Nuclear Resonant Absorption of Gamma Rays by Ca^{40}†

A. C. ECKERT* AND E. F. SHRADER
Case Institute of Technology, Cleveland, Ohio
(Received July 3, 1961)

Ground-state gamma rays ($E_\gamma=10.3$ Mev) emitted by excited Ca^{40} formed in the reaction $K^{39}(p,\gamma)Ca^{40}$ at $E=2.05$ Mev have been resonantly absorbed in calcium. Nuclear resonant absorption occurs when the absorber is placed at such an angle that the energy discrepancy upon emission and absorption of the gamma rays is restored by the Doppler shift resulting from the recoil velocity of the excited Ca^{40} nucleus. The gamma radiation is detected by a scintillation detector mounted behind an angle-defining collimator. The nuclear resonant absorption curve is obtained by observing the transmission through the absorber as the collimator is rotated through the resonant angle. The observed width, which is of the order of 0.8 degree, is a function of the angular divergence of the proton beam and the angular opening of the collimator. From the integral of the absorption as a function of angle and from a yield measurement, the radiation width for a transition to the ground state is determined to be 3.6 ± 0.24 ev, while 5.8 ± 1.8 ev is found for the proton width, and 10.3 ± 1.7 ev for the total width. These values together with the measured angular distribution of the ground-state radiation are consistent with a 2^+ assignment to this level.

I. INTRODUCTION

THE method of nuclear resonant absorption using (p,γ) reactions as a source of gamma radiation, previously employed by Smith and Endt[1] and by Hanna and Schützmeister,[2] has been used to determine the total width and the partial widths of a previously unreported level at 10.3 Mev in Ca^{40}. Although the method has been in use several years, the parameters of only three different levels occurring in three different nuclides have been determined by this method. Seven different levels occurring in three different nuclei, i.e., Al^{27}, S^{32}, and Ca^{40} all reported in the literature to be formed by proton capture and to undergo ground-state gamma-ray transitions, have been investigated in the course of this present work. For none of these levels has nuclear resonant absorption been definitely observed.

II. THEORY

Following the arguments of Smith and Endt, the nuclear absorption in the absorber is given by

$$\mathfrak{A}(E',E_c)=\int_0^\infty \{1-\exp[n(E''-E')]\}\times f(E''-E_c)dE'', \quad \text{(II1)}$$

where

$$E_c=E_0-[E_0^2/2Mc^2]+[E_0(2Mc^2E_p)^{\frac{1}{2}}\cos\theta_c/Mc^2], \quad \text{(II2)}$$

and θ_c is the central angle selected by a collimator between the absorber and a detector. The function $f(E''-E_c)$ is a weighting function which takes into account the angular opening of the collimator and any angular divergence of the proton beam. $f(E''-E_c)$ is normalized such that

$$\int_0^\infty f(E''-E_c)dE''=1. \quad \text{(II3)}$$

The evaluation of $\mathfrak{A}(E',E_c)$ is not possible because an analytic expression for the response function $f(E''-E_c)$ is not known. However, if the absorption integral

$$A=\int \mathfrak{A}(E'',E_c)dE_c, \quad \text{(II4)}$$

is formed, it can be shown that A is independent of the functional form of $f(E''-E_c)$ if the assumption can be made that $f(E''-E_c)$ is zero except in the immediate vicinity of E'' equal to E_c. That this assumption is valid is demonstrated by the fact that an energy resolution of 300 ev is readily obtainable for a collimator opening of one degree or less while the total energy variation due to the Doppler shift from 0° to 90° may be as large as 20 kev. Thus, the area under the absorption curve, expressed in energy units, is given by

$$A=\int_0^\infty \{1-\exp[n\sigma(E''-E')]\}dE''\times\int_0^\infty f(E''-E_c)dE_c. \quad \text{(II5)}$$

Hence,

$$A=\int_0^\infty \{1-\exp[n\sigma(E''-E')]\}dE''. \quad \text{(II6)}$$

Therefore, the area under the absorption curve is independent of the instrumental resolution. However, the shape of the absorption curve $\mathfrak{A}(E',E_c)$ does depend on the resolution in the following manner. For good resolution, hence low counting rate and poor counting statistics, the absorption curve would be narrow but deep; for poor resolution, therefore higher counting rates and better counting statistics, the curve would be

† Work supported in part by the United States Atomic Energy Commission.
* Present address: Thompson Ramo Wooldridge Inc., Cleveland, Ohio.
[1] P. B. Smith and P. M. Endt, Phys. Rev. **110**, 397 (1958); **10**, 1442 (1958).
[2] S. S. Hanna and L. Meyer-Shützmeister, Phys. Rev. **115**, 986 (1959).

broad and shallow. For either case the same area is obtained.

