Nuclear Surface Effects in Muon Capture

PAUL SINGER
*Department of Physics, Technion-Israel Institute of Technology, Haifa, Israel**

(Received July 26, 1961)

Slow-muon capture in heavy nuclei results in a moderately excited nucleus which emits mostly neutrons and to a lesser extent, charged particles. The neutron and alpha emissions can be explained as statistical emission from the compound nucleus formed. The experimentally observed proton emission is ten times higher than that predicted by the same mechanism. It is proposed to take into account clustering of nucleons at the nuclear surface in order to account for the increased proton emission. The capture of the muon by two-nucleon clusters at the surface of AgBr nuclei is calculated and the subsequent direct proton emission evaluated. The experimental findings can be explained with a reasonable strength of correlations assumed.

1. INTRODUCTION

NEGATIVE muons passing through matter are captured in atomic levels as a result of their Coulomb interaction with the positive nuclei. This is possible since the muon can slow down from an energy of a few Mev and cascade down through the levels of the mesonic atom in a total time of approximately 10^{-11} sec, which is much less than the spontaneous disintegration half-life of the free muon (2.21×10^{-6} sec). In heavy nuclei there is then a high probability of muon capture by the nucleus from the K atomic orbit via the weak-interaction process:

$$p\text{ (proton)}+\mu^-\text{ (muon)}\rightarrow N\text{ (neutron)}+\nu\text{ (neutrino)}. \quad (1)$$

The muon capture results in liberation of approximately 100 Mev of energy, most of which is carried away by the neutrino. This picture of the process was first proposed by Tiomno and Wheeler.[1] The excitation energy retained by the capturing nucleus is 15–20 Mev. In light nuclei, the created neutron carrying this energy usually leaves the nucleus without further interaction. In intermediate and heavy nuclei the neutron may divide this energy between the other nucleons and a compound nucleus is formed. The excitation energy is then lost by evaporation of neutrons and to a lesser extent by emission of charged particles. It is quite reasonable to treat the neutron emission as evaporation from a compound nucleus, as it is well known that this is the normal process in nuclear reactions with similar excitation energies.[2] The number of neutrons emitted, 1.5–1.7 neutrons per capture in heavy nuclei,[3] indicates a more complex process than a simple escape from the nucleus of the neutron created in process (1). Nevertheless, although the evaporation process may account for most of the neutron emission, in order to get good agreement with experiment, nuclear correlations and direct emission must also be considered.[4,5]

This article deals with charged particle emission following muon capture. The first experiments[6] showed that with every muon capture in the heavy component of the nuclear emulsion, there appear on the average 0.1 charged particle. Later, an extensive experiment was carried out by Morinaga and Fry[7] and 24 000 stopped muon tracks were examined. Their result is that muon capture in AgBr is accompanied by the appearance of 0.022 proton and 0.005 α particle.

Ishii[8] calculated the charged particle emission from AgBr nuclei excited by muon capture using the statistical model. The nuclear excitation distribution, as calculated by Ishii, depends on the momentum distribution assumed for the nucleons in the nucleus.[9] Ishii considered three possibilities:

1. a Fermi gas distribution at $kT=0$;
2. a Fermi gas distribution at $kT=9$ Mev;
3. the Chew-Goldberger distribution: $F(p)=A/(B+p^2)^2$, where A, B are constants and p is the nucleon momentum in the nucleus.

In Table I the results of Ishii's calculations are compared with those of Morinaga and Fry.[7]

The Chew-Goldberger distribution was proposed[10] in order to account for the deuteron pickup cross section. However, it is fairly clear now that it contains too high a proportion of high momenta and has been shown to be inadequate in further experiments.[11] Therefore, the results obtained with this distribution have no great significance. The Fermi gas distribution at finite temperature ($kT=9$ Mev) gives excellent agreement for α emission. The energy distribution of the emitted α particles, calculated by Ishii, is also in agreement with the measurements of Morinaga and Fry. This is

* Now at Department of Physics, Northwestern University, Evanston, Illinois.
[1] J. Tiomno and J. A. Wheeler, Revs. Modern Phys. **21**, 153 (1949).
[2] P. C. Gugelot, *Nuclear Reactions* (North-Holland Publishing Company, Amsterdam, 1959), Vol. I, Chap. IX. See here for further references.
[3] S. N. Kaplan, B. J. Moyer, and R. V. Pyle, Phys. Rev. **112**, 968 (1958).
[4] P. Singer (to be published).
[5] E. Lubkin, Ann. Phys. **11**, 414 (1960).
[6] E. P. George and J. Evans, Proc. Phys. Soc. (London) **A64**, 193 (1951).
[7] H. Morinaga and W. F. Fry, Nuovo cimento **10**, 308 (1953).
[8] C. Ishii, Progr. Theoret. Phys. (Kyoto) **21**, 663 (1959).
[9] See reference 8 for the details of the calculation, or references 3 and 4 for similar calculations of the excitation energy of the nucleus.
[10] G. F. Chew and M. L. Goldberger, Phys. Rev. **77**, 470 (1950).
[11] K. G. Dedrick, Phys. Rev. **100**, 58 (1955).

TABLE I. Charged particle emission by compound nucleus calculation.

Momentum distribution	Emitted particles per capture (%) alpha	protons
Chew-Goldberger	7.9	2.3
Fermi gas ($kT=0$)	$\sim$0.1	$\sim$0.02
Fermi gas ($kT=9$ Mev)	0.45	0.23
Experiment[a]	0.5	2.2

[a] See reference 7.

quite remarkable since the Fermi gas distribution at 9 Mev is almost identical with the exponential distribution,[12] $f(p)=\exp(-p^2/\alpha^2)$, with $\alpha^2/2M=14$ Mev, and the exponential distribution gives good agreement with the distribution of momenta in nuclei especially for the high components. Brueckner and co-workers[13] analyze several high-energy processes which are dependent on the momentum distribution of the nucleons, such as deuteron pickup, pion capture, high-energy proton-nucleus collision, nuclear photoeffect, and pion production in proton-nucleus collision, and show that all of them are satisfactorily explained by assuming the above-mentioned exponential distribution. This distribution is also obtainable from the Fourier transform of the nuclear-matter wave function, as shown by Brueckner[13] and Tagami[14] for the whole range of momenta.

