is half integral and greater than one, the width of the $(\frac{1}{2},-\frac{1}{2})$ transition gives a measure of the nondiagonal interactions. When the spin is integral, it may be necessary to observe the forbidden transitions in order to obtain a relatively sharp line from which the nondiagonal interactions can be estimated. For ions with an effective spin of $\frac{1}{2}$, the only nondiagonal interactions are the magnetic and electric hyperfine interactions and, of course, the spin-spin interactions which should never be very severe. If the g factor is highly anisotropic, it will be almost impossible to determine the spread of off diagonal interaction as most of the linewidth observed in paramagnetic resonance measurements would be due to the variation of the effective g value. If the hyperfine interaction constants are sensitive to the distortions of the hydrated complex, a spread of values will be encountered. Chromic ions are probably more difficult to orient in frozen solution than most ions because of the large fine structure interactions which are encountered. However, the frozen solution might be useful for the orientation of the nuclei of paramagnetic ions which do not enter the crystal sites. If the ion does enter the crystal sites, nuclear orientation experiments with good quality crystals will be more easily interpreted. If the ion enters the crystal very readily, the presence of occluded solution will be relatively unimportant.

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Inverted Nature and Significance of Negative-Mass Landau Levels*

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Quantum-mechanical considerations on negative-effective-mass carriers in crystals show the following: The energy-level structure of particles with negative effective mass, in the presence of a magnetic field, consists of a system of *inverted* Landau levels. A simple derivation of the level inversion is given. Sharp contradistinctions are found on the behavior of the levels by comparison to that of positive-mass regions of momentum space. The significance of this type of level structure for cyclotron resonance transitions, in particular in the case of the observed small negative-mass branch of heavy holes in Ge is discussed.

INTRODUCTION

THE inverted nature of the harmonic oscillator-like Landau levels for carriers (electrons or holes) with negative effective mass has already been pointed out[1,2] and the implications of such a quantum level structure for nonequilibrium cyclotron resonance work

* This work has been supported in part by the U. S. Air Force, Air Research and Development Command, Air Force Cambridge Research Center under contract.

[1] G. C. Dousmanis, in *Quantum Electronics*, edited by C. H. Townes (Columbia University Press, New York, 1960), p. 458; J. Appl. Phys. **32**, 2005 (1951).
[2] H. Kroemer, International Solid-State Circuit Conference, 1960, University of Pennsylvania (unpublished), p. 81.

have been discussed.[1,3] We give here the mathematical derivation for the level inversion and show that it is a simple consequence of the change of the sign of the effective mass. The fundamental differential between changing the sign of the mass and changing the sign of the charge is pointed out. These points would hardly require proof, but for the fact that an harmonic oscillator Hamiltonian, with a negative mass in the kinetic term but positive force-constant term, implies no bound states for negative-mass particles in the presence of **H**. It will be seen below that, with the Hamiltonian properly written, the bound states arise from the fact that not only the kinetic, but also the potential term is negative. This leads to the inverted Landau level structure of carriers with negative effective mass.

The energy level structure shows that a *single* particle injected into negative-mass Landau level regions in cyclotron resonance will *emit* rather than *absorb* rf power, whereas if injected into the m^+ region it would absorb. It holds for negative-mass regions of the simple as well as re-entrant (warped) band type. Even in the case of an injected particle distribution, as in the negative-mass cyclotron resonance experiments in Ge,[4] an increasing level broadening, decreasing spacing, and increasing level density with decreasing energy imply lifetime (e.g., scattering time) shortening at lower energies. This would make the establishment of nonequilibrium, emissive distributions easier to achieve in m^- rather than m^+ regions of **k** space.

LANDAU LEVEL STRUCTURES—NEGATIVE MASS BRANCH IN WARPED ENERGY SURFACES

An exact quantum treatment of m^- regions would require use of the methods of Luttinger and Kohn[5] that have been applied to usual m^+ regions and to the $k_z=0$ regions (m^+) of warped energy surfaces such as those of heavy holes in Ge and Si. We utilize here much simpler techniques[6] in order to bring out main features that distinguish the quantum properties of m^- from those of m^+ regions of momentum space.

