Properties of Nuclei with Neutrons and Protons in the $1f_{7/2}$ Shell*

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Elliott's generating procedure is used to derive wave functions for nuclei in which the $1f_{7/2}$ level is filling with neutrons and protons. It is assumed that the low-lying states of these nuclei have well-defined isotopic spin and that the configuration is pure. This representation, which mixes states of different seniority, is used to calculate beta-decay transition probabilities, and magnetic-dipole and electric-quadrupole properties of nuclei. The results are in much better agreement with experiment than are those obtained by use of a seniority classification for the nuclear states. In particular, there is a K selection rule which explains the anomalously long half-life for the $Ca^{47} \rightarrow Sc^{47}$ and $Ca^{45} \rightarrow Sc^{45}$ beta decays. The theoretical electric-quadrupole matrix elements are too small by a factor of 3–5, and the theoretical $M1$ lifetimes too short by approximately a factor of 10. For three particles in the $1f_{7/2}$ level (Sc^{43}) or three neutrons in the $1g_{9/2}$ state, the eigenfunctions given by the generator formalism are found to be almost identical with those derived from a conventional shell-model calculation.

INTRODUCTION

THERE is a large body of experimental information about the beta decay, multipole moments, and gamma-ray transition probabilities of nuclei in which the $1f_{7/2}$ neutron and proton levels are being filled. Since these nuclei are near a closed shell, one would not expect them to have a permanent deformation. It is, therefore, of interest to see how far one can go in interpreting the experimental data by assuming a shell-model state with good isotopic spin made up from nucleons in the $f_{7/2}$ level only.

To carry out a conventional shell-model calculation, it is convenient to have the coefficients of fractional parentage. These can be obtained by a chain procedure,[1] but the amount of labor involved is prohibitive except for states of seniority zero or one. In these latter two cases, for which explicit expressions can be obtained for the fractional parentage coefficients,[2] the properties of such states have already been calculated. The theoretical value for the square of the beta-decay matrix element is generally 10 to 100 times the experimental value[3] and in one case ($Ca^{47} \rightarrow Sc^{47}$) is too large by a factor of 10^4. The experimental values of the magnetic moments[4] are often as much as a nuclear magneton away from the theoretical prediction.

In the calculations published to date it has been assumed that the states involved have good seniority—an assumption which is not warranted when both neutrons and protons are filling the $f_{7/2}$ shell. It is possible (and indeed it is found) that small admixtures of states with different seniority may appreciably affect the calculated values in the same way as small admixtures of other j configurations appreciably affect magnetic moments, quadrupole moments, and transition probabilities. Thus we have looked for a different prescription to specify the zero-order shell-model wave functions for nuclei in this region. In the $1p$ shell, Kurath and Pičman[5] have found that the generating procedure of Elliott[6] gives wave functions in remarkably good agreement with those obtained from an interaction calculation. Similarly, at the beginning of the $(1d-2s)$ shell Redlich[7] has found that the method of generator coordinates gives results close to the interaction calculation. This procedure takes into account some of the residual two-body interaction by allowing the well into which particles are put to have a Y_2 deformation. In view of the success of this method for lighter nuclei, it has been used to generate eigenfunctions for the low-lying states of the $f_{7/2}$ nuclei. The results obtained with this representation are in much better accord with experiment than are those found from a seniority classification. This method has the added advantage that calculation of the properties of nuclear levels with spins other than 0^+ or $\frac{7}{2}^-$ is now made fairly simple.

THEORY

In a central potential with Y_2 deformation, a level of spin j is split into $(2j+1)/2$ components—each one being doubly degenerate and characterized by the quantum number k, the projection of the angular momentum on the nuclear symmetry axis.[8] If one considers a single level (i.e., neglects off-diagonal matrix elements between level j and level j') the energy shift is linear in the deformation parameter η. The Nilsson diagram for the $f_{7/2}$ level is shown in Fig. 1. By successively filling the lowest possible levels of Fig. 1, one can construct a normalized N-particle determinant (or linear combination of N-particle determinants) with well-defined isotopic spin. We denote these by $\chi_K(Rx)$, where the coordinate system (Rx) is oriented with respect to the laboratory system x. It is then assumed that from this

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χ_K the low-lying states of the N-particle nucleus can be generated from the Hill-Wheeler integral,

$$\psi_{M(K)}{}^I(x)=(2I+1)C_K{}^I\int dRD_{MK}{}^I(R)\chi_K(Rx), \quad (1)$$

where $C_K{}^I$ is chosen so that $\psi_{M(K)}{}^I(x)$ is normalized and $D_{MK}{}^I(R)$ is the rotation matrix.