It is thus seen that the same distribution can be used to account for the α-particle emission following muon capture, but only accounts for one tenth of the proton emission. This is still justified, as a statistical process is best suited for the α emission following muon capture. On the other hand, for proton emission, still other mechanisms are possible.

It is worthwhile to note at this point that in most nuclear reactions at comparable energies, the α-emission cross section is satisfactorily explained by the statistical process.[15] In contradistinction, in the same reactions, the proton emission is 10 (and sometimes 100) times higher than calculated from a compound nucleus and statistical emission process.[2,16]

In calculating the neutron emission following the muon capture, an effective mass smaller than the free-nucleon mass was used[3,4] in order to obtain agreement with experiment. The effective-mass approximation is not valid in the charged-particle case, since $M^*(p)\rightarrow M$ for momenta near the Fermi level,[17] and charged-particle emission takes place primarily from this region.

Another conceivable process, in addition to the statistical emission, is direct emission from the nucleus. This can occur when the neutron created in (1) interacts directly with a nuclear proton. Unfortunately, such a process seems highly improbable in the light of the results of Elton and Gomes.[18] They tried to explain the large cross section for inelastic scattering of protons with a few tenths of a Mev (which is ten times higher than calculated by assuming a statistical process) by looking for direct knock-on from the nucleus. It was found that the total reflection at the boundary of the nucleus essentially prevents a proton inside the nucleus from leaking outside. Consequently, this process gives a total cross section which is even smaller than for the compound nucleus process.

In the case under consideration, the proton emission is also ten times higher than expected from a statistical emission process. It will be shown that increased emission is obtained if one takes into account nuclear surface effects. Elton and Gomes[18] and Oda and Harada[19] also succeeded in accounting for the high experimental inelastic proton scattering cross section by considering the quasi-elastic scatterings with nucleons in the extreme outer shell of the nucleus.

2. NUCLEON CLUSTERS AT THE NUCLEAR SURFACE

Evidence shall be presented here for the existence of nucleon clusters at the nuclear surface. In Secs. 3 and 4 a model will be proposed which explicitly takes into account the existence of these clusters in calculating the muon capture. It will be shown that this model results in increased proton emission.

The nuclear surface is a low-density region. Therefore, the Pauli principle is less operative here in reducing correlations and accordingly there is a clustering tendency for the nucleons.

Several investigators have dealt with this problem. Tagami[20] has calculated the "healing distance"—the range in which the wave function for a nuclear pair imbedded in nuclear matter becomes identical with the wave function for a free pair—as a function of k, the relative momentum of the pair. He derives the S-state wave function of the pair by using the Bethe-Goldstone equation and assuming a potential consisting of a hard core with radius D. One then obtains, for the healing distance as a function of k,

$$\lambda(k)=D\left[\frac{\pi}{Dk_F}\left(2-K\ln\frac{1+K}{1-K}\right)^{-1}-1\right],\quad K^2\lesssim Dk_F, \quad (2)$$

where $K=k/k_F$, and k_F is the Fermi momentum.

This expression shows that for $k<0.6k_F$, the wave function of the pair is healed within a distance smaller than the mean distance between the nucleons in the

[12] E. Henley, Phys. Rev. **85**, 204 (1952).
[13] K. A. Brueckner, R. J. Eden, and N. Francis, Phys. Rev. **98**, 1445 (1955).
[14] T. Tagami, Progr. Theoret. Phys. (Kyoto) **21**, 533 (1959).
[15] C. B. Fulmer and B. L. Cohen, Phys. Rev. **112**, 1672 (1958).
[16] K. J. Le Couteur, *Nuclear Reactions* (North-Holland Publishing Company, Amsterdam, 1959), Vol. I, Chap. VII.
[17] K. A. Brueckner, *The Many-Body Problem* (Dunod, Paris, 1959), p. 48.
[18] L. R. B. Elton and L. C. Gomes, Phys. Rev. **105**, 1027 (1957).
[19] N. Oda and K. Harada, Nuclear Phys. **7**, 251 (1958).
[20] T. Tagami, Progr. Theoret. Phys. (Kyoto) **21**, 465 (1959).

nucleus. Thus, these nucleons move nearly independently. For k near k_F, $\lambda(k)$ is larger than the mean distance between nucleons, and therefore the pair behaves like a unit. From the local picture of a Thomas-Fermi gas one can conclude that the individual nucleon motion occurs throughout the nuclear volume, but in the surface region there is a strong tendency toward clustering.

The same conclusion is reached by da Providencia.[21] He calculates the energy and the wave function of a finite nucleus by a perturbation method. The perturbation is the difference between the true Hamiltonian

$$H^1=\sum_{i=1}^{A}(-\tfrac{1}{2}\nabla_i^2)+\sum_{i<j=1}^{A}v_{ij}, \tag{3}$$

and the "unperturbed" Hamiltonian

$$H=\sum_{i=1}^{A}(-\tfrac{1}{2}\nabla_i^2+V_i), \tag{4}$$

where V_i is the common average potential.

He then calculates the potential energy density which is divided into two parts, one correlation-free (v_1) and the other expressing the two-particle correlations (v_2). The graph of v_1/v_2 as a function of the distance from the center of the nucleus shows that the surface region is much more correlation-rich than the maximum density region.