If the level is narrow, less than 100 ev, then the Breit-Wigner form of the absorption cross section must be modified to include thermal broadening of the level. The thermal width Δ is given by

$$\Delta = E_0(2kT_e/Mc^2)^{\frac{1}{2}}, \tag{II7}$$

where k is Boltzmann's constant and T_e is an effective temperature determined by the Debye temperature of the absorber. Curves for obtaining T_e are given by Lamb.[3] The arguments of Bethe[4] show that if thermal broadening is included the usual resonant cross section must be replaced by

$$\sigma(t,x) = \sigma_0\psi(t,x), \tag{II8}$$

where

$$\sigma(t,x) = \frac{t}{2\pi^{\frac{1}{2}}}\int_{-\infty}^{\infty}\frac{\exp[-\frac{1}{4}t^2(x-y)^2]dy}{1+y^2}, \tag{II9}$$

$$x = 2\Gamma^{-1}(E-E), \quad t = \Gamma/\Delta.$$

Thus, the absorption area takes on the form

$$\frac{A(t,n\sigma_0)}{\Delta} = \frac{t}{2}\int_{-\infty}^{\infty}\{1-\exp[n\sigma_0\psi(t,x)]\}dx, \tag{II10}$$

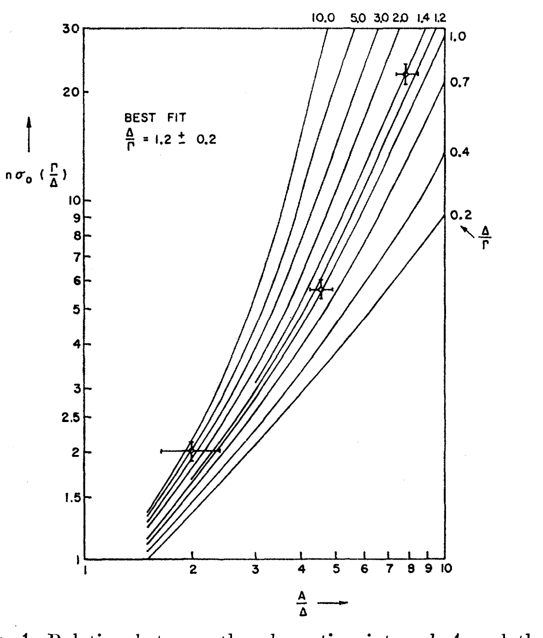

FIG. 1. Relation between the absorption integral A and the level parameters for various values of the ratio Δ/Γ where Δ is the thermal broadening. The points shown are relative experimental values of absorption integral for three different absorber thicknesses. On the log-log plot the points can only be placed in a limited range and still be consistent with a single value of Δ/Γ. The best fit is for $\Delta/\Gamma = 1.2+0.2$. σ_0 is the cross section at resonance.

The values of A/Δ for various values of t and $n\sigma_0$ have been computed[5] and are shown in Fig. 1. In this present work these curves allow the assignment of the value of Γ/Δ to the level in question.

III. EXPERIMENTAL PROCEDURE

The source of protons to initiate the $K^{39}(p,\gamma)Ca^{40}$ reaction was the 3-Mev Van de Graaff linear accelerator of the Case Institute of Technology. The energy stability of this accelerator is 500 ev or less and may be as low as 300 ev at 2 Mev depending on the operating conditions. The possible divergence of the beam was limited by a 1-in. aperture 10 ft from the target and a $\frac{3}{16}$-in. aperture 6 in. in front of the target. Thus a beam divergence of about 0.5° is expected.

The targets were made by evaporating KI onto a tantalum backing which was previously washed in hydrochloric acid. These potassium iodide targets were rather trouble-free and could be left exposed to moist air for several hours without appreciable water absorption. For the resonant absorption experiment targets as thick as 10 kev were used, while for the determination of yield curves targets 1 kev or thinner were used.

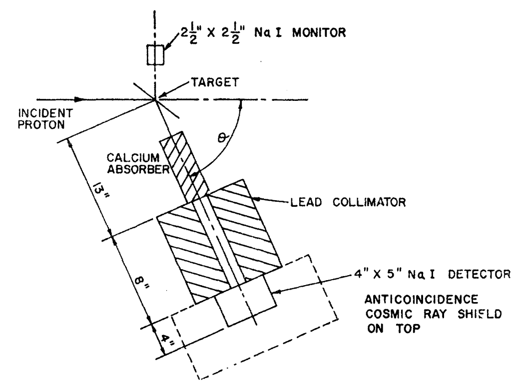

FIG. 2. Experimental arrangement for observing resonant absorption of gamma rays by calcium.

Figure 2 shows the experimental arrangement for observing resonant absorption. A 2.5-in. by 2.5-in. NaI(Tl) crystal was used as the monitor and a 5-in. diameter by 4 in. deep NaI(Tl) crystal was used to detect the photons transmitted through the calcium absorber. Above this detector was a large plastic scintillator which acted as a cosmic-ray anticoincidence shield.

The collimator geometry is shown in more detail in Fig. 3. The entrance or angle defining aperture of the collimator subtends approximately 0.3°. The far aperture of the collimator was large enough that the full

[3] W. E. Lamb, Phys. Rev. **55**, 190 (1939).
[4] H. A. Bethe, Revs. Modern Phys. **9**, 69 (1937).
[5] M. E. Rose, W. Miranker, P. Leak, L. Rosenthal, and J. K. Hendrickson, Westinghouse Electric Corporation Atomic Power Division Report WAPD-SR-506, 1954 (unpublished), Vols. I and II.

target spot is viewed by the detection crystal. The detector and collimator could be rotated as a unit about an axis passing through the target spot. Rotation about an axis not passing through the target spot could result in a variation of counting rate with angle unless the collimator is constructed as shown. To check this point, runs were taken periodically through 5° centered on the expected resonant angle without absorbers to determine that the counting rate was indeed independent of angle.