The determinant χ_K can be further decomposed and written as

$$\chi_K(Rx)=[n_1!n_2!/N!]^{\frac{1}{2}}\mathfrak{A}U_{n_1k_1}(Rx)V_{n_2k_2}(Rx), \quad (2)$$

where n_1 and n_2 are, respectively, the number of protons and neutrons making up the state $(n_1+n_2=N)$, $\mathfrak{A}$ is the antisymmetrization operator, $U_{n_1k_1}(Rx)$ is the normalized n_1-particle proton determinant, and $V_{n_2k_2}(Rx)$ is the neutron determinant $(k_1+k_2=K)$. The determinants U and V are not, of course, angular-momentum eigenfunctions. One can, however, make an angular-momentum decomposition of these determinants. For example,

$$U_{n_1k_1}(Rx)=\sum_{Js} d_{n_1k_1}{}^{Js}u_{k_1}{}^{Js}(Rx)$$
$$=\sum_{Jm_1s} d_{n_1k_1}{}^{Js}D_{m_1k_1}{}^{J}{}^*(R)u_{m_1}{}^{Js}(x), \quad (3)$$

where s stands for any additional label necessary to specify the state J. Using Eq. (3) and its analog for V, one easily shows that

$$\psi_{M(K)}{}^I(x)=[n_1!n_2!/N!]^{\frac{1}{2}}C_K{}^I$$
$$\times\sum_{JLss'} d_{n_1k_1}{}^{Js}d_{n_2k_2}{}^{Ls'}(JLk_1k_2|IK)$$
$$\times\sum_{m_1m_2}(JLm_1m_2|IM)\mathfrak{A}u_{m_1}{}^{Js}(x)v_{m_2}{}^{Ls'}(x), \quad (4)$$

where the bracket is a Clebsch-Gordan coefficient and J refers to the angular momentum of the protons and L to the neutrons. From Eq. (4) it follows that

$$\int\psi_{M(K')}{}^I(x)\psi_{M(K)}{}^I(x)dx$$
$$=C_{K'}{}^IC_K{}^IA(I;n_1k_1,n_1k_1';n_2k_2,n_2k_2'), \quad (5)$$

where

$$A(I;n_1k_1,n_1k_1';n_2k_2,n_2k_2')$$
$$=\sum_{JLss'} d_{n_1k_1}{}^{Js}d_{n_1k_1'}{}^{Js}d_{n_2k_2}{}^{Ls'}d_{n_2k_2'}{}^{Ls'}$$
$$\times(JLk_1k_2|IK)(JLk_1'k_2'|IK'). \quad (6)$$

Thus the normalization coefficient $C_K{}^I$ is determined by the angular momentum decomposition of determinants with neutrons only and protons only. For a single level, this decomposition is easy to find.

For an even-even or odd-A nucleus, the eigenfunctions are assumed to be generated from a single χ_K so that only $k_1=k_1'$, $k_2=k_2'$ comes into the normalization coefficient. However, for an odd-odd nucleus it is possible that a given I state may be generated from two χ's that correspond to the same Nilsson energy. For example, if one assumes positive deformation in Sc^{44}, there are two χ's which could give rise to an $I=2$ ground state. These are $\chi_1=|\pi_{-\frac{1}{2}}\nu_{\frac{1}{2}}\nu_{-\frac{1}{2}}\nu_{\frac{3}{2}}|$ and $\chi_2=|\pi_{\frac{1}{2}}\nu_{\frac{1}{2}}\nu_{-\frac{1}{2}}\nu_{\frac{3}{2}}|$, where π stands for proton, ν for neutron, and the subscripts $\frac{1}{2}$ etc., are values of k for the state in question. To find the appropriate linear combination of χ_1 and χ_2, we follow the suggestion of Kurath and Pičman,[5] namely, to generate from that eigenfunction which diagonalizes the rotational Hamiltonian

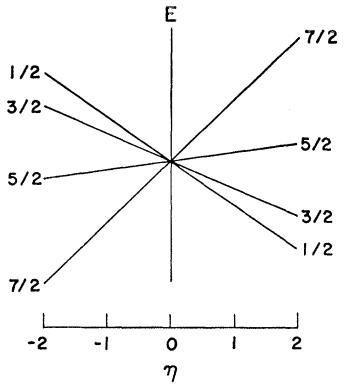

FIG. 1. Energy level diagram for the $1f_{7/2}$ shell as a function of the deformation η.

