

Quantum Theory of Free Carrier Absorption

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The theory of free carrier absorption has been investigated to determine the adequacy of the current quantum theory at large photon energies, and the necessity of a quantum theory at small photon energies. It is found that even at appreciable photon energies virtual states in higher bands make a negligible contribution to the free carrier absorption. It is shown that the quantum theory becomes equivalent to the classical Boltzmann theory of free carrier absorption for photon energies that are small compared to the carrier energies, which is therefore a condition for the validity of the classical theory. The classical theory may, however, give reasonably good agreement with experiment over a much wider range of photon energies, since the inaccuracies involved in the classical theory of free carrier absorption tend to compensate each other. Hot electron effects on free carrier absorption are also discussed.

INTRODUCTION

THE quantum theory of free carrier absorption has been studied by several investigators. Fan *et al.*¹ originally derived expressions for the optical absorption due to free carriers in an attempt to explain the energy and temperature dependence of the absorption in semiconductors at relatively large photon energies. In their work, they pictured the process in which free carrier absorption occurs as a second-order transition involving both the absorption of the light quantum and scattering by either lattice vibrations or impurities.

The work of Fan *et al.* had two deficiencies. The first arose from the neglect of processes in which photons are emitted, although this does not lead to serious discrepancies for photons which are very energetic compared to the thermal energy kT . The second deficiency resulted from considering the terms in the electronic wave function representing the different sequences in which the electromagnetic and scattering perturbations may interact as giving rise to two separate processes. The amplitudes of the wave function representing the final state are coherent regardless of whether the interaction with the photon field or with the scattering mechanism occurs first, since the phases of the different intermediate states involved cancel out.

In more recent work by Lax and Rosenberg,² and also by the author,³ the emission of photons and the coherence of the different sequences of interaction have been taken into account. Lax and Rosenberg have shown that the agreement with experimental data in Ge is thereby improved.

In this paper we shall examine both the adequacy of and the necessity for the current quantum theory of free carrier absorption. The considerations involved will apply mainly to semiconductors, although the conclusion will have some validity for metals and semimetals. At relatively large photon energies we will test the adequacy of the present theory since, for photons of energy comparable to typical band separations, it is conceivable

that virtual transitions through intermediate states in higher bands may become important. We have found that due to a partial cancellation higher bands will usually not contribute significantly to free carrier absorption. We will also consider the validity of the classical Boltzmann expression for free carrier absorption in the region where the photon energy is comparable with the carrier energy. In order to do this we will show how the quantum theory goes over to the classical theory and what approximations are involved in making this transition. Finally, we will discuss the possibility of altering the free carrier absorption by changing the electron distribution with an electric field.

EFFECT OF OTHER BANDS ON FREE CARRIER ABSORPTION

The Hamiltonian H necessary for the description of the transitions occurring in free absorption is given by a sum of the following terms:

$$H = H^E + H^L + H^S + H^{EL} + H^{ES}, \quad (1)$$

where H^E , H^L , and H^S are, respectively, the unperturbed Hamiltonians of the electron in a perfect lattice, the photon or electromagnetic field, and the scattering mechanism. The scattering mechanism is necessary in the treatment of free carrier absorption for the conservation of crystal momentum since, although the emission or absorption of a photon significantly changes the energy of an electron, the same photon has a negligible momentum associated with it. The scattering mechanism may be lattice vibrations, ionized impurities, or some other imperfections. H^{EL} and H^{ES} are the interaction Hamiltonians involving both the coordinates of the electron and the coordinates of the electromagnetic modes or of the scattering mechanism, respectively. H^{EL} is given by⁴

$$H^{EL} = -(eA/mc)\hat{a} \cdot (\hbar/i)\nabla, \quad (2)$$

where $A\hat{a}$ is the electromagnetic vector potential with polarization vector \hat{a} . H^{ES} will remain unspecified, but we will assume that nearly elastic scattering is involved.

¹ H. Y. Fan, W. Spitzer, and R. J. Collins, Phys. Rev. **101**, 566 (1956).

² R. Rosenberg and M. Lax, Phys. Rev. **112**, 843 (1958).

³ W. P. Dumke (unpublished).

⁴ See, for instance, W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, New York, 1944), 2nd ed.

