Measurement of the Neutron-Proton Final-State Interaction in the Electrodisintegration of Deuterium*

H. W. KENDALL, J. I. FRIEDMAN,† E. F. ERICKSON, AND P. A. M. GRAM
High-Energy Physics Laboratory and Department of Physics, Stanford University, Stanford, California
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Measurements have been made of the inelastic differential cross sections for electrodisintegration of deuterium for ϵ in the range from 0 to 12 Mev, where ϵ is the energy in the *n-p* center-of-mass system in the final state. Primary electron energies were in the range from 204 to 500 Mev. The process was studied for momentum transfers from 1.8 to 2.8 f^{-1}. The *n-p* interaction in the unbound state gives rise to a peak in the differential cross sections for ϵ near zero for transitions to the ^1S and ^3S states of the *n-p* system. The high momentum transfers available make the results sensitive to the short range structure of the unbound *n-p* wave functions. The results fail to agree with the predictions of Jankus even at the lowest momentum transfers, when a central attractive force is assumed for the *n-p* force. At the highest momentum transfers the predicted cross sections are approximately 50% greater than the measured ones. The experimental results agree with theory if a repulsive core of the radius required to fit the elastic scattering data is used both in the bound and unbound *n-p* states. The extent to which relativistic corrections alter this conclusion is not known at present.

I. INTRODUCTION

INELASTIC scattering of high-energy electrons from deuterium has been studied experimentally by Yearian and Hofstadter,[1] by Sobottka and others,[2] and, theoretically, by Blankenbecler,[3] Durand,[4] Goldberg,[5] and Jankus.[6] The measurements[1] did not examine with high momentum resolution the details of the scattered electron spectrum in the region of the elastic peak. A neutron-proton interaction in the final state makes its greatest contribution to the cross section in this region. Sobottka[2] examined that part of the spectrum characteristic of scattering from the quasi-free nucleons having very low internal momentum. The results were analyzed to yield the electron-neutron scattering cross sections as a function of the momentum transfer and thus to determine the magnetic structure form factor of the neutron. The *n-p* interaction in the final states introduces corrections of the order of 10% to the impulse-approximation calculations, and hence his measurements cannot be used to determine this interaction.

A knowledge of the *n-p* interaction is important in understanding the two-nucleon interaction and bears on the difficult problem of the interactions that determine the properties of nuclear matter and of finite nuclei. Although high-energy nucleon-nucleon scattering has established the existence of a repulsive core and other features of the nucleon-nucleon potential for high relative energies of the particles, there are few experiments which shed light on the details of the short-range structure of the potential for low relative energies.[7]

In the absence of final-state interactions, the electrodisintegration cross section would go monotonically to zero at disintegration threshold: zero energy in the final *n-p* system. A strong final-state interaction[6] can result in a large enhancement of this cross section near threshold, and measurements of the process at high momentum transfer can be used to study the *n-p* wave function and hence the *n-p* interaction for internucleon spacings of the order of one fermi or less.

In the present experiment we have made measurements of the electrodisintegration process for a number of primary electron energies in the range from 204 to 500 Mev. The four-momentum transfers were in the range from 1.8 to 2.8 f^{-1}, and the cross sections were measured from $\epsilon=0$ to about $\epsilon=12$ Mev, where ϵ is the energy in the recoiling *n-p* system. For this range of ϵ and the momentum transfers of the present experiment, the cross sections depend strongly not only on the initial deuteron wave function but also on the wave function representing the unbound *n-p* system. The latter wave function is sensitive to the *n-p* interaction at very short distances in the final state. We have compared our measurements with the theory of Jankus,[6] modified as described below.

II. ANALYSIS AND DISCUSSION OF DATA

The elastic electron scattering process, $e+d\rightarrow e+d$ contributes some background to measurements of the inelastic process $e+d\rightarrow e+p+n$. Both processes are intrinsically interesting, and absolute cross sections for both were determined in the same series of measure-