The energy levels of particles of mass m in the presence of a magnetic field **H** are of the harmonic oscillator type.[6] Thus, for positive masses,

$$E_1(H)=(n+\tfrac{1}{2})\hbar\omega, \tag{1}$$

with

$$\omega=\pm qH/mc. \tag{2}$$

The choice of signs in (2) is such that ω is always positive. The above result is derived from the basic Hamiltonian and arises from the quantization of the energy in the presence of **H**: One requires the energy-level structure of a particle with charge q and mass m in a magnetic field. We write down in a general form the Hamiltonian and the Schrödinger equation for the particle in a magnetic field given by an appropriate vector potential **A**. Thus

$$\mathcal{H}=\frac{1}{2m}\left(\mathbf{p}-\frac{q}{c}\mathbf{A}\right)^2, \tag{3}$$

or

$$\mathcal{H}=\frac{p^2}{2m}-\frac{q}{2mc}(\mathbf{p}\cdot\mathbf{A}+\mathbf{A}\cdot\mathbf{p})+\frac{q^2}{2mc^2}A^2. \tag{4}$$

The Schrödinger equation is

$$\left[-\frac{\hbar^2}{2m}\nabla^2+\frac{iq\hbar}{2mc}\mathbf{A}\cdot\mathrm{grad}+\frac{iq\hbar}{2mc}(\mathrm{div}\mathbf{A})+\frac{q^2A^2}{2mc^2}\right]\psi=E\psi. \tag{5}$$

The usual vector potential,

$$A_x=0,\quad A_y=Hx,\quad A_z=0, \tag{6}$$

gives constant **H** along the z axis. Thus for the Schrödinger equation, one has

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}-\frac{\hbar^2}{2m}\left(\frac{\partial}{\partial y}-\frac{iqHx}{\hbar c}\right)^2\psi-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial z^2}=E\psi. \tag{7}$$

The substitution

$$\psi(x,y,z)=e^{i(k_yy+k_zz)}u(x) \tag{8}$$

yields

$$-\frac{\hbar^2}{2m}\frac{\partial^2u}{\partial x^2}+\frac{\hbar^2}{2m}\left(k_y-\frac{qHx}{\hbar c}\right)^2u=E_1u, \tag{9}$$

or

$$\frac{\partial^2u}{\partial x^2}-\left(k_y-\frac{qHx}{\hbar c}\right)^2u+\frac{2mE_1}{\hbar^2}u=0, \tag{10}$$

where

$$E_1=E-(\hbar^2k_z^2/2m). \tag{11}$$

The energy of interest, in the xy plane, is given by (10). This is the equation for an harmonic oscillator with center at

$$x_0=c\hbar k_y/qH, \tag{12}$$

and with energies and frequencies given by (1) and (2) above.

Before we deal with the negative-mass solution we note the following: In Eqs. (3)–(5), a reversal of the sign of q does not affect the sign of the Hamiltonian. By contrast, a change of the sign of the mass reverses the sign of the right-hand side of (3) and (4) and also of the Schrödinger equation (5). From the reversal of (3) and (5) for negative m, one would expect bound states of negative energy (inverted Landau levels) as

[3] B. Lax and J. G. Mavroides, in *Solid-State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1960), Vol. 11; (see this review article for a comprehensive list of references).

[4] G. C. Dousmanis, R. C. Duncan, Jr., J. J. Thomas, and R. C. Williams, *Proceedings of the International Conference on Semiconductor Physics, Prague, 1960* (Publishing House, Czechoslovakian Academy of Science, Prague, 1961), p. 603; Phys. Rev. Letters **1**, 404 (1958); G. C. Dousmanis, *ibid.* **1**, 55 (1958).

[5] J. M. Luttinger and W. Kohn, Phys. Rev. **97**, 869 (1955); **102**, 1030 (1956).

[6] See, for instance, R. E. Peierls, *Quantum Theory of Solids* (Oxford University Press, New York, 1956); A. H. Kahn and H. P. R. Frederikse, in *Solid-State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1959), Vol. 9.

will be shown in the following. This may be thought of as resulting from a change in sign of both the kinetic energy *and potential* terms of the Hamiltonian, and the Schrödinger equation.