$$H_{\rm rot}=B\{I^2+J^2-2I_Z{}^2-I_+J_--I_-J_+\}, \quad (7)$$

where J_+, J_- are components of the angular momentum operator of the particles referred to the body-fixed coordinate system and B is a constant related to the moment of inertia of the system. Since B is an over-all multiplicative constant, its value does not affect the diagonalization when the χ_K have the same Nilsson energy. The eigenfunction which diagonalizes Eq. (7) may be written as

$$\phi_M{}^I=\left[\frac{2I+1}{2}\right]^{\frac{1}{2}}\sum_K\alpha_K(1+R_1)D_{MK}{}^I\chi_K, \quad (8)$$

where R_1 is the operator that produces a rotation of 180° about an axis perpendicular to the symmetry (z) axis of the rotor and the α_K are constants. Thus for an odd-odd nucleus Eq. (1) is replaced by

$$\psi_M{}^I(x)=(2I+1)C^I\int dR\sum_K\alpha_KD_{MK}{}^I(R)\chi_K(Rx), \quad (9)$$

with

$$(C^I)^2\sum_{KK'}\alpha_K\alpha_{K'}A(I;n_1k_1,n_1k_1';n_2k_2,n_2k_2')=1. \quad (10)$$

Now that the eigenfunctions have been obtained, it is a simple task to calculate the nuclear properties of the system. As an example, let us consider the beta-decay matrix elements since an interesting selection rule arises

in this case. The calculation of the Fermi part of the interaction is trivial so we restrict ourselves to the Gamow-Teller interaction. In carrying out the calculation, it is convenient to transform $\tau_+\sigma_\mu$ to the (Rx) coordinate system and carry out the operations involved before generating

$$\sum_{i=1}^{N}\tau_+(i)\sigma_\mu(i)=\sum_{i=1}^{N}\tau_+(i)\sum_\lambda D_{\mu\lambda}{}^1(R)\sigma_\lambda(i), \qquad (11)$$

where τ_+ changes a proton to a neutron. Thus

$$\begin{aligned}&\sum_i \tau_+(i)\sigma_\mu(i)\psi_{M_i}{}^{I_i}(x)\\&=(2I_i+1)C^{I_i}\int dR\sum_{K\lambda}\alpha_K D_{\mu\lambda}{}^1(R)D_{M_iK}{}^{I_i}(R)\\&\qquad\times\sum_i\tau_+(i)\sigma_\lambda(i)\chi_K(Rx)\\&=(2I_i+1)C^{I_i}\sum_{IK\lambda}\alpha_K(1I_i\mu M_i|I,\mu+M_i)\\&\quad\times(1I_i\lambda K|I,\lambda+K)\int dR D_{\mu+M_i,\lambda+K}{}^I(R)\\&\qquad\times\sum_i\tau_+(i)\sigma_\lambda(i)\chi_K(Rx). \qquad (12)\end{aligned}$$

The inner product of Eq. (12) with $\psi_{M_f}{}^{I_f}(x)$ picks out the state $I=I_f$. Further

$$\tau_+\sigma_\lambda\pi_m\doteq(9/7)^{\frac{1}{2}}(\tfrac{7}{2}1m\lambda|\tfrac{7}{2},m+\lambda)\nu_{m+\lambda}, \qquad (13)$$

where $\doteq$ means we have taken only the part of $\tau_+\sigma_\lambda\pi_m$ that leaves the nucleon in the $f_{7/2}$ level. Equations (12) and (13) are sufficient to determine the beta-decay matrix elements. Since the operator $\tau_+\sigma_\lambda$ can only change the quantum number k by 0 or ±1, the matrix element vanishes unless the resulting state is unoccupied. This K selection rule explains the anomalously long half-life for the $Ca^{47}\rightarrow Sc^{47}$ beta decay ($\log ft=8.5$) and $Ca^{45}\rightarrow Sc^{45}$ ($\log ft=6$).

To examine the magnetic-dipole and electric-quadrupole properties of the nucleus, it is more convenient to start with the generated wave function, Eq. (4). Since we are dealing with a single-particle operator, we can neglect the antisymmetrization of the wave function. The formulas governing these phenomena are easily calculated by standard Racah algebra and will not be derived here.