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† Now at Massachusetts Institute of Technology, Cambridge, Massachusetts.
[1] M. R. Yearian and R. Hofstadter, Phys. Rev. **111**, 934 (1958).
[2] R. Hofstadter, F. Bumiller, and M. Yearian, Revs. Modern Phys. **30**, 462 (1958); and S. Sobottka, Phys. Rev. **118**, 831 (1960).
[3] R. Blankenbecler, Phys. Rev. **111**, 1684 (1958), and Ph.D. thesis, Stanford University (unpublished).
[4] L. Durand, III, Phys. Rev. **115**, 1020 (1959).
[5] A. Goldberg, Phys. Rev. **112**, 618 (1958).
[6] V. Z. Jankus, Phys. Rev. **102**, 1586 (1956).
[7] See especially the remarks of K. Brueckner in *Proceedings of the International Conference on Nuclear Structure, Kingston, 1960* (University of Toronto Press, Toronto, 1960), pp. 67–75. G. Breit, Phys. Rev. **120**, 287 (1960); and J. K. Perring and R. J. N. Phillips, Nuclear Phys. **23**, 153 (1961). The last two papers contain a number of references to theory and experiments concerning the nucleon-nucleon interaction, and the existence of a repulsive-core potential.

ments by comparison with electron scattering from hydrogen. The elastic scattering results have been reported in reference 8, hereafter referred to as I. The equipment, the programming of the hydrogen comparison runs, the determination of the background, and the application of radiative and bremsstrahlung corrections are discussed in detail there.

Jankus' predictions of the scattered electron spectrum[6] were evaluated with an IBM-610 digital computer. The *n-p* interaction in the *S* states was assumed to be a purely attractive Eckart potential. In this program we used an approximation suggested by Jankus to aid in the evaluation of the integrals $A_{gt(s)}$, using his notation. These predictions were modified to take account of the finite nucleon structure[9] by multiplying them by $|F_p(q)|^2$, where $F_p(q)$ is the proton's structure form factor, and q is the four-momentum transfer in the scattering. In the range of momentum transfers covered in the present experiment, the Dirac and anomalous magnetic form factor of the proton have been determined to be equal within the errors of measurement.[10] In the present analysis we have assumed the equality.

The momentum resolution function of the equipment was determined from a study of the observed elastic peaks of hydrogen and deuterium for such relevant value of the parameters E_0 and θ, where E_0 is the primary electron energy and θ is the laboratory angle of scattered electrons. The theoretically predicted electron spectra were folded into these resolution functions to allow comparison with experiment. We have included only the ^1S and ^3S *n-p* states in our analysis of the final-state interactions. The states for $l>0$ do not contribute to the cross sections at $\epsilon=0$ and do not make substantial contributions for the range of ϵ observed here (cf. the discussions in Sec. III). We evaluated separately the predicted cross sections for transitions to the ^1S, the ^3S, and all states for $l>0$, and these too were folded into the observed resolution functions to allow measurements of the relative contributions to the total cross section of transitions to these different states. Experimentally determined scattered-electron momentum distributions and the theoretical predictions are shown in Figs. 1–7. The dashed curves are visual fits to the experimental data, and the separate contributions from electrodisintegration and elastic scattering are shown as dotted curves. The shapes but not the absolute values of the Jankus predictions were used in making these separations. The shapes of the predicted spectra are not altered significantly on introducing repulsive-core wave functions (see Figs. 8 and 9) and no appreciable error in comparing theory and experiment is introduced by this procedure. The errors of the experimental points are standard deviations arising from counting statistics. The solid curves are theoretical predictions. For the 500-Mev data (Figs. 1–6) the predictions contain about equal contributions from transitions to the ^1S and ^3S final states, corresponding to spin-flip and non-spin-flip transitions, respectively. Figure 7 shows the back-angle scattering to be predominantly spin-flip in nature. If the 145° data were more precise, our separation procedure would be adequate to measure form factors for transitions to the ^1S and ^3S states individually.