A block diagram of the circuitry used is shown in Fig. 4. The pulses from the two detectors were mixed and presented to a 256-channel pulse-height analyzer which had a selective storage feature. The pulses from the monitor were stored in the first group of 128 channels while those pulses from the movable detector were stored in the second group. Thus, the spectra from both detectors were stored simultaneously. A large plastic scintillator placed over the 5-in. crystal and

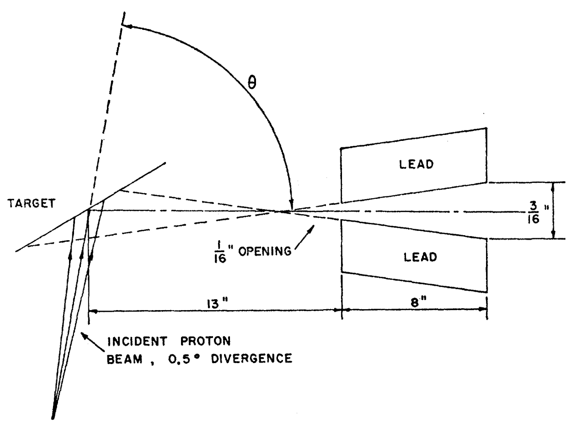

FIG. 3. Details of collimator geometry for observing resonant absorption of gamma rays.

used as an anticoincidence gate reduced the background counts by a factor of three.

The experimental procedure was the following. The movable collimator and detector was set at an angle several resolution widths away from the expected position of resonance and the spectra from the two detectors were recorded. The movable collimator and detector unit was then moved a fraction of a degree closer to the resonance angle and the spectra again recorded. This procedure was continued until the resonance was traversed. During each experimental period at least two traversals were made, one with θ increasing and one with θ decreasing, in an attempt to cancel systematic drifts. For each angular position, the number of counts corrected for background within the 10.3-Mev photopeak of the monitor spectrum was divided into the number of counts within the same photopeak of the spectrum recorded by the movable detector through the collimator as a function of angle. The background, which was essentially all of cosmic-ray

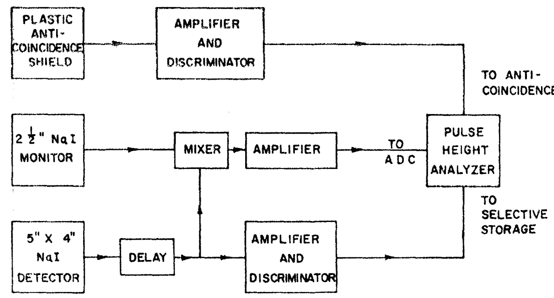

FIG. 4. Block diagram of electronic circuits used in the observation of resonant absorption of gamma rays.

origin, was determined to be independent of machine operation. Thus, knowing the pulse height distribution of the background counts, one can infer the number of background counts under the photopeak from a knowledge of the counts in higher channels.

During the initial phase of experimentation the calcium absorber was in the form of calcium oxide pressed into brick form having a density of 1.4 g/cc as compared to a crystalline density of 3.37 g/cc. The absorber in this form had several disadvantages. First, the presence of the oxygen represented nonresonant attenuation and hence resulted in a reduction of counting statistics. Second, calcium oxide has an appreciable affinity for water. Thus an uncertainty in the composition of the absorber was present. In addition, the Debye temperature of calcium oxide is not known and hence the thermal width of the level cannot be determined. To eliminate these disadvantages an absorber of calcium metal was used during the later phase of the experiment. Only the absorption curves obtained from the calcium metal absorbers were used to determine the parameters of the level being investigated. The results using the calcium oxide absorbers were however in qualitative agreement with those using the calcium metal absorbers if it is assumed that the Debye temperature of calcium oxide is similar to that of magnesium oxide. The ground-state gamma-ray yield as a function of proton energy was measured from 1 Mev to 2.7 Mev using the $2\frac{1}{2}$-in. crystal. The angular distribution of the ground state radiation from the 10.3-Mev level was measured in the conventional manner.

IV. RESULTS

A level in Ca^{40} formed by the resonant capture of protons by K^{39} was reported by Towle *et al.*[6] to occur at $E_p = 1.566$ Mev and to give rise to a strong ground-state gamma-ray transition. The energy of the gamma ray was reported to be 9.90 Mev and the partial width for emission to the ground state was given as $14/(2J+1)$ ev, where J is probably equal to one.[7] It had been also

[6] J. H. Towle, R. Berenbaum, and J. H. Matthews, Proc. Phys. Soc. (London) **A70**, 84 (1957).
[7] R. Berenbaum and J. H. Matthews, Proc. Phys. Soc. (London) **A70**, 445 (1957).

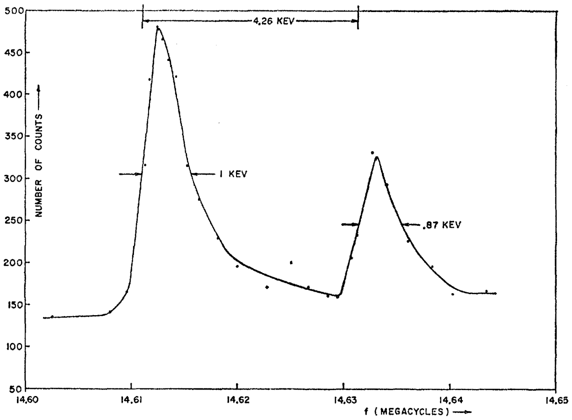

FIG. 5. Structure of gamma-ray yield from the $K^{39}(p,\gamma)Ca^{40}$ reaction at proton energy in the region of 1.56 Mev.