RESULTS

Table I lists the results obtained with this representation for the $1f_{7/2}$-shell nuclei. The calculations have been carried out for both signs of the deformation. It has been assumed that *all* low-lying states of a given even-even or odd-A nucleus can be generated from a single χ_K. For the odd-odd nuclei, states are generated from the appropriate linear combination of the χ_K that diagonalize the rotational Hamiltonian. It is known that the $\frac{5}{2}^-$ and $\frac{7}{2}^-$ levels of odd-A nuclei in this region lie close together (one of them being the ground state). Hence, if this representation has any validity, one would expect to generate from positive deformation near the beginning of the shell and from negative deformation near the end. For example, in Sc^{45} the lowest Nilsson orbital, for negative deformation, is $|\pi_{7/2}\nu_{7/2}\nu_{-7/2}\nu_{5/2}\nu_{-5/2}|$ from which it is impossible to generate a $\frac{5}{2}$ state. For $A<50$, the numerical results for the magnetic moments come closer to experiment when the wave function is obtained from positive deformation; for $A>50$, better agreement is obtained for negative deformation. For beta decay, if one takes into account the magnitude and sign of the correction to the matrix element brought about by admixing small amounts of the $1f_{\frac{5}{2}}$ configuration[9] it is again true that positive deformation gives better agreement with experiment near the beginning of the shell.

A. Magnetic Moments

The magnetic moments calculated with this representation come closer to the experimental values than do those predicted by the seniority eigenfunctions. There is a tendency for the measured magnetic moments of the odd-odd nuclei to lie very close to the value one would obtain by simply coupling the two odd particles to the appropriate spin, i.e., very close to

$$\mu=(I/7)(3+\mu_p+\mu_n). \qquad (14)$$

The many-nucleon calculation leads to only a small correction to Eq. (14) for the $I=6$ states of V^{50} and Mn^{52}. However, for the $I=2$ state of Mn^{52} the correction is large and in disagreement with experiment.

The calculated magnetic moment of Sc^{45} is in good agreement with experiment. If the spin of Ti^{45} is $\frac{7}{2}$, the magnetic moment (which has not been measured) is predicted to be positive for positive deformation and only slightly negative for negative deformation. (If the ground state of Ti^{45} is $\frac{5}{2}$, then $\mu=+0.49$ nm for positive deformation and -0.78 for negative deformation.) It seems to be a characteristic that when the odd nucleon fills a $k=\frac{3}{2}$ orbit the predicted magnetic moment is considerably different from the seniority value. Another example is provided by V^{47} for which seniority predicts 4.65 nm for the magnetic moment of the $\frac{7}{2}$ state while, for positive deformation, this representation gives a value 3.17 nm.

In general, the admixing of states of higher seniority tends to correct the magnetic moment in the desired direction. Since j-configuration mixing gives a contribution in the same direction (although in many cases the correction is too small[4]) it may be argued that the generating procedure mixes in somewhat too much of these states of higher seniority. It actually takes only a small admixture to change the magnetic moment by an appreciable amount. For example, with positive de-

[9] R. J. Blin-Stoyle and C. A. Caine, Phys. Rev. **105**, 1810 (1957).

TABLE I. Various properties of nuclei in the $1f_{7/2}$ shell, calculated by use of seniority eigenfunctions and wave functions generated from either positive or negative deformation. Where the spins of the states involved are known, they are italicized. F means the transition is forbidden and ··· means that one of the states involved cannot be generated from the lowest Nilsson orbital. Unless otherwise stated the experimental data are taken from the Nuclear Data Sheets[a] and Way *et al.*[b]