The installation of a Burroughs-220 computer at Stanford enabled us to compute the values of the Jankus predictions without the approximation discussed above and to investigate, in addition, the effect of introducing

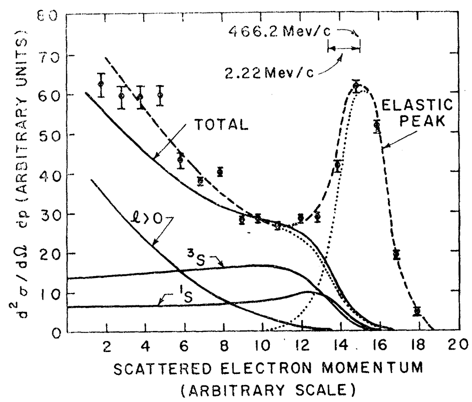

FIG. 1. Scattered electron spectrum for $E_0=500$ Mev, $\theta=43°$, and $q=1.80$ f^{-1}. In this and the following figures through Fig. 7, the dashed lines are fits to the observed data using the shape of the Jankus predictions as modified by the finite structure of the proton and the experimental momentum resolution function. The dotted curves show the elastic peak shape and, within a normalization, the experimental resolution functions. The solid curves are the Jankus predictions modified as described in the text for the effects of the finite structure of the proton and the momentum resolution function. These predictions make use of the approximation suggested by Jankus. The consequences of this approximation are discussed in the text: They lower the cross sections by about 7%. The separate predictions for transitions to the ^3S, the ^1S, and the $l>0$ states of the *n-p* system are shown. The errors shown are discussed in the text and, in greater detail, in reference 8.

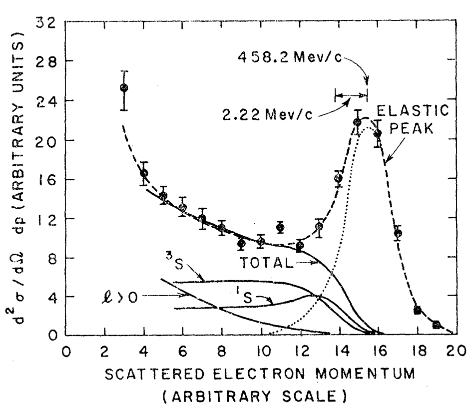

FIG. 2. Scattered electron spectrum for $E_0=500$ Mev, $\theta=48.5°$, and $q=2.0$ f^{-1}. See the caption for Fig. 1.

[8] J. Friedman, H. Kendall, and P. A. M. Gram, Phys. Rev. **120**, 992 (1960), (referred to above as I). This paper contains references to earlier experiments on electron scattering from deuterium.
[9] D. R. Yennie, M. Lévy, and D. G. Ravenhall, Revs. Modern Phys. **29**, 144 (1957).
[10] F. Bumiller (private communication). See also reference 2.

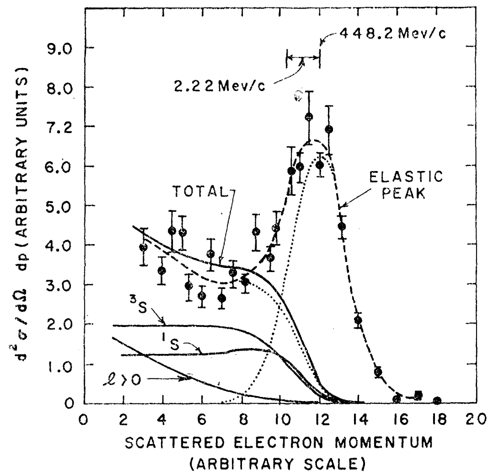

FIG. 3. Scattered electron spectrum for $E_0=500$ Mev, $\theta=55°$, and $q=2.24$ f^{-1}. See the caption for Fig. 1.

a repulsive-core wave function in either the deuteron's bound state or in both the bound and unbound states. The approximation made in the earlier calculations was found to lower the predicted cross sections by about 7%. A typical series of predictions is shown in Figs. 8 and 9 where the introduction of a repulsive-core wave function in both initial and final states is seen to reduce the cross section by about 1.6 times the amount predicted using the core in the initial state only. In the calculation of the final-state interaction, wave functions corresponding to an Eckart potential[6] have been used in both the bound and unbound states of zero angular momentum. A repulsive core of radius $r_0=0.42$ f was introduced either into the bound-state wave function alone or into both the bound and unbound wave functions by substituting for r (the n-p separation) the quantity $r-r_0$, and then adjusting the parameters of the potentials to match the low energy n-p scattering data. This value of the repulsive-core radius is in qualitative