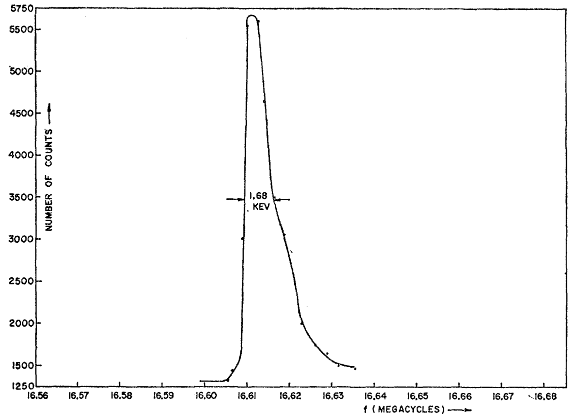

FIG. 7. The 2.05-Mev resonance in the $K^{39}(p,\gamma)Ca^{40}$ reaction.

reported that the total width is less than one kev. This information indicated that there was a good possibility for this level to display nuclear resonant absorption. Consequently, the region in the vicinity of the expected resonance angle was carefully scanned using a 6-in. sample of calcium oxide absorber, but no resonant absorption was definitely observed. Upon later examination of a (p,γ) yield curve over this level, using a target on the order of 15-kev thick, it was noted that there appeared to be structure in this resonance. Upon making a yield curve determination using a one-kev thick potassium iodide target, it was found that this level reported as a single line is a doublet separated by 4.26 kev (see Fig. 5), both levels decaying predominantly by ground-state transitions.

The yield of the ground-state gamma radiation from $E_p=1.45$ to 2.1 Mev from the $K^{39}(p,\gamma)Ca^{40}$ reaction using a 10-kev thick potassium iodide target is shown in Fig. 6. At $E_p=2.05$ Mev is a previously unreported level at 10.3 Mev in Ca^{40}. Figure 7 shows the yield from this level using a thin target. The yield goes from base to peak in 370 ev, which is comparable to the attainable energy resolution of the Van de Graaff. Because of the high yield and the apparent narrow width of this level, it appeared to be a good candidate for nuclear resonant absorption.

In the region up to 3 Mev, several more well-resolved resonances[8] were found, but for each the yield was much lower and the ground-state transition not as prominent as for the 2.05-Mev level. These levels were not investigated because of marginal results one could expect from such levels.

Figure 8 shows the gamma-ray spectrum from the 10.3-Mev level using a $2\frac{1}{2}$-in. diameter by $2\frac{1}{2}$-in. long NaI scintillation detector. Analysis of this spectrum indicates that $(80\pm8)\%$ of the time the level decays to the ground state, the remaining 20% going to the first-excited state resulting in a 7.0-Mev gamma ray.

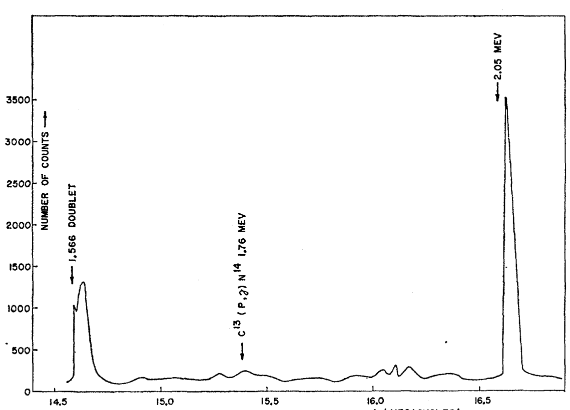

FIG. 6. Ground state gamma-ray yield from the $K^{39}(p,\gamma)Ca^{40}$ reaction from 1.5 Mev to 2.1 Mev.

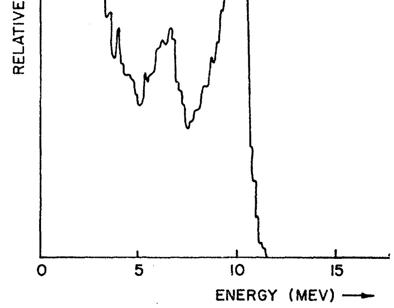

FIG. 8. Gamma-ray spectrum for the $K^{39}(p,\gamma)Ca^{40}$ reaction at the $E_p=2.05$ Mev resonance. Spectrum was obtained using a $2\frac{1}{2}$-in. by $2\frac{1}{2}$-in. NaI crystal spectrometer.

[8] A preliminary report of a more exhaustive investigation of the $K^{39}(p,\gamma)Ca^{40}$ reaction has been given by R. L. Zimmerman and M. Moe, Bull. Am. Phys. Soc. 6, 47 (1961).

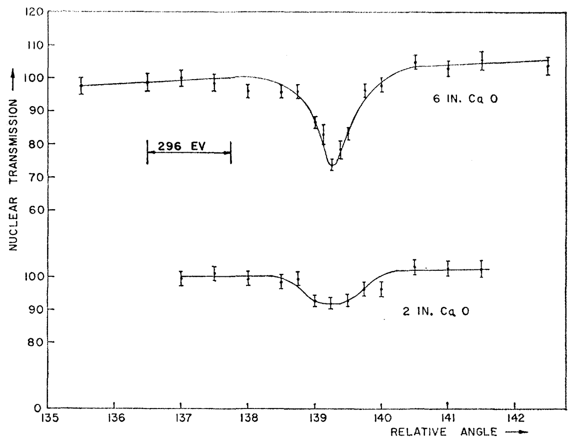

FIG. 9. Nuclear resonant absorption by the 10.3-Mev level of Ca^{40}. Top curve is the transmission by 6 inches of CaO of radiation from the $K^{39}(p,\gamma)Ca^{40}$ reaction. Bottom curve is obtained using 2 in. of CaO. Note break in transmission scale between curves.