The Hamiltonian, with the use of the vector potential defined above, can be written as

$$\mathcal{H}=\frac{1}{2m}\left[p_x^2+\left(p_y-\frac{qHx}{c}\right)^2+p_z^2\right]. \tag{13}$$

A change in the sign of q changes the origin of the harmonic oscillator from cp_y/qH to $-cp_y/qH$, whereas the change in the sign of the mass changes the sign of $\mathcal{H}$. The other change involved in changing q for a particle of given mass is the direction of rotation in the magnetic field. Likewise particles of the same charge, but opposite mass will, of course, rotate in opposite direction in the magnetic field.

For m negative, from (3), (5), and (6) one has

$$\frac{\hbar^2}{2|m|}\frac{\partial^2\psi}{\partial x^2}+\frac{\hbar^2}{2|m|}\left(\frac{\partial}{\partial y}-\frac{iqHx}{\hbar c}\right)^2\psi+\frac{\hbar^2}{2|m|}\frac{\partial^2\psi}{\partial z^2}=E\psi. \tag{14}$$

Comparison with (7) shows that all terms except $E\psi$ have changed sign. The expression corresponding to (10) becomes, from (8) and (14):

$$\frac{\partial^2 u}{\partial x_2}-\left(k_y-\frac{qHx}{\hbar c}\right)^2 u-\frac{2|m|E_1}{\hbar^2}u=0. \tag{15}$$

Substituting

$$E_1=-\mathcal{E}, \tag{16}$$

we obtain

$$\frac{\partial^2 u}{\partial x^2}-\left(k_y-\frac{qHx}{\hbar c}\right)^2 u+\frac{2|m|\mathcal{E}}{\hbar^2}u=0, \tag{17}$$

which is of identical form with (10) except that it involves $\mathcal{E}$ instead of E_1. Then for m^-, and for the motion on the xy plane,

$$-\mathcal{E}=E_1=-(n+\tfrac{1}{2})\hbar\omega, \tag{18}$$

$$\omega=\pm qH/mc, \tag{19}$$

where, as in (2), the sign in (19) is chosen so that ω is positive.

The particles in the crystal are not free, but move in the periodic lattice potential. The extension of the argument follows the one given earlier for the m^+ case[6] and will not be discussed here. One uses the effective mass m^* instead of m. Equations (1) and (18) apply, of course, to the constant-m (parabolic) regions of the bands. As m^* increases in the nonparabolic regions the levels become closely spaced. The "reversal" of energy levels in m^- regions[1] for bands of a simple type[7] is shown in Fig. 1. The variation of the level spacing and width with energy has been discussed by Brailsford[8] for the m^+ case. The levels not only get closer together, but also *broaden* as one approaches from either side the middle of the band. This *lattice* broadening is a consequence of the second term in (10) and (14) and would imply, aside from other causes, a decrease in lifetime (for example, the collision time) in these regions. The spacing decreases and broadening increases as one proceeds to the band center, but the overlap occurs mostly in a region near the center.

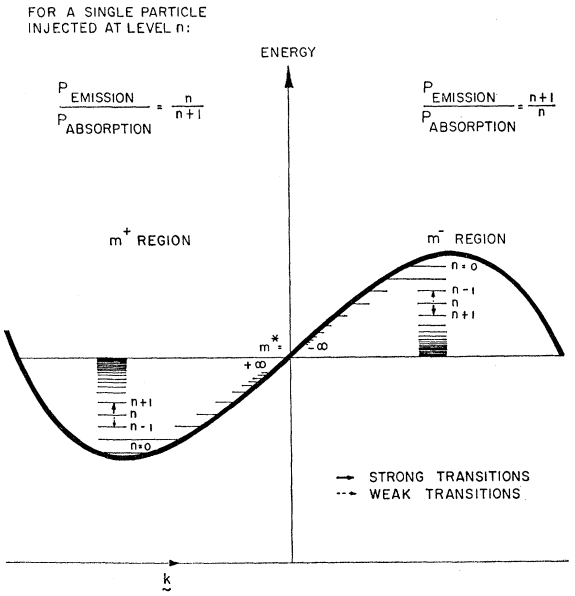

FIG. 1. Landau levels in positive- and negative-mass regions of a simple band (see text). In a conduction band the right-hand side would represent the usually underpopulated m^- electron regions at high particle energies (past the inflection point). The same diagram is applicable to a valence band with the vertical axis representing increasing hole energy. The magnetic field is applied on the plane of the paper and is perpendicular to the **k** vector shown. Note the inverted nature of levels and matrix elements for rf electric dipole transitions in the m^- region.