A	Quantity calculated	Seniority	Deformation +	Deformation −	Experiment
43	Magnetic moment of *7/2* ground state of Sc^{43} (nm)	4.08	4.78	4.94	
	Quadrupole moment of *7/2* ground state of Sc^{43} (barns)	−0.070	−0.073	−0.069	
	Quadrupole moment of *7/2* ground state of Ti^{43} (barns)	−0.051	−0.089	−0.120	
	log*ft* for β decay $Sc^{43} \rightarrow Ca^{43}$ (*7/2* → *7/2*)	3.88	4.21	5.04	4.9
	log*ft* for β decay $Sc^{43} \rightarrow Ca^{43}$ (*7/2* → *5/2*)	*F*	4.82	4.25	5.0
	log*ft* for β decay $Ti^{43} \rightarrow Sc^{43}$ (*7/2* → *7/2*)	3.59	3.49	3.46	3.4[c]
	*M*1 lifetime of 138-kev transition in Sc^{43} (5/2 → *7/2*) (nsec)	0.011	0.015	···	
44	log*ft* for electron capture $Ti^{44} \rightarrow Sc^{44}$ (*0* → 1)		3.81	3.88	4.5
	log*ft* for β decay $Sc^{44} \rightarrow Ca^{44}$ (*2* → *2*)		4.28	4.30	5.3
45	Magnetic moment of *7/2* ground state of Sc^{45} (nm)	5.11	4.61	4.74	4.756
	Magnetic moment of 7/2 ground state of Ti^{45} (nm)	−1.34	+0.74	−0.01	
	Quadrupole moment of *7/2* ground state of Sc^{45} (barns)	−0.090	−0.068	−0.066	−0.22
	log*ft* for β decay $Ca^{45} \rightarrow Sc^{45}$ (*7/2* → *7/2*)	3.98	*F*	*F*	6.0
	log*ft* for β decay $Ti^{45} \rightarrow Sc^{45}$ (*7/2* → *7/2*)	4.61	3.96	4.66	4.6
	log*ft* for β decay $Ti^{45} \rightarrow Sc^{45}$ (7/2 → 5/2)		4.82	···	5.4[d]
46	*B*(*E*2) for Coulomb excitation of 887-kev state in Ti^{46} (*0* → *2*)		0.003	0.006	0.083[e]
	log*ft* for β decay $Sc^{46} \rightarrow Ti^{46}$ (*4* → *4*)		*F*	*F*	6.2
	log*ft* for β decay $V^{46} \rightarrow Ti^{46}$ (0 → *0*)		3.48	3.48	3.5
47	Magnetic moment of *5/2* ground state of Ti^{47} (nm)		−0.71	+0.17	−0.787
	B(*E*2) for Coulomb excitation of 160-kev state in Ti^{47} (*5/2* → *7/2*)		0.0013	0.0016	0.040
	log*ft* for β decay $Ca^{47} \rightarrow Sc^{47}$ (*7/2* → *7/2*)	4.25	*F*	*F*	8.5
	log*ft* for β decay $Sc^{47} \rightarrow Ti^{47}$ (*7/2* → *5/2*)		*F*	*F*	6.1
	log*ft* for β decay $Sc^{47} \rightarrow Ti^{47}$ (*7/2* → *7/2*)	4.39	*F*	*F*	5.3
	log*ft* for β decay ground state $V^{47} \rightarrow Ti^{47}$ (5/2 → *5/2*)		4.38	4.57	4.9
	*M*1 lifetime of 160-kev transition in Ti^{47} (*7/2* → *5/2*) (nsec)		0.036	0.063	0.326±0.1[f]
48	*B*(*E*2) for Coulomb excitation of 990-kev state in Ti^{48} (*0* → *2*)		0.003	0.007	0.070[e]
	log*ft* for β decay $Cr^{48} \rightarrow V^{48}$ (*0* → 1)		3.66	3.66	⩾4.1
	log*ft* for β decay from ground state $V^{48} \rightarrow Ti^{48}$ (4 → *4*)		*F*	*F*	6.1
49	Magnetic moment of *7/2* ground state of Ti^{49}	−1.67	−1.18	−1.18	−1.1022
	Magnetic moment of *7/2* ground state of V^{49}	5.45	4.04	3.47	4.46
	log*ft* for β decay $Sc^{49} \rightarrow Ti^{49}$ (*7/2* → *7/2*)	4.25	*F*	*F*	5.8
	log*ft* for β decay $V^{49} \rightarrow Ti^{49}$ (*7/2* → *7/2*)	4.39	*F*	*F*	6.1
	log*ft* for β decay from ground state $Cr^{49} \rightarrow V^{49}$ (5/2 → *7/2*)		5.43	4.12	5.0
	log*ft* for β decay from ground state $Cr^{49} \rightarrow V^{49}$ (5/2 → 5/2)		4.57	4.38	5.4
	log*ft* for β decay from ground state $Cr^{49} \rightarrow$ 152-kev state V^{49} (5/2 → 3/2)		4.08	···	4.9
	*M*1 lifetime of 152 kev → 89 kev transition in V^{49} (3/2 → 5/2) (nsec)		4.36	···	27.4±8[g]
	*M*1 lifetime of 89-kev transition in V^{49} (5/2 → *7/2*) (nsec)		0.28	0.16	0.8±0.5[g]
50	Magnetic moment of *I*=*6* ground state of V^{50} (nm)		3.18	3.26	3.341
	log*ft* for β decay $Mn^{50} \rightarrow Cr^{50}$ (0 → *0*)		3.48	3.48	3.4
51	log*ft* for β decay from ground state $Cr^{51} \rightarrow V^{51}$ (*7/2* → *7/2*)	3.98	*F*	*F*	5.4
	log*ft* for β decay from ground state $Cr^{51} \rightarrow V^{51}$ (*7/2* → *5/2*)		*F*	*F*	5.8
	log*ft* for β decay from ground state $Mn^{51} \rightarrow Cr^{51}$ (7/2 → 7/2)	4.61	4.66	3.96	5.2
52	Magnetic moment of *I*=*6* ground state of Mn^{52} (nm)		2.63	3.07	±3.08
	Magnetic moment 390-kev state of Mn^{52} (*I*=2) (nm)		−0.36	−0.35	1.04[h]
	log*ft* for β decay $Fe^{52} \rightarrow Mn^{52}$ (*0* → 1)		3.88	3.81	4.7
	log*ft* for β decay from ground state $Mn^{52} \rightarrow Cr^{52}$ (*6* → *6*)		*F*	5.01	5.6
	log*ft* for β decay from 390-kev state $Mn^{52} \rightarrow Cr^{52}$ (*2* → *2*)		4.30	4.28	5.3
	*M*1 lifetime 555 kev → 390 kev transition in Mn^{52} (1 → 2) (nsec)		1.8×10^{-3}	1.6×10^{-3}	<0.05[h]
53	log*ft* for β decay $Fe^{53} \rightarrow Mn^{53}$ (7/2 → *7/2*)	3.88	5.04	4.21	5.1
	log*ft* for β decay $Fe^{53} \rightarrow Mn^{53}$ (7/2 → *5/2*)		4.25	4.82	5.3