The resonant absorption curves for two thicknesses of calcium oxide absorber and three thicknesses of calcium metal absorber are shown in Figs. 9 and 10. A total of approximately 1500-μa hours was required to obtain data for the five curves. Table I indicates the measured area for each absorber thickness.

The experimental angular distribution of the 10.3-Mev gamma rays is shown in Fig. 11. The least-squares fit, up to terms in $\cos^4\theta$, is

$$W(\theta)=1-(0.17\pm0.02)\cos^2\theta+(0.03\pm0.03)\cos^4\theta.$$

V. DISCUSSION OF RESULTS

The three areas for the calcium metal absorbers are found to best fit the curves designated by $\Delta/\Gamma=1.2\pm0.2$ in Fig. 1. Using the value $\Delta=12.4$ ev as determined from Eq. (II7), the following results are obtained:

$$\Gamma=10.3\pm1.7 \text{ ev}.$$

This value of Δ/Γ permits a determination of σ_0 and therefore of $(2J+1)\Gamma_{\gamma^0}=18.2\pm1.2$ ev. An absolute yield measurement gives the result that

$$(2J+1)\Gamma_{\gamma^0}\Gamma_p/\Gamma=9.5\pm1 \text{ ev},$$

while spectral analysis indicates a branching ratio of

$$\Gamma_{\gamma^0}/\Gamma=0.80\pm0.08.$$

TABLE I. Experimental results of the resonant absorption of the 10.3-Mev gamma ray in absorbers of varying composition and size.

Absorber composition	Length (inches)	n(nuc./cm^2)	Max. absorption (%)	Area (ev)
CaO	2.0	7.90×10^{22}	9 ± 2	24 ± 4
	6.0	22.6×10^{22}	28 ± 2	59 ± 4
Ca metal	1.0	5.94×10^{22}	12 ± 2	25 ± 5
	2.75	16.1×10^{22}	24 ± 3	56 ± 4
	11.25	65.6×10^{22}	44 ± 4	97 ± 8

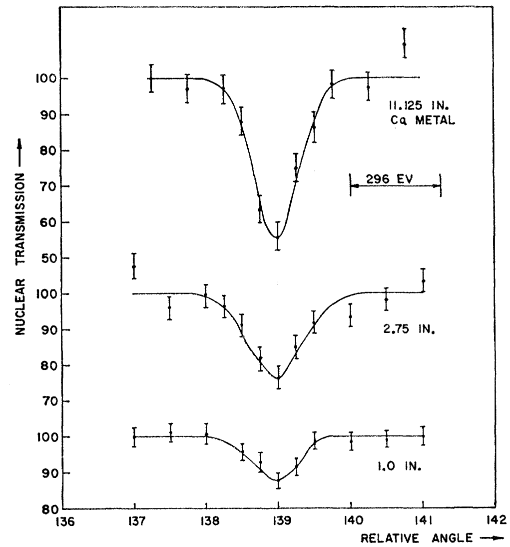

FIG. 10. Nuclear resonant absorption by the 10.3-Mev level of Ca^{40}. From top to bottom the curves are for transmission by 11.125, 2.75, and 1.0 in. of calcium metal, respectively, of radiation from the $K^{39}(p,\gamma)Ca^{40}$ reaction. Note transmission scale breaks between curves.

To calculate the partial widths Γ_γ, Γ_{γ^0}, and Γ_p requires a knowledge of J. Since the ground-and first-excited states of Ca^{40} are known to be 0^+ states, the spin of the 10.3 Mev state is likely to be 1 or 2. A $J=2$ assignment gives a consistent set of partial widths while a $J=1$ assignment gives values for the partial widths which are physical only by taking the extremes of the experimental errors. It was hoped that the angular distribution of the gamma rays would settle this question. However using the tables of Ferguson and Rutledge,[9] the analysis of the angular distribution is not unique (see Appendix 1) since channel spin mixing, $S=1$ or 2, and l mixing are possible, l being the angular-momentum quantum number of the incident proton. It is shown there that the level assignments $J=1^{\pm}$ and $2^{\pm}$ are consistent with the measured angular distribution, although $J=2^{\pm}$ is favored.

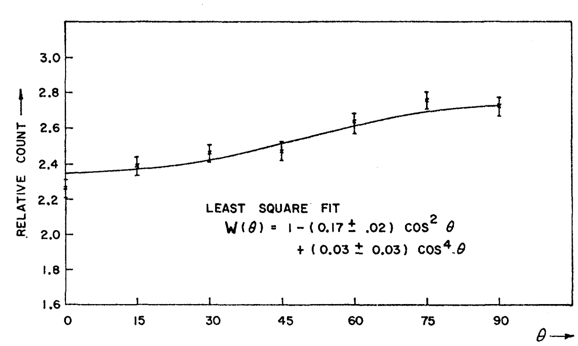

FIG. 11. Angular distribution of ground state radiation from the 10.3-Mev level of Ca^{40} excited in the $K^{39}(p,\gamma)Ca^{40}$ reaction.

[9] A. J. Ferguson and A. R. Rutledge, Chalk River Laboratory Report CRP-615 (unpublished).