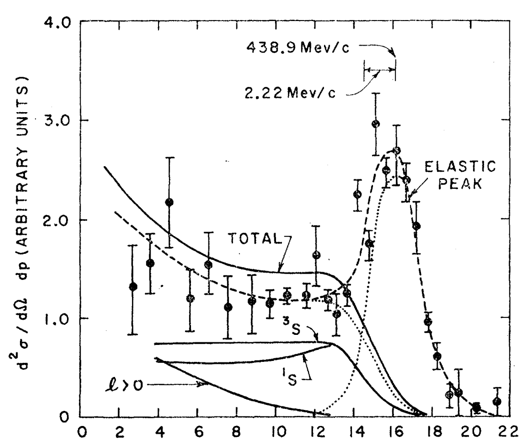

FIG. 4. Scattered electron spectrum for $E_0=500$ Mev, $\theta=61°$, and $q=2.40$ f^{-1}. See the caption for Fig. 1.

agreement with that found necessary in the elastic scattering measurements (see I). No attempt was made to evaluate the effect of the core on the transitions to the unbound states of angular momentum greater than zero, for an estimate of Jankus[6] indicates this effect to be small and our data do not contain significant information on these transitions. Figures 10 and 11 compare the various versions of the theory with the experimental results for a value of ϵ of approximately 2 Mev. Figure 10 includes the data at relatively small angles and the yields include approximately equal contributions from spin-flip and non-spin-flip deuteron disintegration. Figure 11 includes the data taken at $\theta=145°$ where the

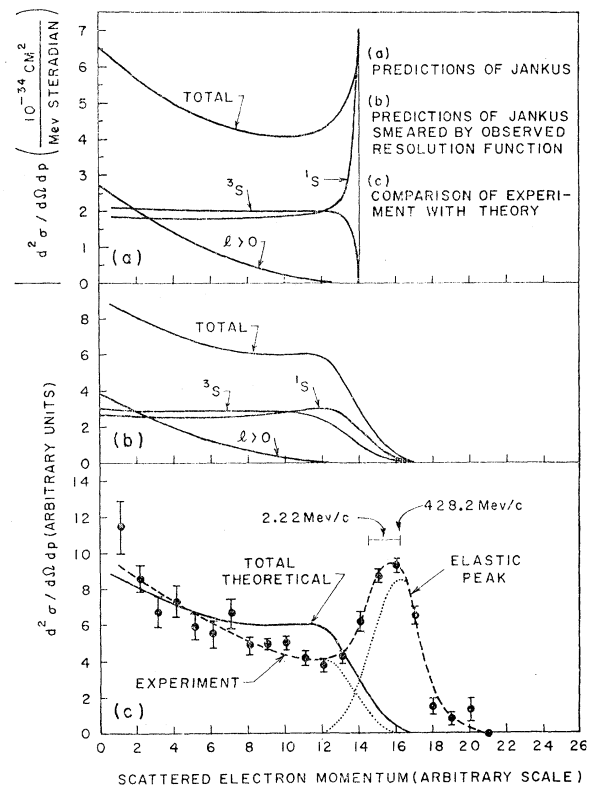

FIG. 5. Scattered electron spectra for $E_0=500$ Mev, $\theta=67.5°$, and $q=2.60$ f^{-1}. In addition to the curves discussed in the caption for Fig. 1 [cf. (b) and (c)], (a) shows the Jankus prediction unmodified by the experimental resolution function. The momentum scale is the same for the three spectra.

yields are predominantly from spin-flip disintegration. The errors shown include contributions from counting statistics and from errors introduced in the data processing. A more complete discussion of the error analysis is made in I.

The relativistic expression of the energy gap G between the elastic peak and disintegration threshold is

$$G=\frac{D+D^2/2Mc^2}{1+(2E_0/Mc^2)\sin^2(\theta/2)},$$

where D and M are the deuteron's binding energy and rest mass. Jankus' nonrelativistic treatment of the deuteron-proton kinematics overestimates G by approximately the amount $(p/2.43)^4$, in Mev, where p is the three-momentum transfer in the scattering in units of reciprocal fermis. In the absence of a well-defined procedure to correct for the nonrelativistic kinematics, we have taken G to be equal to D. An estimate of the error attendant on this treatment was included in the errors shown in Figs. 10 and 11.