Wilkinson[22] has pointed out the role the surface nucleon clusters play in K^- capture. As is known,[23] K^- capture by the nucleus occurs primarily from the $5g$ atomic level where the half-life for nuclear capture is ten times smaller than for the competing electromagnetic transition. This means that the capture takes place at a distance 2×10^{-13} cm from where the nuclear density is half the value at the nucleus center, i.e., 80% of the captures occur in a region containing less than 10% of the nuclear matter. When the K^- capture is by a single nucleon, the reaction is $K^-+N\rightarrow Y+\pi$, where Y is a hyperon with approximately 60 Mev energy. When a multinucleon capture occurs, the process is $K^-+2N\rightarrow Y+N$ and the hyperon has 150 Mev. The latest experiments[24] show that nearly 50% of the captures in AgBr are of the second type. This result is highly significant to our problem.

The experiment of Hodgson[25] on the (p,α) reaction should also be considered. According to him it seems that a surface nucleon of AgBr is found in a cluster 40% of the time.

[21] J. da Providencia, Proc. Phys. Soc. (London) **77**, 81 (1961).
[22] D. H. Wilkinson, Phil. Mag. **4**, 215 (1959).
[23] P. B. Jones, Phil. Mag. **3**, 33 (1958).
[24] M. Nikolic *et al.*, Helv. Phys. Acta **33**, 221 (1960).
[25] P. E. Hodgson, Nuclear Phys. **8**, 1 (1958).

3. MODEL FOR DIRECT PROTON EMISSION

In an improved independent-particle model, the effect of the correlations throughout the nuclear volume should be accounted for. This entails the use of a realistic momentum distribution and an effective mass for the nucleons,[3,4] the latter being due to the dependence of the average nuclear potential on the momentum.

The surface correlations require special treatment. Here, they manifest themselves mainly by nucleon clustering. The most frequent cluster is the two-nucleon one. When a proton in such a cluster captures a muon, the process cannot be treated as a simple one-particle capture as in (1). Because of the proximity of the second nucleon and the strong internucleon interaction, the capturing proton is immediately scattered by the second nucleon. Thus, the capture is really effected by the pair, not by an isolated nucleon. The other nucleons, which are relatively remote from the pair, essentially do not participate in the process. Therefore, the elementary capture process in this case is (N is a nucleon)

$$\mu^-+2N\rightarrow 2N'+\nu. \tag{5}$$

This kind of capture causes two energetic nucleons to appear at the nuclear surface, with a fair probability of direct escape. Naturally, the capture by an isolated surface proton will also give a neutron with a fair probability for direct emission, but this does not affect the proton emission. Capture by a two-proton cluster results in the appearance of a proton and a neutron, and we shall consider these protons when trying to account for the high observed proton emission.

As mentioned previously, Elton and Gomes have shown[18] that the large total reflection at the nuclear boundary practically prevents protons from escaping, once they are in the nucleus. A good probability of escape exists only for those protons which do not encounter this obstacle. The potential in the nuclear surface region is given by

$$V=V_C+V_N, \tag{6}$$

where V_C is the Coulomb potential and V_N the Woods-Saxon potential[19,26]

$$V_N(r)=-V_0\{\exp[(r-R_1)/d]+1\}^{-1}, \tag{7}$$

with

$$V_0=40\text{ Mev},\quad d=0.5\times10^{-13}\text{ cm},\qquad R_1=1.35A^{\frac{1}{3}}\times10^{-13}\text{ cm}. \tag{8}$$

The nuclear matter extends beyond R_c, which is defined by

$$V(R_c)=0. \tag{9}$$

In this classically forbidden region the problem of total reflection no longer exists. Therefore, we will assume, in analogy to Elton and Gomes, who calculated

[26] R. D. Woods and D. S. Saxon, Phys. Rev. **95**, 577 (1954); M. A. Melkanoff *et al.*, *ibid.* **101**, 507 (1956).

the scattering of protons by nucleons moving in this region, that only the protons appearing after captures by clusters in this region may be directly emitted from the nucleus.

The number of protons in this region can be estimated on the basis of the charge distribution in the nucleus, $\rho(x)$, obtained from high-energy electron-nucleus scattering[27]:

$$\rho(x)=\frac{Z}{4\pi r_1^3 N_0}\begin{cases}1-\frac{1}{2}\exp[-n(1-x)], & x<1\\ \frac{1}{2}\exp[-n(x-1)], & x>1\end{cases} \tag{10}$$

where $x=r/r_1$ and N_0 is a normalization constant (so that $4\pi\int\rho(r)r^2dr=Z$) and is given by

$$N_0=1/3+2/n^2+\exp(-n)/n^3; \tag{11}$$

n and r_1 are constants which must be specified for each nucleus. The potential created by this distribution is

$$V_C(r)=(e^2Z/r_1)J(x), \tag{12}$$

where

$$\begin{aligned} J(x)&=N_0^{-1}\left[\frac{1}{n^2}+\frac{1}{2}-\frac{1}{6}x^2+\frac{e^{-n}}{n^2}\left(\frac{1-e^{nx}}{nx}+\frac{1}{2}e^{nx}\right)\right], & x<1\\ &=\frac{1}{x}N_0^{-1}\left[e^{-n(x-1)}\left(\frac{1}{x}+\frac{n}{2}\right)\bigg/n^3\right], & x>1.\end{aligned} \tag{13}$$

We are interested in the Ag and Br nuclei which were studied by Morinaga and Fry.[7] The constants n and r_1 for these nuclei are taken from the report of Ford and Wills[28]:

$$\begin{aligned}\text{Ag: } & r_1=5.14\times10^{-13}\text{ cm}, & n=7.20;\\ \text{Br: } & r_1=4.63\times10^{-13}\text{ cm}, & n=6.06.\end{aligned} \tag{14}$$

By using (7), (12), (13), and (14) one finds for R_c

$$\begin{aligned}\text{Br: } & R_c=6.53\times10^{-13}\text{ cm};\\ \text{Ag: } & R_c=7.00\times10^{-13}\text{ cm}.\end{aligned} \tag{15}$$

It is now possible to find the number of protons, $\bar{N}$, in the region beyond R_c:

$$\bar{N}=\int_{R_c}^{\infty}\rho(\mathbf{r})d\mathbf{r}. \tag{16}$$

From (10) and (16) one obtains

$$\bar{N}=\frac{Ze^{-n(x_c-1)}}{2n^3N_0}[n^2x_c^2+2nx_c+2], \tag{17}$$

where $x_c=R_c/r_1$. One obtains:

$$\text{Br: } \bar{N}=1.48;\quad \text{Ag: } \bar{N}=1.52. \tag{18}$$

We will use an average value, $\bar{N}=1.5$, for both nuclei, as this is sufficiently accurate for our purpose, and we will confine our calculations to Ag only.