The magnetic energy levels are also drawn in a schematic way in Fig. 2 for the case of heavy holes in Ge and Si. The meaning of k_y and k_z is clear from the accompanying figure of the re-entrant contours. In these cases m_x and m_y are negative and fairly equal to each other in the central portion of the re-entrancy (shaded part, Fig. 2). m_z is positive, but the energy parallel to $\mathbf{H}(k_z)$ is not quantized and is separable so that the level inversion in directions perpendicular to k_z is not basically affected. Normal Landau levels are indicated in regions B (m^+) and inverted levels are shown in the m^- region (Fig. 2). Note that in both Figs. 1 and 2 the upper energy limit in the m^- region consists of the lowest quantum number energy level ($n=0$).

In the re-entrant case of Ge and Si we shall limit the discussion to a mere delineation of the various regions

[7] See, for instance, W. Shockley, *Electrons and Holes in Semiconductors* (D. Van Nostrand Company, Inc., Princeton, New Jersey, 1951), Chap. 5.

[8] A. D. Brailsford, Proc. Phys. Soc. (London) **A70**, 275 (1957).

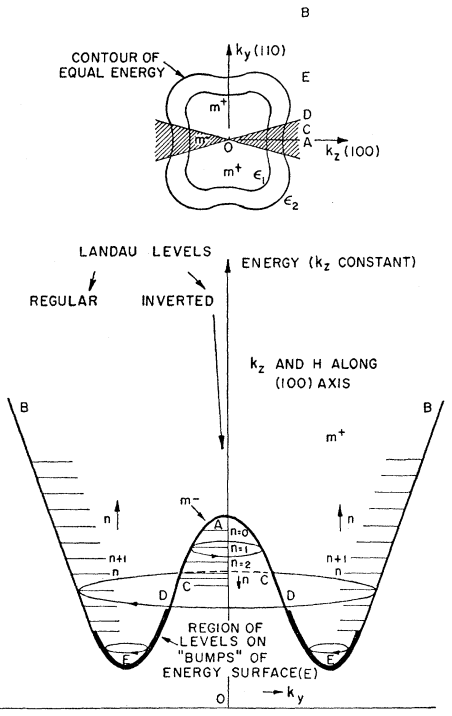

FIG. 2. Schematic diagram of energy vs the k_y axis for a constant value of k_z in bands of the type of heavy holes in Ge and Si. These bands have a small negative mass (re-entrant) region at low energies. The Landau levels are inverted in the central part of the re-entrancy. The classical orbits are not circular, but are shown as such only to illustrate the sense of particle rotation in the three branches of the energy spectrum. The upper part of the figure shows two contours of equal energy ($\epsilon_2 > \epsilon_1$), and the shaded region denotes that of the negative "transverse" mass, computed in directions perpendicular to, and radially outwards from, the [100] axis (see Fig. 64 of reference 3). In the region of levels with fairly equal spacing, which are of main interest to resonance (central part of shaded region), this mass is about equal to the value of the Shockley "tube" mass. The "hump" (energy separation AE) is zero at the $k_z=0$ (reference 4), and increases as one proceeds to higher contour energies along the k_z axis. It is estimated that approximately as much as 45% of momentum space is taken up by the eight "bumps" (E) on the energy surface in Ge. By contrast the volume associated with the two shaded regions is only 4% of the total [the other four similar regions on the energy surface are to be counted as belonging to the remaining main part (B) of the positive-mass region for $\mathbf{E}_{rf}$ perpendicular to the [100] axis (k_z above)]. These relative sizes for the three branches apply to volume in momentum space and are not to be taken as accurate representations of particle distributions under the influence of $\mathbf{H}$ and nonequilibrium conditions. The opposite rotation of the m^- particles allows their experimental separation from the two other branches of the energy spectrum by use of circularly polarized microwaves.