[a] *Nuclear Data Sheets*, National Academy of Sciences, National Research Council (U. S. Government Printing Office, Washington, D. C., 1959).
[b] *Nuclear Level Schemes, A=40—A=92*, compiled by K. Way, R. W. King, C. L. McGinnis, and R. van Lieshout, Atomic Energy Commission Report TID-5300 (U. S. Government Printing Office, Washington, D. C., 1955).
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[g] Preliminary results of R. E. Holland and F. J. Lynch. I should like to thank Dr. Holland for communicating these results prior to publication.
[h] R. W. Bauer, M. Deutsch, G. S. Mutchler, and D. G. Simons, Phys. Rev. **120**, 946 (1960).

formation, the generated $\frac{7}{2}$ state of Sc^{43} can be written as

$$\psi^{7/2}(\text{gen}) = (1001/1089)^{1/2}\psi^{7/2}(v=1) - (78/1089)^{1/2}\psi^{7/2}(v=3) - (10/1089)^{1/2}\psi^{7/2}(v=3), \quad (15)$$

where v is the seniority of the state and the second seniority-three state is the one which has a zero fractional-parentage coefficient for the parent with $J=1$, $T=0$. This small admixture changes the magnetic moment from the seniority value of 4.08 nm to 4.78 nm.

B. Beta Decay

To calculate the ft value for beta decay, we have used

$$ft = \frac{6100}{\langle 1\rangle^2 + 1.4\langle \boldsymbol{\sigma}\rangle^2}, \quad (16)$$

where $\langle 1\rangle$ is the Fermi matrix element, $\langle \boldsymbol{\sigma}\rangle^2$ is the square (appropriately averaged) of the Gamow-Teller matrix element, the constant 6100 is taken from the O^{14} decay,[10] and 1.4 is approximately the square of the ratio of the axial-vector to the vector coupling constants.[11]

One can look more closely at the K selection rule discussed in the last section and talk of orders of K forbiddenness. For example, the $Ca^{47} \rightarrow Sc^{47}$ decay has two orders of K forbiddenness, since one has to move a single proton or single neutron up through two Nilsson levels to get a nonvanishing matrix element. One would, therefore, expect that this should be a well obeyed selection rule—and indeed it is. On the other hand, all but one ($Sc^{49} \rightarrow Ti^{49}$) of the other forbidden transitions given in Table I have only one order of K forbiddenness and their $\log ft$ values are correspondingly smaller. The worst violation of this selection rule is the $Sc^{47} \rightarrow Ti^{47}$ ($\frac{7}{2} \rightarrow \frac{7}{2}$) transition which has $\log ft = 5.3$. Even this small value should not be difficult to obtain since it only requires that $\langle \sigma\rangle^2 = 0.022$. (This is to be compared with the $Sc^{43} \rightarrow Ca^{43}$ calculation in which the theoretical value of $\langle \sigma\rangle^2$ is 4/7.)