III. CONCLUSION

It can be seen from Figs. 1–7 and Figs. 10 and 11 that for high q the Jankus theory fails to predict the inelastic cross section correctly for ϵ near zero. The predicted cross section at $q=2.8$ f^{-1} is about 50% above the measured value. The effect of the final-state interaction becomes less important as ϵ increases, and the agreement of experiment with the Jankus predictions improves, as shown by the work of Yearian and Hofstadter[1] and by Sobottka.[2]

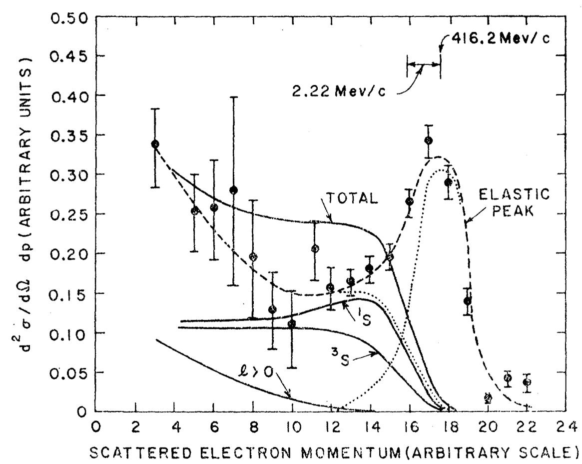

FIG. 6. Scattered electron spectrum for $E_0=500$ Mev, $\theta=75°$, and $q=2.8$ f^{-1}. See the caption for Fig. 1.

For the values of ϵ studied in the present experiment, transitions to the ^3S and ^1S states of the n-p system comprise the major contributions to the cross sections. The inelastic cross section for the $M1$ spin-flip transitions to the ^1S state is sharply peaked at $\epsilon=0$, whereas the cross section for the non-spin-flip transition goes to zero at $\epsilon=0$ and reaches maximum when ϵ is a few Mev. The details of the peak were not determined in the present experiment, as in all cases the peak was narrower than the experimental resolution function. Jankus' theoretical predictions, unmodified by the experimental resolution functions, are shown in Figs. 5, 8, and 9.

In the theoretical predictions used here to compare with the experimental results, a number of approximations in calculation of the final-state interaction were made which are expected to introduce some uncer-

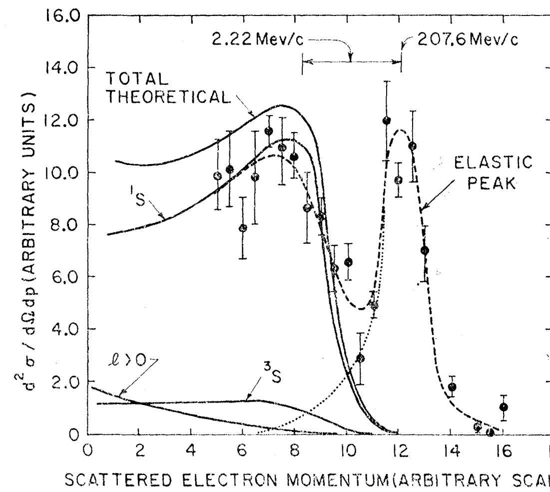

FIG. 7. Scattered electron spectrum for $E_0=260$ Mev, $\theta=145°$, and $q=2.25$ f^{-1}. Notice the small contribution from transitions to the ^3S n-p state to the inelastic differential cross section. See the caption for Fig. 1.

tainties in the predicted values. Jankus has made numerical estimates of the errors consequent on several of these approximations:

(1) No exchange currents were considered. For the irrotational part of the current density, Jankus showed that a Serber-type velocity-dependent potential leads to correction terms important only for large ϵ, for the interaction becomes part of the electric dipole term and leads to final ^3P states. These transitions are strongly reduced near $\epsilon=0$ as a consequence of the angular momentum barrier. The exchange-current contribution from the solenoidal current density has been evaluated