of momentum space that give resonances of their own,[1,3] i.e., groups of energy levels which despite level-spacing variation within each group, contribute to a single resonance with the usual energies and power levels in cyclotron resonance.[3] Figure 2 shows schematically the three *main* quantum structures with H along the [100] axis for a constant value of k_z. The re-entrant contours are shown and the shaded region is that of the negative *transverse* mass as usually defined (see caption of Fig. 2). This mass is constant near the center of the shaded region, hence the level spacing there is fairly constant (see Fig. 64, reference 3). Its value near the central cone region, that is significant for resonance, is about the same as that of the Shockley "tube" mass.[3] The m^- orbits are fairly circular at the center, but become distorted and broadened as one approaches 17° in Ge (inflection point D, Fig. 2). The classical trajectories have been discussed by Kagan.[9] For a plane perpendicular to k_z the m^- orbits go only near 17° in Ge for k_y along the [110] axis, but as far as 34° for k_y parallel to [100] axis, before one runs into the region of levels associated with the "bumps" of the energy surface (regions E, Fig. 2). In this transition region, both the quantum picture[1] and the classical[9] one suggest a behavior resembling that of open orbits. Thus only the region within 17° with such values for $\mathbf{H}$, contour energy and k_z so that at least a few equally spaced levels (with the smallest quantum numbers) are within it, can be considered as a *resonant* negative-mass region. (There is no "hump" on the energy vs k_y curves, i.e., no negative-mass region if one restricts the picture[4,10] to $k_z=0$.) Figure 2 applies to the cases where the re-entrancy (energy separation AD, for example, ~7% of contour energy in Ge) is large enough so that at least a few levels with the smallest quantum numbers are fairly equally spaced. The structure for energy surfaces that are axially symmetric about the k_z axis, and for the region close to $k_z=0$ has been examined semiclassically on a model that extends uniformly the m^- region to 34° from the [100] axis.[11]

A further point essential to the existence of discrete negative-mass energy levels may be noted here. In the negative-mass regions of Figs. 1 and 2 the intersections of a plane perpendicular to $\mathbf{H}$ with the energy surface are closed curves: In Fig. 2 the curves for all three branches of the energy spectrum lie in the same zone and are closed, and in Fig. 1 they are closed by using periodic continuation in $\mathbf{k}$ space. This leads to discrete energy levels in the main parts of these regions as indicated.

We point out here that, as indicated from the behavior of the m^- orbits, at angles larger than about 15° in Ge, the energy surface is anything but symmetric about the [100] axis. The third quantum branch, associated with the "bumps" on the energy surface, occupies an appreciable amount of momentum space, although the levels, with $\mathbf{H}$ along the [100] axis, are not well defined since the orbits run around E (Fig. 2) and from the contour shape, they cannot easily conserve energy and k_z. Their mass is close to that of the usual heavy holes[9] (region B). We compute that roughly 45% of the

[9] Yu. Kagan, J. Exptl. Theoret. Phys. U.S.S.R. **38**, 1854 (1960) [translation: Soviet Phys.-JETP **11**, 1333 (1960)].

[10] For an extension to the case of $k_z \neq 0$ see R. F. Wallis and H. J. Bowlden, Phys. Rev. **118**, 456 (1960).

[11] B. Rosenblum and R. C. Duncan, Jr., in *Proceedings of the International Conference on Semiconductor Physics, Prague 1960* (Publishing House, Czechoslovakian Academy of Science, Prague, 1961), p. 606.

volume in momentum space is occupied by these regions (near [111] axes) in the absence of **H**.

DISCUSSION

Consider one of the consequences of the inversion of the energy levels in both the case of the simple band (Fig. 1) and that of re-entrancy (Fig. 2): A single particle injected into the equally spaced part of the m^- region will emit rather than absorb rf power under resonance, since the matrix element for a downward (emissive) transition [proportional to $(n+1)^{\frac{1}{2}}$] is larger than that for an upward transition (proportional to $n^{\frac{1}{2}}$). By contrast, if the particle is injected into the m^+ region it will absorb rf, since the transition probability to the higher than lower state is larger, and despite the fact that there are empty levels below it. This is sufficient quantum-mechanical proof that a single m^- particle is emissive.