Aside from the above cases, the beta-decay matrix elements are generally larger than needed to fit experiment. An important effect has been neglected in this calculation, namely, the effect of mixing in small amounts of the $1f_{\frac{5}{2}}$ configuration. For the Ti^{43}, Sc^{43}, Ca^{43} triad this has been included by Blin-Stoyle and Caine,[9] who find that such admixtures have a large effect on the transition probabilities and tend to make the $\log ft$ values larger than given by the seniority calculation. By using a combination of seniority mixing and j-configuration mixing, it should now be possible to fit the $\log ft$ values for the $A=43$ triad with a more reasonable choice of the separation of the $1f_{7/2}$ and $1f_{5/2}$ single-particle levels. For heavier nuclei in this region one would also expect that configuration mixing would tend to increase the $\log ft$ value. (In the case of the K-forbidden transitions such j-configuration mixing would, of course, decrease the $\log ft$ value.) Since our calculation neglects this effect, the calculated $\log ft$ values should be expected to be smaller than the experimental ones. For this reason one would say that generating from positive deformation gives better agreement for beta decay near the beginning of the shell.

There are several cases in which the spins of the initial or final nucleus in the decay are unknown. In these cases the values assumed for the unknown spins are those that seem to give the more consistent results. For example, in the electron capture $Ti^{44} \rightarrow Sc^{44}$, the spin of the final state is either 0 or 1. From the lowest Nilsson orbital there is no way of getting a spin-0 state; so we have chosen $I_f = 1$. For Ti^{45} we have taken the spin of the ground state to be $\frac{7}{2}$ so that $\log ft = 3.96$ for decay to the $\frac{7}{2}$ state of Sc^{45}. This disagreement with experiment is about the same as for the Sc^{43} decay. On the other hand, a spin of $\frac{5}{2}$ for Ti^{45} would lead to $\log ft = 4.54$.

C. $M1$ Lifetimes

There is little experimental information on $M1$ lifetimes in this region. In Ti^{47} the calculated lifetime is too short by a factor of ten for positive deformation and a factor of five when negative deformation is used. The same situation seems to persist in V^{49}.

Although there is no experimental value for the lifetime of the 138-kev state[12] in Sc^{43}, it is instructive to compare the lifetime obtained from the generated wave functions for both the ground state and the 138-kev level (assumed spin of $\frac{5}{2}$) with that calculated from the seniority eigenfunction for the $\frac{7}{2}$ level and the generated wave function for the $\frac{5}{2}$ state.[13] In this case the "seniority calculation" gives an even shorter lifetime for the state than does the generator calculation: $\tau_{\text{sen}} = 0.11 \times 10^{-10}$ sec, compared to $\tau_{\text{gen}} = 0.15 \times 10^{-10}$ sec.

D. Quadrupole Properties

The quadrupole matrix element has been calculated on the assumption that $\langle r^2\rangle = (1.1A^{\frac{1}{3}} \times 10^{-13}\ \text{cm})^2$. The theoretical result for the quadrupole moment of Sc^{45} is only a third of the experimental value. The experimental values of $B(E2)$ in the titanium isotopes are consistently 20–30 times the theoretical ones. Such a result need not necessarily imply that these nuclei are strongly deformed. Arima and Horie,[14] Arima and Yoshida,[15] and Kurath[16] have shown that weak configuration mixing can appreciably influence the quadrupole matrix ele-

[10] D. L. Hendrie and J. B. Gerhart, Phys. Rev. **121**, 846 (1961).
[11] See, for example, M. Gell-Mann, Revs. Modern Phys. **31**, 834 (1959).
[12] G. J. McCallum, A. T. G. Ferguson, and G. S. Mani, Nuclear Phys. **17**, 116 (1960).
[13] In Sc^{43} there are two possible $\frac{5}{2}^-$ levels [B. H. Flowers, Proc. Roy. Soc. **A212**, 248 (1952)]. Thus the seniority classification is not sufficient to determine the eigenfunction.
[14] H. Horie and A. Arima, Phys. Rev. **99**, 778 (1955).
[15] A. Arima and S. Yoshida, Nuclear Phys. **12**, 139 (1959).
[16] D. Kurath, Nuclear Phys. **14**, 398 (1960).

ments so that an enhancement factor of 3–5 should not be impossible to obtain.