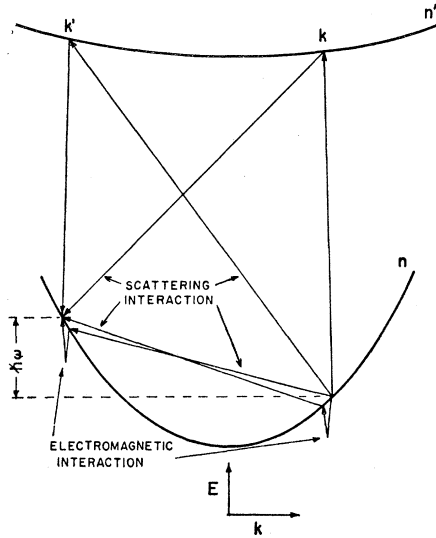


FIG. 1. States involved in the second order transition characteristic of free carrier absorption. The wave-vector change due to the photon has been somewhat exaggerated for the sake of clarity.

The optical absorption constant may be calculated from the time rate of change of the number of photons N in an electromagnetic mode. If n is the index of refraction, the absorption constant K is given by

$$K = -(n/c)(1/N)(dN/dt). \quad (3)$$

$-dN/dt$ is simply given by the sum of the probabilities $\Phi_{kk'}$ for transitions in which photons are absorbed minus a similar sum of the probabilities $\Phi_{kk'}^+$ for transitions in which photons are emitted; that is

$$-\frac{dN}{dt} = \int \int (\Phi_{kk'} - \Phi_{kk'}^+) f_k (1 - f_{k'}) \frac{2d\mathbf{k}d\mathbf{k}'}{(2\pi)^6}. \quad (4)$$

Let us now calculate the transition probability for a second-order transition between two states \mathbf{k} and \mathbf{k}' close to an energy minimum in the n th band. In addition to this band there will be other bands n' . The second-order probability amplitude $C_{nk'nk}$ in the final state $n\mathbf{k}'$ at a time t after turning the perturbations on is given by⁴

$$C_{nk'nk} = \frac{\exp[-i(E_{nk} - E_{nk'} \pm \hbar\omega)t/\hbar] - 1}{E_{nk} - E_{nk'} \pm \hbar\omega} M^\mp, \quad (5)$$

where

$$M^\mp = \frac{H_{nk'nk}^{ES} H_{nk'nk}^{EL}}{\mp \hbar\omega} + \sum_{n' \neq n} \frac{H_{nk'n'k}^{ES} H_{n'k'nk}^{EL}}{E_{n'k} - E_{nk} \mp \hbar\omega} + \frac{H_{nk'nk'}^{EL} H_{nk'nk}^{ES}}{\pm \hbar\omega} + \sum_{n' \neq n} \frac{H_{nk'n'k'}^{EL} H_{n'k'nk}^{ES}}{E_{n'k'} - E_{nk}}. \quad (6)$$

The first two terms in M correspond to treating the electromagnetic interaction first and therefore involve intermediate states of wave number almost identical with \mathbf{k} , the wave number of the initial state. The last two terms correspond to treating the scattering interaction first and therefore the wave number of the intermediate state is almost identical with \mathbf{k}' , the wave number of the final state. The two terms involving sums over n' correspond to the intermediate states being in higher bands. We have neglected to give explicitly the phases of the terms in M because they are identical for all four terms. This follows from the fact that, in every term, the same initial and final states and the same electromagnetic mode and scattering component are involved, while both the intermediate states and their complex conjugates always appear together. The contributions to the process, and the states involved, are pictured in Fig. 1.

The matrix elements in M for the electromagnetic interaction are given by a product of the matrix elements of the momentum operator between electronic states and of the vector potential between electromagnetic states. The nonvanishing matrix elements between electromagnetic states are⁴

$$H^{EL} = -\frac{e}{nm} \left(\frac{2\pi N}{\omega} \right)^{\frac{1}{2}} \mathbf{P} \cdot \hat{\mathbf{a}} \quad (7)$$

for the absorption of a photon, and

$$H^{EL} = -\frac{e}{nm} \left(\frac{2\pi \hbar(N+1)}{\omega} \right)^{\frac{1}{2}} \mathbf{P} \cdot \hat{\mathbf{a}} \quad (8)$$

for the emission of a photon. Because we are interested in only the induced-component of the emission we will ignore the 1 appearing in the matrix element for the emission process. The always present spontaneous emission just balances out the effects of transitions induced by the blackbody radiation.