Even when a *distribution* of particles is injected into the m^- region, its very structure (decreasing level spacing, increasing density of states, and increasing level broadening with decreasing E_1) favors establishment of emissive distribution.[1,4,12] In the warped surfaces of Ge and Si such an emissive distribution may be limited to directions perpendicular to the [100] axis, i.e., may consist of overpopulation of the central cone region as optically excited carriers,[1,3] for instance, lose energy by phonon emission in the anisotropic band. It is not necessary for such a distribution to be an increasing function of k_z or of the total energy. What is required is that it be more dense at point A than at C or D (Fig. 2). Thus in this kind of effect partial population inversion along particular directions of momentum space is required. The overpopulation of the central cone region may be brought about by anisotropic scattering[13] in the warped surface. In the present picture the very lattice broadening of the levels and their increasing density suggest lifetime (for example, scattering time) shortening[1,3] as the energy decreases in directions perpendicular to the [100] axis. This implies overpopulation of the upper levels (region A, Fig. 2) under excitation. All of these effects have a common cause, i.e., the band anisotropy.

Quantum-mechanical considerations showing that m^- regions are not expected to show much resonant absorption even at equilibrium have already been discussed.[1,11] We wish to emphasize here the significance of "anharmonicity," that was pointed out quite early,[12] in the re-entrant region: For each k_z value (Fig. 2) the significant properties of this region arise from the fact that, once H is applied, no states exist near point A (high conductance region) but at $n=0$, 1, etc., which are in regions of lower conductance, *because of the change of curvature, even at these low quantum numbers.*[12] In a parabolic band the small number of levels and the fact that the quantum numbers are small would not be significant. If the curvature were the same, essentially comparable absorption may be expected at equilibrium from both a classical and a quantum calculation. However, in the present case, a classical calculation would be assuming a high positive conductance as if the particles were near A and would overestimate the result of such absorption. Even the semiclassical treatments with the axial approximation would tend to overestimate resonant absorption at equilibrium for the real Ge surface since they underestimate the anharmonicity if they uniformly extend the negative-mass region to 34° from the [100] axis.

The main m^+ region B (k_z close to zero) consists (except for the unequal spacing of the lowest levels that yield the structures called quantum effects[3]) of essentially a single harmonic oscillator system with equal spacing up to energies at least two orders of magnitude larger than the Landau level spacing (microwave range). In sharp contrast the m^- region consists of a whole series of anharmonic oscillator systems, one for each value of k_z. Only one of these systems is represented in Fig. 2. The anharmonicity, for a given band and value of the magnetic field, is a function of energy of the contour, and is higher at lower energies.

The deviation of the band from parabolic behavior is of much greater significance in m^- cases than in the m^+ ones: The negative-mass population is concentrated in the nonparabolic (lower energy) and nonresonant, low rf conductance, region under near-equilibrium conditions. By contrast most of the m^+ population occupies at equilibrium the parabolic regions that are at lower energies, are resonant, and strongly absorptive at equilibrium.

Application of a magnetic field along the re-entrant axis in warped surfaces creates a particle redistribution between m^+ and m^- regions of momentum space.[1,4] This redistribution will not be treated here, but we may point out that the m^- region of unequally-spaced levels (Figs. 1 and 2) gives, under thermal equilibrium conditions, a *paramagnetic* contribution to the magnetic susceptibility, in contrast to the usual Landau diamagnetism of the m^+ regions of holes or electrons.

In conclusion, we have pointed out basic differences in the behavior of negative-mass Landau levels in comparison to that of positive-mass levels by examining qualitatively properties of the former. These include: (1) The reversal of energy levels, electric dipole moment

[12] That only a few levels with the smallest quantum numbers would have, in the actual case, sufficiently equal spacing to be of importance in cyclotron resonance in Ge, along with the inverted nature of the levels, was orally presented at the American Physical Society meeting in New York, January 1959 [Bull. Am. Phys. Soc. 4, 28 (1959)]. The level inversion was independently discovered by Dr. Kroemer (private communication). In this work (reference 2), in reference 10, and in work by Fisrov, related to this subject, the axial approximation is used [Yu. A. Firsov, Soviet Phys., Solid State 1, 48 (1959)]. See also R. C. Williams and F. Herman, *Proceedings of the International Conference on Semiconduction Physics, Prague, 1960* (Publishing House, Czechoslovakian Academy of Science, Prague, 1961), p. 599.