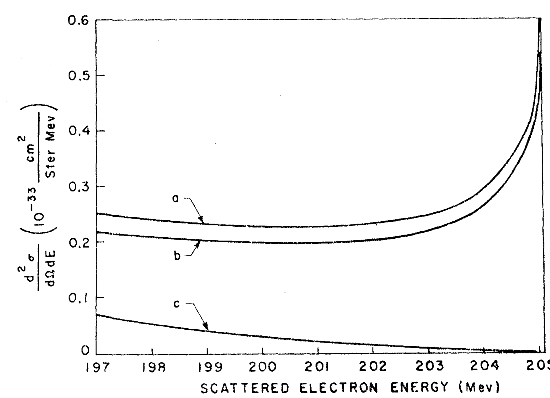

FIG. 8. This figure shows the theoretical predictions for the scattered electron spectrum near disintegration threshold for electron-deuteron scattering for $E_0=260$ Mev and $\theta=145°$. The calculations were made with the exact Jankus expression [Eqs. (9) and (10) of reference 6] and show the effect of introducing a repulsive core of radius 0.42 f either into just the deuteron bound state [curve (a)] or into both the bound and unbound states [curve (b)]. Curve (c) shows the contribution to the cross section for final states of angular momentum greater than zero, using a central attractive force in the initial state, but including no final-state interaction.

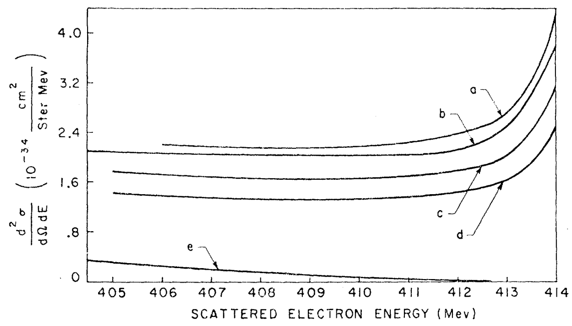

FIG. 9. This figure shows the theoretical predictions for the scattered electron spectrum near disintegration threshold for electron-deuteron scattering for $E_0=500$ Mev and $\theta=75°$. Curve (a) shows the predictions using the exact Jankus result and assuming central attractive forces in both the bound and unbound states. Curve (b) is the same as (a) except that an approximation suggested by Jankus was used to evaluate the cross sections. Curve (c) shows the predictions of the exact Jankus expression assuming a repulsive core of radius 0.42 f in the deuteron's bound state but an attractive potential operative in the unbound ^1S and ^3S states. Curve (d) is the same as (c) except that the core was included in both the bound and unbound states. Curve (e) shows the contributions from states of angular momentum greater than zero. In (e) a central attractive force was used in the deuteron's ground state and no interaction in the final state.

by Berger[11] and Jankus[6] for Hulthén wave functions describing the deuteron's ground state. This leads to transitions to ^1D states (negligible for ϵ near zero) and to ^1S states. These latter transitions were estimated to contribute about 10% to the magnetic dipole transitions for small ϵ.

(2) We included no effects from transitions to final states of $l>0$. Final-state interactions are negligible for final states of $l>2$ from the effects of the angular momentum barrier. For the $l=1$ final states, the centrifugal force term is of the same order of magnitude as a Wigner-type n-p force. The Born approximation was used by Jankus to estimate the magnitude of the corrections to the cross sections from the $l=1$, $l=2$, and $l=3$ final-state interactions. A strong P-state force was approximated using a Wigner potential. The corrections for $\epsilon=10$ Mev for scattering at $\theta=60°$ at a primary electron energy of 350 Mev were about 20% and went to zero approximately quadratically as ϵ approached zero. If the force is approximated by a Serber potential which removes the interaction in states of odd-l, the correction is much smaller, amounting to about 3% for $\epsilon=10$ Mev for the above conditions and also approaching zero quadratically as ϵ approaches zero. The Serber force is deduced from n-p scattering experiments at several hundred Mev. It is not clear whether this or the Wigner force is more realistic at $\epsilon\approx0$.

(3) Consideration of tensor forces acting in the n-p S states was omitted because Jankus has shown that a tensor interaction in the ^3S state gives rise to correction terms which nearly cancel.

One of the important approximations in the original Jankus expression was the failure to include a repulsive-core n-p potential. Jankus estimated the consequence of assuming an extreme repulsive-core potential corresponding, in our notation, to $r_0=0.7$ f. The assumption leads to a decrease of the predicted cross section of about 20% at $q=1.7$ f^{-1}, a result roughly independent of ϵ for small ϵ and a greater decrease than is consistent with the present experimental results. It can be seen from Figs. 10 and 11 that a value of $r_0=0.42$ f (with Eckart wave functions) gives a satisfactory fit to the data. It is known from the work described in I that the Hulthén wave function is inadequate to describe the deuteron's ground state. A Gartenhaus repulsive-core potential is sufficient to obtain agreement with the elastic scattering results, although it has been suggested that relativistic corrections might provide an equally satisfactory explanation of the observed elastic cross sections.