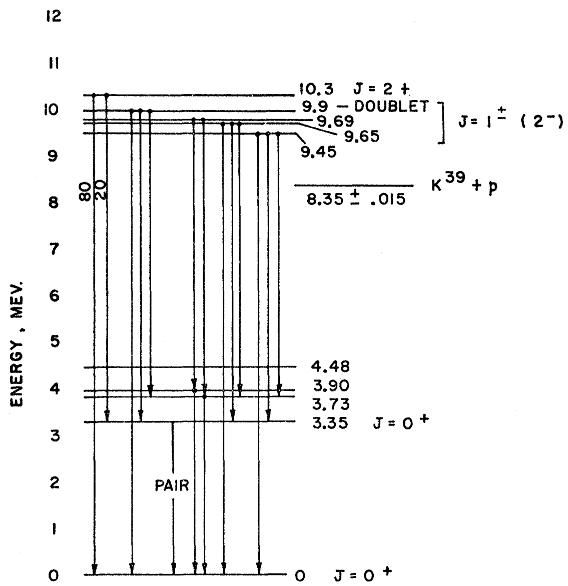

FIG. 12. The energy-level diagram of Ca^{40}.

The proton energy at resonance is determined to be $E_p=2.05\pm0.01$ Mev relative to the $E_p=0.993$ Mev resonance of the $Al^{27}(p,\gamma)Si^{28}$ reaction. Using the value $Q=8.85$ Mev for the $K^{39}(p,\gamma)Ca^{40}$ reaction, the excitation energy E_0 of the Ca^{40} level is calculated to be 10.3 Mev. Table II gives a summary of results and Fig. 12 the known level structure of Ca^{40}. In order to determine the parity of the state, additional information is required, since even though the angular distribution measurement is consistent with the assignment of $J=2$ it does not in itself determine the parity.

Additional information may be obtained by considering the following equation,

$$\Gamma_p=(2k/\pi K)v_l D, \qquad \text{(V1)}$$

where k is the wave number of the incident proton, K is the proton wave number inside the nucleus ($K=10^{13}$ cm^{-1}), v_e is the penetration factor for the Coulomb barrier (computed with $r=1.2\times10^{-13}$ cm), and D is a proportionality constant having the units of energy.[1]

Equation (V1) may be rewritten in the form

$$\Gamma_p=2kv_l\gamma,$$

where γ is the reduced width. According to Vogt[10] the reduced width in the vicinity of $A=40$ is $\gamma=10^{-8}$ ev cm. For $\Gamma_p=5.8$ ev and $k=3.13\times10^{12}$ cm^{-1} at a proton energy of 2 Mev,

$$v_l=\Gamma_p/2k\gamma=9.26\times10^{-5}.$$

Using the equation of Mott and Massey[11] the penetration factors for three different values of l have been calculated to be

$$v_0=3.95\times10^{-3},$$
$$v_1=3.74\times10^{-4},$$
$$v_2=5.64\times10^{-5},$$

and thus the value of $l=2$ yields a penetration factor which is in fair agreement with the expected value of the reduced width in the vicinity of $A=39$. If the formation of the compound nucleus is through d-wave capture, then the parity of the state is positive.

It should be pointed out, however, that to obtain the observed angular distribution the $J=2^+$ assignment requires an $l=0$, $l=2$ mixing in the order of 10 to 1 while consideration of the required penetration factor to yield the measured value of Γ_p requires that $l=2$ should predominate. This apparent discrepancy cannot be explained.

A 2^+ level assignment is strengthened by the additional information obtained through consideration of the radiation width, Γ_{γ^0}. The single-particle widths according to Weisskopf[12] are

$$\Gamma_w(E2)=0.79 \text{ ev}, \quad M^2=4.5,$$
$$\Gamma_w(M2)=2.06\ 10^{-2} \text{ ev}, \quad M^2=1700,$$

where

$$M^2=\Gamma_{\gamma^0}/\Gamma_w,$$

and is the strength of the transition measured in Weisskopf units. For $M2$ transitions measured values of M^2 vary from 10^{-3} to 1. Thus, it is unlikely that the transition is $M2$. For $E2$ transitions, however, observed values of M^2 range from 1 to 1000. Thus, the transition is most likely $E2$ and the 2^+ level assignment is supported.

In conclusion, a 2^+ level assignment is most consistent with the data, however, a 2^- or $1^\pm$ level assignment cannot be completely excluded.

TABLE II. Parameters of the 10.3-Mev level in Ca^{40}.

E_0	Excitation energy	$10.3\ \pm0.01$ Mev
E_p	Proton energy	2.05 ± 0.005 Mev
Γ	Total width	$10.3\ \pm1.7$ ev
Γ_p	Proton width	$5.8\ \pm1.8$ ev
Γ_{γ^0}	Gamma width to ground state	$3.6\ \pm0.24$ ev
Γ_γ	Total gamma width	$4.5\ \pm0.55$ ev
J^π	Spin and parity	2^+ $(2^-,1^\pm)$

APPENDIX 1. ANGULAR DISTRIBUTION ANALYSIS

The angular distribution of the gamma rays emitted from a (p,γ) reaction may be written in the form

$$W(\theta)=\sum_{i=0}^{n} A_i P_{2i}(\cos\theta),$$

where the $P_{2i}(\cos\theta)$ are the Legendre polynomials.

[10] E. Vogt, *Nuclear Reactions*, edited by P. M. Endt and M. Demeur (North Holland Publishing Company, Amsterdam, 1959), Vol. 1.