DISCUSSION

That the generating procedure leads to wave functions in excellent agreement with the interaction calculation is not too surprising in the $1p$ shell. In that case only the $L=0$ and $L=2$ parts of the Slater decomposition of the residual interaction come in and the method of generator coordinates replaces the $L=2$ part by an average field. However, in the $1f_{7/2}$ shell one "explores" the Slater decomposition up to $L=6$ and there is no guarantee that an average $L=2$ field can come close to reproducing the interaction calculation. To gain some insight into the validity of the generator formalism, one can compare the wave function given by this procedure to that found by Kurath[17] who did a shell model calculation for Sc^{43} on the assumption that the residual two-body interaction could be approximated by a delta function. The results generated from positive deformation are almost identical to his. The shell-model calculation leads to a magnetic moment of 4.82 nm for the ground state of Sc^{43}, whereas the generating procedure yields 4.78 nm. For the beta decay of Ti^{43} and Sc^{43}, Kurath found 0.72 and 0.24, respectively, for $\langle\sigma\rangle^2$ while our calculation gives 0.70 and 0.27.

A second simple instance in which seniority mixing may be important is in the $g_{9/2}$ level since two 9/2 states are possible when three identical nucleons are in this level.[18] From the experimental data on Zr^{90}, Talmi and Unna[19] have deduced the magnitudes of the $(g_{9/2})^2$ pairings. From these values one can calculate the amount of seniority mixing in the $\frac{9}{2}$ level of the three-particle system. The admixture turns out to be small. If one generates the $I=9/2$ state associated with this configuration for positive deformation, one finds

$$\psi^{9/2}(\text{gen})=(7293/7310)^{1/2}\psi^{9/2}(v=1)-(17/7310)^{1/2}\psi^{9/2}(v=3), \quad (17)$$

which has an overlap of 0.997 with the Talmi-Unna wave function.

There is one place in the $f_{7/2}$ shell where the generating procedure gives results in disagreement with the interaction calculation. For four identical nucleons (Ca^{44} or Cr^{52}) there are two possible $I=2$ states. It has been shown[20] for this case that seniority is a good quantum number even when a two-body interaction is introduced. If we generate the Ca^{44} wave function from either positive or negative deformation, the $I=2$ state so obtained is a mixture of the seniority-two and seniority-four eigenfunctions

$$5(4!)^{-\frac{1}{2}}\int dR D_{M0}{}^2(R)\,|\nu_{1/2}\nu_{-1/2}\nu_{3/2}\nu_{-3/2}| = -\left[\frac{88}{231}\right]^{\frac{1}{2}}\phi_M{}^2(v=2)-\left[\frac{18}{231}\right]^{\frac{1}{2}}\phi_M{}^2(v=4), \quad (18)$$

$$5(4!)^{-\frac{1}{2}}\int dR D_{M0}{}^2(R)\,|\nu_{7/2}\nu_{-7/2}\nu_{5/2}\nu_{-5/2}| = \left[\frac{88}{231}\right]^{\frac{1}{2}}\phi_M{}^2(v=2)-\left[\frac{18}{231}\right]^{\frac{1}{2}}\phi_M{}^2(v=4).$$

The beta decay matrix elements given in Table I were calculated by assuming the seniority two eigenfunction for Ca^{44} and Cr^{52}.

This calculation has been restricted to the $f_{7/2}$ level alone. That such an assumption is not fully justified is clearly evidenced by the position of the first excited $I=2$ state in Ca^{42} (1.53 Mev) and Ca^{44} (1.16 Mev). If the shell were pure, these two levels should have the same excitation energy. One would, therefore, expect that although the predominant contribution to the wave function comes from the $(f_{7/2})^n$ configuration, mixtures of the $1f_{\frac{5}{2}}$, $2p_{\frac{3}{2}}$, and $2p_{\frac{1}{2}}$ levels would be important. This suggests generating from the intrinsic wave functions of the Nilsson calculation,[8] in which such admixtures are present. The effect of admixing these excited states is to favor prolate deformation near the beginning of the shell and oblate deformation near the end. This is in agreement with what we have empirically found, whereas Fig. 1 by itself would suggest the opposite.

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