For $n' \neq n$ there will be nonvanishing matrix elements of \mathbf{P} of zero order in \mathbf{k} and we will simplify the notation and represent these matrix elements by $\mathbf{P}_{n'n}$. The intraband matrix elements of \mathbf{P} vanish at an extremum and are given⁵ by $\mathbf{P}_{nk'nk} = m\mathbf{v}$, the group velocity of the state times the free electron mass.

We will assume that the scattering mechanisms that we are interested in are expandable in the Fourier components of $\mathbf{k}' - \mathbf{k} = \Delta\mathbf{k}$. That is,

$$H^{ES} = \sum_{\Delta\mathbf{k}} V_{\Delta\mathbf{k}} e^{i\Delta\mathbf{k} \cdot \mathbf{r}}. \quad (9)$$

Since we are interested in the scattering between states relatively close together in \mathbf{k} space the scattering matrix elements simplify in the following way:

$$H_{n'k'nk}^{ES} = V_{\Delta\mathbf{k}} \langle u_{n'k'} | u_{nk} \rangle. \quad (10)$$

⁵ See A. H. Wilson, *The Theory of Metals* (Cambridge University Press, New York, 1953), 2nd ed.

To lowest nonvanishing order in \mathbf{k} , the intraband matrix element is

$$H_{n\mathbf{k}'n\mathbf{k}} = V_{\Delta\mathbf{k}}. \quad (11)$$

For $n' \neq n$ we may expand, using first-order perturbation theory, the relevant $u_{n\mathbf{k}}$'s in terms of the u 's at \mathbf{k}' . The perturbation is $(\hbar/m)(\mathbf{k}-\mathbf{k}') \cdot \mathbf{P}$. For example, to obtain $H_{n'\mathbf{k}'n\mathbf{k}}$ we first express $u_{n\mathbf{k}}$ as

$$u_{n\mathbf{k}} = u_{n\mathbf{k}'} + \sum_{n' \neq n} \frac{\hbar(\mathbf{k}-\mathbf{k}') \cdot \mathbf{P}_{n'n}}{m(E_n - E_{n'})} u_{n'\mathbf{k}'}, \quad (12)$$

and since

$$\langle u_{n'\mathbf{k}} | u_{n\mathbf{k}} \rangle = \delta_{n'n},$$

we obtain

$$H_{n'\mathbf{k}'n\mathbf{k}}^{ES} = V_{\Delta\mathbf{k}} \frac{\hbar(\mathbf{k}-\mathbf{k}') \cdot \mathbf{P}_{n'n}}{m(E_{n'} - E_n)}. \quad (13)$$

Similarly,

$$H_{n\mathbf{k}'n'\mathbf{k}}^{ES} = V_{\Delta\mathbf{k}} \frac{\hbar(\mathbf{k}-\mathbf{k}') \cdot \mathbf{P}_{nn'}}{m(E_{n'} - E_n)}. \quad (14)$$

Finally, because \mathbf{P} is a Hermitian operator we have the useful relationship that

$$P_{n'n} = -P_{nn'}^*. \quad (15)$$

If we now substitute Eqs. (7)-(15) for the matrix elements in M , we obtain the following expression

$$M = \frac{e}{nm} \left(\frac{2\pi\hbar N}{\omega} \right)^{\frac{1}{2}} V_{\Delta\mathbf{k}} \left[\frac{(\mathbf{v}-\mathbf{v}') \cdot \hbar(\mathbf{k}-\mathbf{k}')}{\mp\hbar\omega} + \frac{1}{m^2} \sum_{n' \neq n} \frac{\mathbf{P}_{n'n} \mathbf{P}_{n'n}}{E_n - E_{n'}} \times \left(\frac{1}{E_{n'} - E_n \mp \hbar\omega} - \frac{1}{E_{n'} - E_n} \right) \right] \cdot \mathbf{a}. \quad (16)$$

Of the two terms in M , the first arises from considering intermediate states in the n th band and the second gives the effect of intermediate states in all of the other bands. This second term includes contributions from both sequences of the photon and scattering interactions. These two contributions are almost equal, but are of opposite signs with the result that most of the expected contribution to free carrier absorption due to intermediate electronic states in higher (or lower) bands cancels out.