It is now necessary to calculate the capture probability of the muon by a quasi-free nucleon cluster moving in the nuclear surface region, and evaluate the number of directly emitted protons.

4. PSEUDODEUTERON MODEL CALCULATION

The two nucleon clusters are of three kinds with regard to composition, namely two-proton, two-neutron, and neutron-proton clusters. The second kind cannot absorb a muon, so it does not contribute to our problem. We shall call two-nucleon clusters, *pseudodeuterons*.[29] We shall assume that the pseudodeuterons are in an S-state. This is the state in which the nucleons are closest and makes the greatest contribution to our problem. It is also in accordance with the description of the nuclear interaction in terms of Serber forces which lead to nuclear interactions in even states only. We shall further distinguish between three types of pseudodeuterons: a proton-proton cluster which has to be in a singlet spin state, and a neutron-proton cluster which can be in either a triplet or singlet state. We shall designate the different pseudodeuterons as

$$\text{(a) } [p\text{-}p]^0,\quad \text{(b) } [n\text{-}p]^0,\quad \text{(c) } [n\text{-}p]^1, \tag{19}$$

where the superscript refers to the spin state.

The wave function of the capturing nucleus is written as

$$\Psi(1,2,\cdots,A)=\exp(i\mathbf{K}'\cdot\mathbf{R}')\psi(r)\varphi(3,\cdots,A), \tag{20}$$

where the system is not antisymmetrized between the nucleons $(3,4,\cdots,A)$ and the pseudodeuteron nucleons. This indicates the absence of interaction between the pseudodeuteron and the other nucleons. The wave function of the pseudodeuteron is written as a product of the center-of-mass motion part and the part dependent on the relative coordinates of the pseudodeuteron nucleons, $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$. The wave function φ does not change in the process and is thus eliminated from the calculations. The capturing pseudodeuteron is quasi-free, as the two nucleons are very close. The probability of other nucleons also being close is negligible. Formally, the matrix-element H_{fi} for capture is similar to the corresponding expression for muon capture by a deuteron.[30,31] (We will use the same notation as reference 31.) The probability of quantum transition in the first order of perturbation theory is

$$\omega=2\pi\sum\nolimits_f|H_{fi}|^2\rho_f,\quad (\hbar=c=1), \tag{21}$$

where ρ_f is the density of final states and where in the

[27] R. Hofstadter, Ann. Rev. Nuclear Sci. **7**, 231 (1957).
[28] K. W. Ford and J. G. Wills, Los Alamos Scientific Laboratory Report, LAMS-2387, 1960 (unpublished), and J. G. Wills (private communication).

[29] The term pseudodeuteron is occasionally used for a neutron-proton in an S-triplet-state nuclear cluster.
[30] A Rudik, Doklady Acad. Nauk SSSR **92**, 739 (1953).
[31] H. Überall and L. Wolfenstein, Nuovo cimento **10**, 136 (1958).

case under consideration

$$H_{fi}=\int \exp[-i\mathbf{K}\cdot(\mathbf{r}_1+\mathbf{r}_2)/2]F^*(\mathbf{q},\mathbf{r})J_f{}^\dagger\chi_f{}^\dagger \times[\tau_-(1)V(1)+\tau_-(2)V(2)]J_i\chi_i\psi(r)d\mathbf{r}_1d\mathbf{r}_2. \quad (22)$$

The calculation is carried out within the c.m. system of the pseudodeuteron. The final spatial wave function of the two nucleons is resolved into the c.m. motion with momentum $\mathbf{K}=\mathbf{p}_1+\mathbf{p}_2$ and a part F dependent on the relative coordinates and the relative momentum $\mathbf{q}=\frac{1}{2}(\mathbf{p}_1-\mathbf{p}_2)$. χ_i, χ_f are the initial and final spin functions and J_i, J_f are corresponding isospin functions. We have chosen the Universal Fermi Interaction of the $(A-V)$ type[32] and thus

$$V(i)=\exp[-i\mathbf{k}_\nu\cdot\mathbf{r}_i](S+\mathbf{T}\cdot\boldsymbol{\sigma})\varphi_\mu(\mathbf{r}_i), \quad (23)$$

$$S=C_V\psi_\nu{}^\dagger(1+\gamma_5)\psi_\mu, \quad \mathbf{T}=C_A\psi_\nu{}^\dagger(1+\gamma_5)\boldsymbol{\sigma}\psi_\mu, \quad (24)$$

where $\mathbf{k}_\nu$ is the momentum of the neutrino, φ_μ the spatial muon wave function, ψ_ν, ψ_μ the spinor wave functions of the neutrino and the muon, and C_V, C_A the coefficients of the vector and pseudovector interactions. τ_- is the isospin operator which converts a proton into a neutron and vanishes when operating on a neutron. As there is not enough information available on the wave functions of the different clusters, the same spatial wave function will be assumed for all three types of pseudodeuterons. The antisymmetrization requirement of the two-nucleon wave function, together with the assumption that $\psi(\mathbf{r})$ is in an S state, leads to the following wave functions for the pseudodeuterons in their c.m. system:

$$\begin{aligned}[p\text{-}p]^0&: J_p(1)J_p(2)\chi_S\psi(r),\\ [n\text{-}p]^0&: 2^{-\frac{1}{2}}[J_p(1)J_n(2)+J_n(1)J_p(2)]\chi_S\psi(r), \quad (25)\\ [n\text{-}p]^1&: 2^{-\frac{1}{2}}[J_p(1)J_n(2)-J_n(1)J_p(2)]\chi_T\psi(r),\end{aligned}$$

where χ_S, χ_T are the spin wave functions for the singlet and triplet states, respectively. In order to evaluate (21) we shall utilize, with the appropriate corrections, the calculation of the similar expression for the deuteron carried out by Überall and Wolfenstein.[31] They obtained the following expression for the muon capture probability by the deuteron[33]:

$$\omega_D=K_VI_t+K_A(2I_t+I_s), \quad (26)$$

$$I_i=\int pdpd\Omega\int J_i{}^*J_ik_\nu dk_\nu, \quad (27)$$

and

$$J_i=\int F_i{}^*(\mathbf{p}+\mathbf{k}_\nu/2,\mathbf{r})\exp[-i\mathbf{k}_\nu\cdot\mathbf{r}/2]\psi_D(r)d\mathbf{r}, \quad (28)$$

where i is either singlet (s) or triplet (t) and p the momentum of the observed neutron, and where

$$K_V=\frac{|C_V|^2}{\pi a_D{}^3}\frac{M}{4(2\pi)^4};\quad K_A=\frac{|C_A|^2}{\pi a_D{}^3}\frac{M}{4(2\pi)^4}. \quad (29)$$

a_D is the Bohr radius of the μ-mesonic atom of deuterium and M the nucleon mass.

Überall and Wolfenstein assumed that the two nucleons in the final state interact only when in an S state. They calculated ω_D with and without this assumption. In the first case, $F(\mathbf{q},\mathbf{r})$ is taken as an adequately symmetrized plane wave, and for the second possibility an interacting S state is added. They showed that inclusion of the interaction does not change the capture probability by more than 20%. Therefore, we shall neglect interaction in the final state and use the values obtained by Überall and Wolfenstein for this case.

If we antisymmetrize the wave function for the two nucleons following the muon capture, we obtain the following final states, summarized in Table II, by using the interaction (23), (24). The transitions induced by the vector interaction (Fermi type) are given in the column V and those induced by the pseudovector interaction (Gamow-Teller type) in the column A. The space wave function of the two nucleons is either an S or P state, while the spin state is indicated by the superscript.

In order to obtain the capture probability for the pseudodeuteron, we have to introduce three corrections in the expression for H_{fi} used by Überall and Wolfenstein in deriving (26):

(a) Instead of $\psi_D(\mathbf{r})$ we must use the spatial wave function $\psi(\mathbf{r})$ of the pseudodeuteron.

(b) We must use the wave functions χ and J appropriate to our case.

(c) Überall and Wolfenstein have used the value of the muon wave function at the origin $\varphi_{\mu,D}(0)$ in their derivation since the muon wave function hardly changes within the deuteron volume.

In this case the capture is done by protons in Ag which moves outside R_c defined in (9). The nuclear

TABLE II. Final states after muon capture by pseudodeuteron.

	Final state			
	V		A	
Initial state	S	P	S	P
$[n\text{-}p]^1$	—	$[n-n]^1$	$[n-n]^0$	$[n-n]^1$
$[n\text{-}p]^0$	$[n-n]^0$	—	—	$[n-n]^1$
$[p\text{-}p]^0$	$[n-p]^0$	$[n-p]^0$	$[n-p]^1$	$[n-p]^1$

[32] Corrections for strong interactions are neglected since they do not seriously affect the value of the capture probability. See reference 31.

[33] See reference 31 for full details on the derivation of (26) from (21) and (22), which involves appropriate summations and averages over spin states and the conservation of momentum and energy for the process.

density outside R_c decreases very rapidly, while $\varphi_{\mu,\mathrm{Ag}}$ changes much more slowly. Therefore, we shall approximate the muon wave function by its value at R_c, $\varphi_{\mu,\mathrm{Ag}}(R_c)$.

Accordingly, we have to multiply the result of Überall and Wolfenstein by $\alpha=|\varphi_{\mu,\mathrm{Ag}}(R_c)|^2/|\varphi_{\mu,\mathrm{D}}(0)|^2$.

We shall show, by using considerations first mentioned by Levinger[34] in his pseudodeuteron model calculation for the nuclear photoeffect, that we can compensate for the difference between ψ_D and ψ by multiplying the result for the deuteron capture by $\beta=|C_1|^2/|C_2|^2$, where C_1, C_2 are the normalization constants of ψ and ψ_D.[35]

By using the spin and isospin wave functions given in (25), and summing over the possible final states given in Table II, we obtain, bearing in mind the above considerations,

$$\omega_{[n\text{-}p]^1}=\alpha\beta[K_VI_t+K_A(2I_t+I_s)], \tag{30}$$

$$\omega_{[n\text{-}p]^0}=\alpha\beta[K_VI_s+2K_AI_t], \tag{31}$$

$$\omega_{[p\text{-}p]^0}=\alpha\beta[2K_VI_s+4K_AI_t]. \tag{32}$$

Now we shall justify the "β-constant approximation." We shall use for the pseudodeuterons the approximate Hulthén wave function for an S state due to a Yukawa potential.[34,36]

$$\psi(r)=\frac{(4\pi)^{\frac{1}{2}}}{(\alpha^2+k^2)^{\frac{1}{2}}v^{\frac{1}{2}}}\frac{\sin(kr+\delta)/\sin\delta-e^{-\mu r}}{r}. \tag{33}$$