[11] N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, New York, 1949), p. 55.

[12] V. F. Weisskopf, Phys. Rev. 83, 1073 (1951).

The reaction to be considered is the following:

$$K^{39}+p \rightarrow (Ca^{40})^* \rightarrow Ca^{40}+\gamma,$$

where $(Ca^{40})^*$ represents the 10.3-Mev level in Ca^{40}; the angular momentum J and the parity of which are to be determined. The ground states of K^{39} and Ca^{40} are known to be $\frac{3}{2}^+$ and 0^+, respectively. Table III gives the possible values of S (channel spin) and l for different assumed values of J and parity of the excited state.

The tables of Ferguson and Rutledge[9] have been used to determine the coefficients A_i which are listed in Table IV. In Table IV f is the measure of the channel spin mixing and is the ratio of the square of the matrix element for $S=1$ to the square of the matrix element for $S=2$. x and y are the squared matrix element ratios for the two possible l values (l,l'). Consideration of Gamow penetration factors indicates that the smaller of the two possible values should be favored and hence the values of x and y would be expected to be less than unity.

The task to be performed is to determine the values of x, y, and f required to fit the measured distribution for the four cases above. Thus the feasibility of each assumed J value and parity may be determined.

The measured distribution is

$$W(\theta)=1-(0.17\pm0.02)\cos^2\theta+(0.03\pm0.03)\cos^4\theta.$$

The results are as follows.

$J=1^+$: For this case no unique solution exists.

TABLE III. Possible J^π values of the 10.3-Mev level in Ca^{40} and required channel spin and proton orbital angular momentum.

J^π	S	l	Radiation to ground state
1^+	1	0,2	$M1$
	2	2	
1^-	1	1	$E1$
	2	1,3	
2^+	1	2	$E2$
	2	0,2	
2^-	1	1,3	$M2$
	2	1,3	

However,

$$f=0.587x^2-2.24x-0.27,$$

Hence, $x\geqslant 3.9$. Therefore, $l=2$ is favored over $l=0$.

$J=1^-$: For this case no unque solution exists. However,

$$f(0.27x^2-1.47x-0.03)=0.63,$$

or $x\geqslant 5.4$. Thus $l=3$ is favored over $l=1$.

$J=2^+$: There exists a unique solution which is

$$x=0.096,\quad f=252.$$

Hence, $l=0$ and $S=2$ are favored and a small admixture of $l=2$ is required to fit the distribution.

$J=2^-$: The exact solution for this case is formidable, however it may be demonstrated easily that solutions for x and y less than unity exist.

TABLE IV. Angular correlation coefficients for photons emitted from the 10.3-Mev level in Ca^{40} for various values of J^π. f is channel spin mixing and x, y are the squared matrix element ratios for the two possible values of l.

J^π	$W(S,l,l')$	Relative probability
1^+	$W(1,0,0)=1$	1
	$W(1,0,2)=5^{\frac{1}{2}}(0.63246)P_2$	x
	$W(1,2,2)=1-5^{\frac{1}{2}}(0.22361)P_2$	x^2
	$W(2,2,2)=1+5(0.22361)P_2$	f
	$W=W(1,0,0)+xW(1,0,2)+x^2W(1,2,2)+fW(2,2,2)$	
1^-	$W(1,1,1)=1+0.5P_2$	1
	$W(2,1,1)=1-5^{\frac{1}{2}}(0.04472)P_2$	f
	$W(2,1,3)=5^{\frac{1}{2}}(0.65727)P_2$	xf
	$W(2,3,3)=1-5^{\frac{1}{2}}(0.17889)P_2$	x^2f
	$W=W(1,1,1)+fW(2,1,1)+xfW(2,1,3)+x^2fW(2,3,3)$	
2^+	$W(1,2,2)=1+0.357P_2+1.141P_4$	1
	$W(2,0,0)=1$	f
	$W(2,0,2)=1.195P_2$	xf
	$W(2,2,2)=1-1.53P_2-0.491P_4$	x^2f
	$W=W(1,2,2)+fW(2,0,0)+xfW(2,0,2)+x^2fW(2,2,2)$	
2^-	$W(1,1,1)=1+5^{\frac{1}{2}}(0.22361)P_2$	
	$W(1,1,3)=-5^{\frac{1}{2}}(0.15649)P_2+3(0.93314)P_4$	
	$W(1,3,3)=1-5^{\frac{1}{2}}(0.25555)P_2-3(0.19048)P_4$	
	$W(2,1,1)=1-5^{\frac{1}{2}}(0.2236)P_2$	
	$W(2,1,3)=-5^{\frac{1}{2}}(0.3833)P_2-3(0.38095)P_4$	
	$W(2,3,3)=1+5^{\frac{1}{2}}(0.06389)P_2+3(0.28571)P_4$	
	$W=W(1,1,1)+xW(1,1,3)+x^2W(1,3,3)+fW(2,1,1)+yfW(2,1,3)+y^2fW$	

Our conclusions are as follows:

For $J=1^-$ the required l mixing is such that the higher values are favored. It is expected that this is not the case.

For $J=2^\pm$ the smaller l values are favored. Thus, it is expected that the assignment of $J=2$ is best, however no assumption about the parity of the state may be made.