Durand[12] has recently made a detailed study of the final-state interaction for the inelastic scattering process considered here and his work contains a detailed critique of the theoretical situation, and a comparison of our results with a slightly different treatment of the repulsive core. His results are similar to ours. The extent to which a completely relativistic treatment of the calculation including the kinematics and retardation effects of the potential would modify the predicted cross sections is unknown, as no such treatment has been completed.

In conclusion we would like to remark that the analyses presented here are really of an exploratory nature. Although it is pleasing to find that the similar repulsive-core assumptions that are required to fit the elastic scattering data (reported in I) serve to remove the discrepancies of the simple Jankus theory for the scattering near disintegration threshold, it should be noted that a more thorough treatment of the relativistic

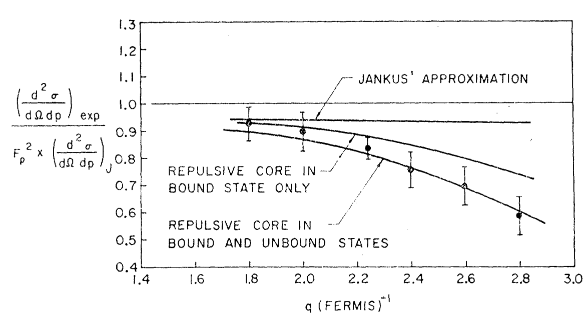

FIG. 10. The points plotted here show the ratio of observed inelastic cross sections at $E_0=500$ Mev (see Figs. 1–6) to the predictions of the Jankus theory, modified by the inclusion of the proton form factors and the experimental resolution of the equipment. The ratio is given as a function of q for ϵ approximately 2 Mev. The solid lines show the predictions of the Jankus calculation incorporating repulsive-core wave functions either in just the bound state or in the bound and the ^1S and ^3S unbound states; predictions of the approximate theory [see text following Eq. (12) of reference 6] are also shown. The observed yields have about equal contributions from spin-flip and non-spin-flip processes leading to ^1S and ^3S final n-p states, respectively. Errors are discussed in the text.

[11] J. M. Berger, Phys. Rev. 94, 1698 (1954).

[12] L. Durand, III, Phys. Rev. 123, 1393 (1961).

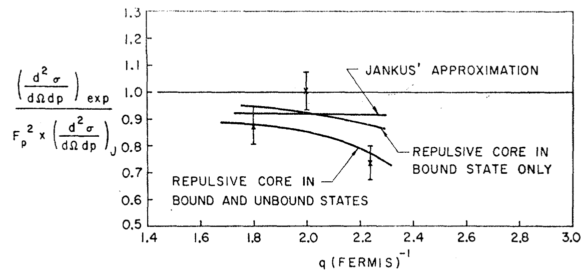

FIG. 11. This figure displays the data for $\theta=145°$ from Fig. 7 and two other points at $\theta=145°$: $q=1.8$ f^{-1} and $q=2.0$ f^{-1}. The major contributions to the yields at this large angle are from the spin-flip disintegration of the deuteron to the final ^1S n-p state. See the caption for Fig. 10.

corrections is required before quantitative deductions are possible on the basis of the present measurements. The agreement between the predictions, assuming a repulsive-core potential, and our results supports the view that the n-p interaction can in fact be described by a potential which has similar characteristics to that necessary to describe both the high-energy n-p scattering experiments and the known uniform density of nuclear matter as seen in the $A^{\frac{1}{3}}$ dependence of the radii of heavy nuclei. This agreement suggests a detailed study to investigate the theory of the final-state interaction using phase shifts determined from high-energy nucleon scattering experiments and from the semiphenomenological fits to medium energy photo-disintegration cross sections.[13]

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[13] See reference 12 and also M. L. Rustgi, W. Zernik, G. Breit, and D. J. Andrews, Phys. Rev. **120**, 1881 (1960); and W. Zernick, M. L. Rustgi, and G. Breit, *ibid.* **114**, 1358 (1959) which contain a number of references to earlier work on the description of the n-p system at medium and high energy.