$\mathbf{k}=\frac{1}{2}(\mathbf{k}_1-\mathbf{k}_2)$ is the wave number for the relative motion of the two nucleons and μ^{-1} is the nuclear force range. Outside this range, $\psi(r)$ tends to the S term of a plane wave and the normalization gives one pair in the volume v which in our case is the volume of the relevant region for capture. α^{-1} is the scattering length and δ the phase shift; they are related by

$$\cot\delta=-\alpha/k+\tfrac{1}{2}r_0k, \tag{34}$$

where r_0 is the effective range. We are interested in the case when the two nucleons are close, so we expand $\psi(r)$ for the range $kr\ll1$. One obtains

$$\psi(r)\simeq(4\pi/v)^{\frac{1}{2}}(\alpha^2+k^2)^{-\frac{1}{2}}r^{-1}(1-\alpha r-e^{-\mu r}). \tag{35}$$

The deuteron wave function from the effective-range theory is

$$\psi_\mathrm{D}(r)=[2\bar{\alpha}/(1-\bar{\alpha}r_0)]^{\frac{1}{2}}r^{-1}\times[\exp(-\bar{\alpha}r)-\exp(-\mu r)], \tag{36}$$

where $\bar{\alpha}^{-1}$ is the "deuteron radius" which is related to the scattering length of the triplet state by

$$\alpha=\bar{\alpha}-\tfrac{1}{2}\bar{\alpha}^2r_0. \tag{37}$$

At small distances,

$$\psi_\mathrm{D}(r)\simeq[2\bar{\alpha}/(1-\bar{\alpha}r_0)]^{\frac{1}{2}}r^{-1}(1-\bar{\alpha}r-e^{-\mu r}). \tag{38}$$

α and $\bar{\alpha}$ are nearly equal, and hence ψ and ψ_D are proportional in the relevant region. Indeed, it is a known result of the effective-range theory that the wave function is not very sensitive to the deuteron energy. This means that we can use the result of Überall and Wolfenstein for the deuteron by multiplying it by

$$\beta=2\pi(1-\bar{\alpha}r_0)/\bar{\alpha}\langle\alpha^2+k^2\rangle v.$$

$\langle\ \rangle$ means that we have to average over the different possibilities for $(\mathbf{k}_1-\mathbf{k}_2)$.

By using the values of K_V, K_A, I_s, and I_t given by Überall and Wolfenstein we obtain for the capture probability by the different pseudodeuterons, using (26), (30), (31), and (32),

$$\omega_{[p\text{-}p]^0}=1.74\omega_\mathrm{D}\frac{|\varphi_{\mu,\mathrm{Ag}}(R_c)|^2}{|\varphi_{\mu,\mathrm{D}}(0)|^2}\frac{2\pi(1-\bar{\alpha}r_0)}{\bar{\alpha}\langle\alpha^2+k^2\rangle v}, \tag{39}$$

$$\omega_{[n\text{-}p]^0}=0.87\omega_\mathrm{D}\frac{|\varphi_{\mu,\mathrm{Ag}}(R_c)|^2}{|\varphi_{\mu,\mathrm{D}}(0)|^2}\frac{2\pi(1-\bar{\alpha}r_0)}{\bar{\alpha}\langle\alpha^2+k^2\rangle v}, \tag{40}$$

$$\omega_{[n\text{-}p]^1}=\omega_\mathrm{D}\frac{|\varphi_{\mu,\mathrm{Ag}}(R_c)|^2}{|\varphi_{\mu,\mathrm{D}}(0)|^2}\frac{2\pi(1-\bar{\alpha}r_0)}{\bar{\alpha}\langle\alpha^2+k^2\rangle v}. \tag{41}$$

In order to calculate $\langle\alpha^2+k^2\rangle$ we shall assume the following momentum distribution $\rho(k_i)$ for the nucleons in the surface region of the nucleus

$$\rho(k_i)=C\exp(-k_i^2/b^2),\qquad b^2\hbar^2/2M=4\text{ Mev}. \tag{42}$$

This distribution is obtained[18] by assuming that the tail of the nuclear wave function is given by $\eta(r)=Ae^{-ar}/r$. A is a normalization constant and $a=(2MB)^{\frac{1}{2}}/\hbar$, where B is the separation energy equal to 8 Mev. If one takes the Fourier transform of

$$\eta(r)=0,\quad r<R_c;\quad \eta(r)=Ae^{-ar}/r,\quad r>R_c; \tag{43}$$

one gets

$$\eta(p)=\frac{4\pi A}{p(a^2+p^2)}(a\sin pR_c-p\cos pR_c)e^{-aR_c}. \tag{44}$$

The momentum distribution is given $\rho(p)=|\eta(p)|^2$. It can be shown[18] that a very good approximation for $\rho(p)$ is (42) with $b^2=a^2/2$. If we average $\langle\alpha^2+k^2\rangle$ with

---

[34] J. S. Levinger, Phys. Rev. 84, 43 (1951).

[35] It should be noted that in deriving (26) from (21), summation over the final states is effected with due allowance for conservation of energy, and the binding energy of the quasi-free pseudodeuteron is not necessarily equal to that of the deuteron. An additional calculation by Überall and Wolfenstein[31] for muon capture by a deuteron, using an average value for the neutrino energy, $\bar{k}_\nu$, and summing over the final states by using the completeness theorem, showed that a change of a few Mev in the binding energy does not appreciably affect the result, in view of the large value of $\bar{k}_\nu$. This means that our approximation should not involve misleading errors.