PHYSICAL REVIEW VOLUME 124, NUMBER 5 DECEMBER 1, 1961

Energy Levels of Na^{21} and Mg^{22}†

F. Ajzenberg–Selove
Haverford College, Haverford, Pennsylvania

AND

L. Cranberg and F. S. Dietrich*
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico

(Received July 31, 1961)

Neon gas has been bombarded with 2.4, 3.1, 4.6, and 6.1-Mev deuterons and with 3.4- and 4.5-Mev He^3 particles. Time-of-flight measurements of neutron groups indicate excited states of Na^{21} at 0.37 ± 0.04, 1.69 ± 0.05, 2.83 ± 0.04, 3.89 ± 0.05, 4.86 ± 0.06, and 5.01 ± 0.05 Mev, in addition to the well-known states at 2.43, 3.57, 4.18, 4.31, and 4.49 Mev. Angular distributions show strong direct interaction features. Mg^{22}, here reported for the first time, has a mass excess $(M-A)$ of -0.14 ± 0.08 Mev (C^{12} reference) from $Q=-0.043\pm0.08$ Mev for $Ne^{20}(He^3,n)Mg^{22}$. The first excited state is at 0.995 ± 0.04 Mev. No other states are observed with $E_x<2.5$ Mev.

I. INTRODUCTION

In Ne^{21}, studies of the proton groups from the reactions $F^{19}(He^3,p)Ne^{21}$ (Hinds and Middleton[1]) and $Ne^{20}(d,p)Ne^{21}$ (Freeman[2]) have shown the existence of eleven states with excitation energies E_x less than 5 Mev. In the mirror nucleus Na^{21}, only eight states had been reported in the same energy interval when this work was begun. Thus this experiment was undertaken to locate the remaining states, to determine, if possible, their J^π, and to study the interaction mechanisms involved in the $Ne^{20}(d,n)Na^{21}$ reaction at deuteron energies at and above the Coulomb barrier.

The information[3] concerning the Na^{21} states come from two types of studies: the bound states have been observed in the $Ne^{20}(d,n)Na^{21}$ reaction, using plate,[4] spectrometer,[5] and counter-ratio[6] techniques; the unbound states have been determined by studying the interactions[7] of Ne^{20} and protons [$Ne^{20}(p,\gamma)Na^{21}$,[5,8,9] $Ne^{20}(p,p)Ne^{20}$,[10] and[11] $Ne^{20}(p,p')Ne^{20*}$]. The evidence for the observed states may be summarized as follows:

Ground state. The mass of Na^{21} is known[12] to 32 kev. The superallowed character of the β^+ decay to the ground state of Ne^{21} shows $J^\pi=\frac{3}{2}^+$.

*0.37-*Mev *state.* This state has been observed in $Ne^{20}(d,n)Na^{21}$. At $E_d=1$ Mev ($\theta=0°$ and 90°), Swann and Mandeville[4] reported a state with $Q=-0.17\pm0.05$ Mev which they tentatively assigned to the ground-state reaction. Angular distributions[5] at $E_d=2.7$, 4.5, and 4.9 show $l_p=2$ and therefore $J^\pi=(\frac{3}{2},\frac{5}{2})^+$ for this state.

*1.73-*Mev *state.* This state was previously reported to be located at 1.46 ± 0.04 Mev by Marion, Slattery, and Chapman.[6] They used the counter ratio technique for locating the corresponding threshold in the (d,n) reaction. While we are in agreement with the other results reported by Marion *et al.* (see below), we disagree by over 200 kev with this determination. It may

† Work performed under the auspices of the U. S. Atomic Energy Commission and the National Science Foundation.

* Now at California Institute of Technology, Pasadena, California.

[1] S. Hinds and R. Middleton, Proc. Phys. Soc. (London) **74**, 779 (1959).

[2] J. M. Freeman, Phys. Rev. **120**, 1436 (1960).

[3] N. B. Gove, C. L. McGinnis, R. Nakasima, and K. Way, *Nuclear Level Schemes, Landolt-Börnstein Critical Tables* (Springer-Verlag, Berlin, 1961). We are indebted to Dr. F. Everling for this information concerning the $A=21$ and 22 level schemes.

[4] C. P. Swann and C. E. Mandeville, Phys. Rev. **87**, 215(A) (1952).

[5] R. E. Benenson and L. J. Lidofsky, Phys. Rev. **123**, 939 (1961).

[6] J. B. Marion, J. C. Slattery, and R. A. Chapman, Phys. Rev. **103**, 676 (1956).

[7] See P. M. Endt and C. M. Braams, Revs. Modern Phys. **29**, 68 (1957) for a review of the early work.

[8] N. W. Tanner, Phys. Rev. **114**, 1060 (1959); G. C. Thomas and N. W. Tanner, Proc. Phys. Soc. (London) **75**, 498 (1960).

[9] Valter, Gonchar, Lvov, and Tsytko, Iszv. Akad. Nauk SSSR, Ser. Fiz. **23**, 228 (1959).

[10] W. Haeberli, A. Galonsky, E. Goldberg, and R. Douglas, Phys. Rev. **91**, 438(A) (1953); W. Haeberli, *ibid.* **99**, 640(A) (1955).

[11] A. Galonsky, W. Haeberli, E. Goldberg, and R. Douglas, Phys. Rev. **91**, 439(A) (1953); M. C. Cox, J. J. van Loef, and D. A. Lind, *ibid.* **93**, 925(A) (1954).

[12] F. Everling, L. A. König, J. H. E. Mattauch, and A. H. Wapstra, Nuclear Phys. **18**, 529 (1960).