[13] Anisotropic scattering has been considered in detail by D. O. North of these Laboratories (private communication).

matrix elements, and transition probabilities in the m^- region. A *single* particle injected into the m^- region will emit rather than absorb rf power in cyclotron resonance. (2) The Landau levels contract to a closely spaced system with decreasing energy, again in contrast to the m^+ cases. (3) The upper energy states are discrete and fairly well defined, whereas the lower states are *broadened* and their *density* is higher. (4) The anharmonicity of the oscillator-type levels is of much greater significance in the m^- rather than the m^+ case. This is particularly so in the re-entrant case whose small extent allows only a few equally-spaced levels with the smallest quantum numbers to be of significance in rf resonant transitions. All these factors will affect the relaxation processes in these systems (shorten the relaxation of lower states) and make the establishment of emissive distribution under excitation more likely in m^-, rather than m^+ regions, as suggested earlier.[1,4,13] A decreasing population with increasing k_y (Fig. 2) i.e., overpopulation of the central cone region (region A, Fig. 2), is necessary for observation of the emissive resonance.[4] This may be considered as resulting from anisotropic scattering[13] or lifetime decrease in the states with decreasing energy (increasing k_y) suggested by the increasing level density, level broadening, and other factors discussed here.

ACKNOWLEDGMENT

The author takes pleasure in acknowledging his indebtness to several colleagues at RCA Laboratories for their interest and many helpful discussions on this subject.

Note added in proof. We make some further remarks on statements made above and their relation to recent work here and elsewhere.

The mention of the carrier scattering, rather than recombination time, is based on recent work[4,14,15] on the behavior of the emissive negative mass resonance in Ge with increasing rf signal power. These measurements indicate that the characteristic time of the emissive distribution is larger (as expected), but closer in magnitude to the scattering time ($=0.8\times10^{-10}$ sec) rather than the recombination time ($>10^{-7}$ sec). Further details on this work and its analysis[14] falls outside the limited scope of the present paper. The same holds for some computations[15] (based on well-known work,[16] and not directly involving the Landau structure) that indicate that for a group of phonon emission processes, the carrier energy loss is larger for decay (towards $\mathbf{k}=0$) along the DO (and EO), rather than the AO, directions of momentum space (Fig. 2). These differences arise from the band anisotropy. We simply point out here that the differences in density of quantized states, Landau level broadening, etc. between regions A and C, D arise from the same cause, and would lead to similar conclusions regarding the establishment of anisotropic distributions in $\mathbf{k}$ space. This, of course, is not to be taken as "proof" of connection between the lifetimes of the quantized states and the scattering time that is affected by a wide variety of solid state processes.

The limitations of classical transport calculations apply strictly to these portions of energy-momentum space where the Landau spacing is unequal, the number of levels small, and the quantum numbers small, such as is the case in the re-entrancy. The success of classical treatments in the case of other regions or effects (main m^+ region B, harmonics of heavy holes, etc.) should not be underestimated.

The exact fractional volume of momentum space, taken up by particles in the eight "bumps" of this energy surface at equilibrium, is subject to some arbitrariness as to the solid angle one assigns to them, and to some extent in slight uncertainties[3] in the band-structure constants A, B, and C. Thus the figure quoted above (45%) has an accuracy of about 15%. What is emphasized here is the *large* fraction they take up. And it is interesting to note that an appreciable effect of this warping on the temperature dependence of the mobility has been very recently computed.[17]

That the observation of effects arising from the re-entrancy is affected by impurities has been indicated, aside from the early impact ionization studies with shallow states,[3] from more recent work[18] involving deeper acceptors.

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[14] RCA Laboratories Report, March 15, 1961 (unpublished).

[15] B. W. Faughnan, G. C. Dousmanis, R. C. Duncan, Jr., and R. M. Josephs (unpublished).

[16] J. M. Ziman, *Electrons and Phonons* (Oxford University Press, New York, 1960), Chap. X.

[17] G. Tauber, in *Proceedings of the International Conference on Semiconductor Physics, Prague, 1960* (Publishing House, Czechoslovakian Academy of Science, Prague, 1961), p. 653.

[18] D. M. S. Bagguley, R. A. Straddling, and J. S. S. Whiting, Proc. Roy. Soc. (London) **A232**, 340 (1961).