The relative magnitudes of the two terms in M can be deduced as follows. The “ f ” sum⁵

$$\frac{1}{m} \sum_{n' \neq n} \frac{\mathbf{P}_{n'n} \mathbf{P}_{n'n}}{E_n - E_{n'}}$$

is equal to $\frac{1}{2}(m/m^* - 1) \cong \frac{1}{2}m/m^*$, where m^* is the effective mass tensor. Assuming parabolic bands, $\hbar(\mathbf{k}-\mathbf{k}') = m^*(\mathbf{v}-\mathbf{v}')$. If we replace $E_{n'} - E_n$ in the difference between the two energy denominators by some average value ΔE and assume that $\hbar\omega \ll \Delta E$, the second term becomes approximately

$$\frac{(\mathbf{v}-\mathbf{v}')}{2} \left(\frac{1}{\Delta E \mp \hbar\omega} - \frac{1}{\Delta E} \right) \cong \frac{1}{2} \frac{(\mathbf{v}-\mathbf{v}')}{\pm \hbar\omega} \left(\frac{\hbar\omega}{\Delta E} \right)^2,$$

which is smaller than the first term in M by a factor of $\frac{1}{2}(\hbar\omega/\Delta E)^2$. It can indeed be shown that for the “simple two-band model” of a semiconductor the second term in M vanishes. In any case, the contribution to free electron absorption of intermediate states in other bands should be negligible as long as $\hbar\omega \ll \Delta E$. At photon energies comparable to the gap energy, however, one would expect virtual states in higher bands to make a large contribution to free carrier absorption, which contribution could be calculated only if one had rather detailed knowledge of the band structure and scattering mechanisms.

RELATIONSHIP OF THE CLASSICAL AND QUANTUM THEORIES

Using the standard procedures of time-dependent perturbation theory, one may obtain from the probability amplitude in the final state the transition probability for transitions to this state from other electronic states. In the previous section we demonstrated that we may neglect higher bands n' , and we will therefore drop the band index. We will also assume an isotropic effective mass. The transition probability may therefore be written⁴

$$\begin{aligned} \mathcal{P}_{\mathbf{k}\mathbf{k}'} &= (2\pi/\hbar) M^2 \delta(E_{\mathbf{k}'} - E_{\mathbf{k}} \mp \hbar\omega) \\ &= \frac{2\pi}{\hbar} |V_{\Delta\mathbf{k}}|^2 \frac{e^2}{n^2} \frac{2\pi N}{\hbar\omega^3} [(\mathbf{v}-\mathbf{v}') \cdot \hat{\mathbf{a}}]^2 \\ &\quad \times \delta(E_{\mathbf{k}'} - E_{\mathbf{k}} \mp \hbar\omega). \end{aligned} \quad (17)$$

If we average over the directions of polarization $\hat{\mathbf{a}}$, and recognize that $(2\pi/\hbar) |V_{\Delta\mathbf{k}}|^2$ is the usual probability per unit time $W_{\mathbf{k}\mathbf{k}'}$ for scattering between states at the same energy, we may write

$$\mathcal{P}_{\mathbf{k}\mathbf{k}'} = W_{\mathbf{k}\mathbf{k}'} \frac{2\pi N \hbar e^2 (\mathbf{v}-\mathbf{v}')^2}{3 n^2 \omega^2 \hbar} \delta(E_{\mathbf{k}'} - E_{\mathbf{k}} \mp \hbar\omega). \quad (18)$$

The absorption constant, which may now be calculated using Eq. (4), is given by

$$\begin{aligned} K &= \frac{4\pi}{3} \frac{e^2}{n\hbar\omega^3} \int \int W_{\mathbf{k}\mathbf{k}'} |\mathbf{v}-\mathbf{v}'|^2 f_{\mathbf{k}} (1-f_{\mathbf{k}'}) \\ &\quad \times [\delta(E_{\mathbf{k}} - E_{\mathbf{k}'} + \hbar\omega) - \delta(E_{\mathbf{k}} - E_{\mathbf{k}'} - \hbar\omega)] \frac{d\mathbf{k} d\mathbf{k}'}{(2\pi)^6}. \end{aligned} \quad (19)$$