[36] E. G. Betrametti and G. Tomasini, Nuovo cimento 18, 688 (1960).

this distribution, one obtains (see Appendix)

$$\left\langle\frac{1}{\alpha^2+\frac{1}{4}|\mathbf{k}_1-\mathbf{k}_2|^2}\right\rangle=\frac{16\alpha}{(2\pi)^{\frac{3}{2}}b^3}\left\{\frac{\pi^{\frac{1}{2}}}{2\lambda^{\frac{1}{2}}}+\frac{\pi e^{\lambda}}{2}[\Phi(\lambda^{\frac{1}{2}})-1]\right\}, \quad (45)$$

where $\lambda=2\alpha^2/b^2$ and $\Phi(x)$ is the error integral. If we use[37] the value of the scattering length for the triplet state,[38] $\alpha=0.185\times10^{13}$ cm^{-1}, we obtain

$$\left\langle\frac{1}{\alpha^2+\frac{1}{4}|\mathbf{k}_1-\mathbf{k}_2|^2}\right\rangle=8.26\times10^{-26}\ \mathrm{cm}^{-2}. \quad (46)$$

Now we can calculate $\omega_{[p\text{-}p]^0}$ which is relevant for our purpose. Making the reasonable assumption[39] that the number of protons and neutrons is equal in the region outside R_c, we have on the average three nucleons there. We take for v,

$$v=[4\pi(1.2\times10^{-13})^3/3]A(3/A),$$

where the nuclear radius is given by $R=1.2\times10^{-13}A^{\frac{1}{3}}$ cm. Also[38] $r_0=1.70\times10^{-13}$ cm; $\bar{\alpha}=0.23\times10^{13}$ cm^{-1}. By using these constants, we obtain[40]

$$\beta=\frac{2\pi(1-\bar{\alpha}r_0)}{\bar{\alpha}\langle\alpha^2+k^2\rangle v}=5.2. \quad (47)$$

The value of the muon wave function at the origin of the deuteron system is

$$|\varphi_{\mu,\mathrm{D}}(0)|^2=\frac{1}{\pi a_{\mathrm{D}}^3}=\frac{1}{\pi}\left[\frac{\hbar^2}{m_e e^2}\frac{m_e(1+m_\mu/M_d)}{m_\mu}\right]^{-3}$$
$$=1.61\times10^{31}\ \mathrm{cm}^{-3}, \quad (48)$$

where m_e, m_μ, M_d are the masses of the electron, muon, and deuteron. The value for $\varphi_{\mu,\mathrm{Ag}}(R_c)$ is taken from the numerically calculated wave function for Ag of Ford and Wills[28]

$$|\varphi_{\mu,\mathrm{Ag}}(R_c=7\times10^{-13}\ \mathrm{cm})|^2=1.31\times10^{35}\ \mathrm{cm}^{-3}. \quad (49)$$

Using these numbers, we obtain for (39)

$$\omega_{[p\text{-}p]^0}=7.36\times10^4\omega_{\mathrm{D}}. \quad (50)$$

The calculated[31] value of ω_{D} is 88.23 sec^{-1} and the experimental value for muon capture[41] in Ag, $\omega_{\mathrm{Ag}}=112.5\times10^5$ sec^{-1}. Thus we obtain

$$\omega_{[p\text{-}p]^0}(\text{theoretical})/\omega_{\mathrm{Ag}}(\text{experimental})=0.58; \quad (51)$$

We shall now evaluate the number of directly emitted protons following a pseudodeuteron-type capture in Ag. In the capture region there are on the average 1.5 protons, e.g., 0.75 proton pairs. Statistically, one-fourth of them are in the relevant singlet state. We shall also introduce a parameter γ representing the time interval during which a proton singlet S state pair behaves like a "pseudodeuteron," i.e., when the protons are very close to each other. It shall also be assumed that the energetic proton created in the capture process leaves the nucleus in half the cases, i.e., when the momentum is in a direction away from the inner region. Then, the number of protons N_p directly emitted from Ag following pseudodeuteron capture in the region beyond R_c is

$$N_p=\gamma\times0.58\times0.75\times\tfrac{1}{4}\times\tfrac{1}{2}=0.055\gamma. \quad (52)$$

In order to account for the experimental results of Morinaga and Fry,[7] $N_p(\exp)=0.022$, we must put $\gamma=0.4$. This is in agreement with Hodgson's finding,[25] that the nucleon in the surface region spends 40% of its time in a cluster.

5. DISCUSSION

In this work, a model of direct interaction was proposed which may be responsible for most of the proton emission following muon capture.

As the compound nucleus picture is inadequate for an explanation of the experimental results, we had to look for another possible mechanism. Nucleon clustering at the nuclear surface seems to provide an explanation for the increased proton emission. This picture is also in agreement with K^- multinucleon capture in the nuclear surface region.

The calculations discussed in this paper are semi-qualitative and could be improved. Our main purpose was to ascertain whether the proposed mechanism could significantly improve the results for proton emission processes in muon capture by heavy nuclei. It seems that the answer is in the affirmative.

In order to improve the results further, more accurate pseudodeuteron wave functions should be used. The fact that the phase space available to the two nucleons is not exactly the same as for free particles should be taken into account. In fact, one of the nucleons enters the nucleus and therefore many states are forbidden to it by the Pauli principle. This will reduce our result. On the other hand, we assumed that in only half the cases did the proton leave the nucleus. Since the protons are outside R_c, it seems reasonable that a proton in this region should have a higher escape probability owing to Coulomb repulsion.

[37] As mentioned previously, no distinction is made between the spatial wave functions of the different pseudodeuterons, and we assume that they are fairly well described by the approximate triplet Hulthén S wave function.

[38] L. Hulthén and M. Sugawara, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, Chap. 1.

[39] L. Wilets, Revs. Modern Phys. **30**, 542, (1958).

[40] This result can be compared with the calculations for pion capture by pseudodeuterons throughout the nuclear volume. In this case the two-nucleon capture is imposed by the large amount of energy released, in order to have conservation of momentum. K. A. Brueckner, R. Serber, and K. M. Watson, Phys. Rev. **84**, 258 (1951) and N. C. Francis and K. M. Watson, Am. J. Phys. **21**, 659 (1953) evaluated the Γ factor giving the increased pseudodeuteron capture probability compared to the deuteron capture owing to the closeness of the nucleons. They obtained $\Gamma\simeq4-10$ which is comparable with our result (47).

[41] J. C. Sens, Phys. Rev. **113**, 679 (1959).