If in the second term of the integral we interchange the subscripts, which are symmetrical everywhere except in the δ functions, we obtain the somewhat simpler result that

$$\begin{aligned} K &= \frac{4\pi}{3} \frac{e^2}{n\omega^3 \hbar} \int \int W_{\mathbf{k}\mathbf{k}'} |\mathbf{v}-\mathbf{v}'|^2 (f_{\mathbf{k}} - f_{\mathbf{k}'}) \\ &\quad \times \delta(E_{\mathbf{k}} - E_{\mathbf{k}'} + \hbar\omega) \frac{d\mathbf{k} d\mathbf{k}'}{(2\pi)^6}. \end{aligned} \quad (20)$$

It is worthwhile to point out several features of this

expression for K . The probability of a transition depends upon the factor $(\mathbf{v}-\mathbf{v}')^2$ which reduces the frequency of transitions involving a small change in \mathbf{v} . In transport theory the calculation of the relaxation time for momentum transfer involves a similar factor $(1-\cos\theta)$, for weighing the effectiveness of a transition through an angle θ in \mathbf{k} space, since small-angle scattering does not reduce the current as effectively as does scattering through a larger angle.

If we assume that the photon energy is small compared to the energy of the carriers involved, then we may make the following approximation:

$$(\mathbf{v}-\mathbf{v}')^2 = v^2 + v'^2 - 2\mathbf{v}\cdot\mathbf{v}' \cong 2v^2(1-\mathbf{v}\cdot\mathbf{v}'/v^2). \quad (21)$$

The relaxation time for momentum transfer τ is given by the usual expression⁶

$$\frac{1}{\tau} = \int W_{\mathbf{k}\mathbf{k}'} \delta(E_{\mathbf{k}} - E_{\mathbf{k}'}) (1 - \mathbf{v}\cdot\mathbf{v}'/v^2) \frac{d\mathbf{k}}{(2\pi)^3}. \quad (22)$$

If we now assume that $\hbar\omega \ll E_{\mathbf{k}}$ and use Eqs. (21) and (22) in Eq. (20), we obtain

$$K = \frac{8\pi e^2}{3cn\hbar\omega^3} \int \frac{v^2}{\tau} [f(E) - f(E+\hbar\omega)] \frac{d\mathbf{k}}{(2\pi)^3}. \quad (23)$$

The factor $f(E) - f(E+\hbar\omega)$ accounts for the dependence of the absorption constant on the net absorption rate between levels at energies E and $E+\hbar\omega$. For photons of rather large energies $\hbar\omega \ll kT$ a negligible number of photon emission processes will be possible, and for a nondegenerate distribution it will be possible to neglect $f(E+\hbar\omega)$ in Eq. (23). At longer wavelengths where $\hbar\omega \sim kT$, it is necessary to retain $f(E+\hbar\omega)$ since photon emission processes will be of considerable importance. The neglect of emission processes by Fan *et al.*¹ resulted in a severe overestimation of the absorption constant at longer wavelengths. For $\hbar\omega \ll kT$, one may make the approximation that

$$f(E) - f(E+\hbar\omega) = -\frac{\partial f}{\partial E} \hbar\omega. \quad (24)$$

If we substitute Eq. (24) in Eq. (23) we obtain

$$K = \frac{8\pi e^2}{3cn\omega^2} \int \frac{v^2}{\tau} \frac{\partial f}{\partial E} \frac{d\mathbf{k}}{(2\pi)^3}, \quad (25)$$

which is identical to the classical expression⁷ for free carrier absorption obtained from the Boltzmann equation for $\omega\tau \gg 1$.⁸ One might therefore conclude that the

criterion for the applicability of the classical theory of free electron absorption would be for $\hbar\omega \ll kT$. While this criterion is correct for a classical carrier distribution such as in nondegenerate semiconductors, it is too stringent a requirement for a degenerate distribution where $kT \ll E_F$, the Fermi level. For a degenerate distribution the absorption constant in Eq. (23) becomes