The surface pseudodeuteron capture should also affect the neutron emission. It has been shown[3,4] that most of the neutron emission occurs as evaporation from a compound nucleus. However, the theoretical result is 20% lower than the experimental emission. The disagreement can be partly explained by taking into account the neutron-pair emission following capture by the n-p pseudodeuterons.

Two energetic neutrons appear following such a capture. They could be emitted directly, which would give a multiplicity of two; or both of them could penetrate the nucleus which would then lead to a multiplicity of $\simeq 1.3$ as calculated for Ag from the compound nucleus model[4]; or one of them could be emitted directly while the other penetrated the nucleus, which would again lead to a multiplicity of at least 2. The most recent experimental result[42] is 1.55 ± 0.06 neutrons per muon capture in Ag. If we take into account the mechanism described above, the average emission calculated using the compound nucleus model would be increased.

The relevant captures are those by $[n\text{-}p]^0$ and $[n\text{-}p]^1$. The capture probability is obtained from (40), (41), (47), (48), and (49). We find that

$$\omega_{[n-p]^0}+\omega_{[n-p]^1}=\left(\frac{0.87}{4}+\frac{3}{4}\right)\times\frac{1.31\times10^{35}}{1.61\times10^{31}}\times5.2\omega_{\rm D}$$
$$=4.10\times10^4\omega_{\rm D}. \quad (53)$$

By using the values of $\omega_{\rm D}$ and $\omega_{\rm Ag}$ mentioned before, we obtain

$$(\omega_{[n-p]^0}+\omega_{[n-p]^1})/\omega_{\rm Ag}=0.32. \quad (54)$$

The number of neutron-proton pairs is $N\times P$, and for $N=P=1.5$ we have 2.25 pairs. We shall also assume that half the pairs are in an even-parity state and can be treated as pseudodeuterons. On the basis of these numbers, and using the same value for γ as in the proton emission, i.e., $\gamma=0.4$, we see that in $0.32\times2.25\times\frac{1}{2}\times0.4=14.4\%$ of the captures the muon is captured by a pseudodeuteron and approximately two neutrons are emitted. Combining this with the 1.27 average emission from compound nucleus processes in the other 85.6% of the cases, we obtain 1.37 neutrons emitted per capture in Ag. If the number of neutrons in the region $r>R_c$ is larger than assumed ($N=P$), then this figure can be increased still further.

To conclude, the approximate calculations presented show that there is a net surface effect causing an increased proton emission and having a non-negligible effect on the neutron emission.

ACKNOWLEDGMENT

The author wishes to express his sincere thanks to Dr. A. B. Volkov for his constant helpful advice, for fruitful discussions, and for critical reading of the manuscript.

[42] S. N. Kaplan (private communication, 1961).

APPENDIX

We have to find the average of the expression,

$$[\alpha^2+\tfrac{1}{4}|\mathbf{k}_1-\mathbf{k}_2|^2]^{-1},$$

when the normalized distributions of $\mathbf{k}_1$, $\mathbf{k}_2$ are

$$\rho(\mathbf{k}_i)d^3k_i=(1/\pi^{\frac{3}{2}}b^3)\exp(-k_i^2/b^2)d^3k_i. \quad (A1)$$

It is convenient to introduce new variables, $\mathbf{k}$ and $\mathbf{K}$, defined by

$$\tfrac{1}{2}(\mathbf{k}_1+\mathbf{k}_2)=\mathbf{K},\quad \tfrac{1}{2}(\mathbf{k}_1-\mathbf{k}_2)=\mathbf{k}. \quad (A2)$$

From the Jacobian of the transformation we have

$$d^3k_1d^3k_2=8d^3Kd^3k. \quad (A3)$$

Therefore,

$$\left\langle\frac{1}{\alpha^2+\frac{1}{4}|\mathbf{k}_1-\mathbf{k}_2|^2}\right\rangle=\frac{16\pi^2}{\pi^3b^6}8\int_0^\infty\exp(-2K^2/b^2)K^2dK$$
$$\times\int_0^\infty\exp(-2k^2/b^2)\frac{k^2}{\alpha^2+k^2}dk$$
$$=\frac{16\alpha}{(2\pi)^{\frac{1}{2}}b^3}\int_0^\infty\exp(-\lambda y^2)\frac{y^2}{1+y^2}dy, \quad (A4)$$

where we have put $k^2=\alpha^2y^2$ and

$$\lambda=2\alpha^2/b^2. \quad (A5)$$

We use the result[43] (for $\lambda>0$)

$$G=\int_0^\infty\frac{\exp(-\lambda x^2)}{x^2+1}dx=\frac{\pi}{2}e^\lambda[1-\Phi(\lambda^{\frac{1}{2}})], \quad (A6)$$

where Φ is the error integral, defined as

$$\Phi(x)=\frac{2}{\pi^{\frac{1}{2}}}\int_0^x\exp(-t^2)dt. \quad (A7)$$

Denoting the integral in (A4) by I, we see that

$$I=-dG/d\lambda. \quad (A8)$$

Finally,

$$\left\langle\frac{1}{\alpha^2+\frac{1}{4}|\mathbf{k}_1-\mathbf{k}_2|^2}\right\rangle$$
$$=\frac{16\alpha}{(2\pi)^{\frac{1}{2}}b^3}\left\{\frac{\pi^{\frac{1}{2}}}{2\lambda^{\frac{1}{2}}}+\frac{\pi e^\lambda}{2}[\Phi(\lambda^{\frac{1}{2}})-1]\right\}, \quad (A9)$$

obtained by using the expression for the derivative of the error integral,

$$\Phi'(x^{\frac{1}{2}})=(2/\pi^{\frac{1}{2}})e^{-x}. \quad (A10)$$

[43] W. Gröbner and N. Hofreiter, *Integraltafel* (Springer-Verlag, Berlin, 1950), Vol. 2, p. 66.