$$K = \frac{8\pi e^2}{3cn\omega^2} \left[-\rho(E) \right]_{E_F}, \quad (26)$$

where $\rho(E)$ is the density of states in energy for carriers of one spin orientation. This is also the result of the classical Boltzmann theory of free carrier absorption for a degenerate distribution. If we have parabolic bands and designate the number of carriers per unit volume by N_c , then

$$K = \frac{4\pi}{nc} \frac{e^2 N_c}{\omega^2 \tau m^*}. \quad (27)$$

For a degenerate distribution then, the criterion for the applicability of the classical theory of free carrier absorption is that $\hbar\omega \ll E_F$.

An idea of the range of validity of the classical theory of free electron absorption may be obtained from the wavelengths of those photons whose energy is equal to kT or E_F . At room temperature $kT = 0.026$ eV which corresponds to the energy of a photon of wavelength 48μ . Fermi levels in degenerate semiconductors are typically of the order of 0.1 eV, which corresponds to a wavelength of approximately 12μ . In metals, the Fermi levels are characteristically of the order of several electron volts, so that the classical theory of free electron absorption should apply over a much wider range than in semiconductors.

In actual practice, however, the classical theory of free electron absorption may give fairly good agreement with experimental data for photons of much higher energy than our conditions for its validity would suggest. If we assume that the scattering probability $W_{\mathbf{k}\mathbf{k}'}$ is independent of \mathbf{k} and \mathbf{k}' , corresponding to isotropic scattering, then the differences between the classical and quantum theories of free carrier absorption stem from two sources. First, the replacement in the classical limit of $f(E) - f(E+\hbar\omega)$ by $-(\partial f/\partial E)\hbar\omega$ in the nondegenerate case and of $\int [f(E) - f(E+\hbar\omega)] d\mathbf{k}/(2\pi)^3$ by $\rho(E_F)\hbar\omega$ in the degenerate case both cause the classical theory to overestimate the net population of electrons contributing to absorption, for photon energies $\hbar\omega \approx kT$ or E_F , respectively. Another source of error arises from the fact that in the quantum theory, both the density of final states and optical matrix elements at the final states (for absorption processes) will increase with increasing photon energy. By comparison, the classical theory, which does not take into account the change of electron energy, underestimates the contribution to the absorption due to each electron.

⁶ See R. Peierls, *Quantum Theory of Solids* (Oxford University Press, New York, 1955), p. 119.

⁷ H. Y. Fan, in *Solid-State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1955), Vol. 1.

⁸ The condition that $\omega\tau \gg 1$ has its analog in the requirement of the present quantum theory that the line broadening be small compared to the photon energy. This requirement is necessitated by the neglect of line broadening effects, which neglect in turn makes the quantum theory of free carrier absorption inapplicable at very long wavelengths.

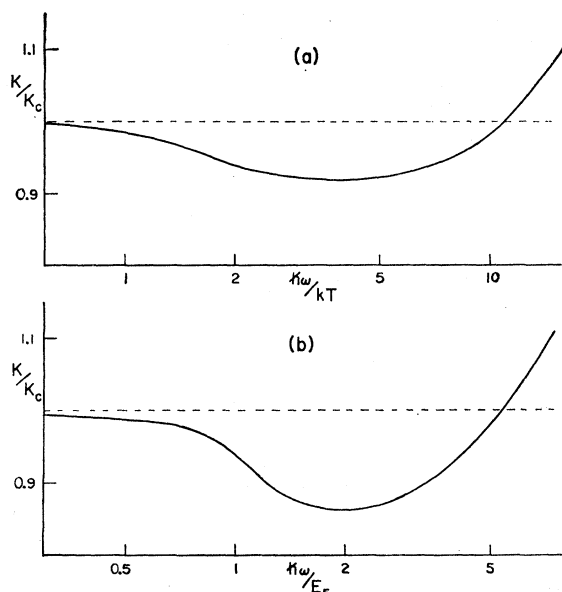


FIG. 2. Corrections to the classical theory of free carrier absorption for isotropic scattering, calculated from the quantum theory. Case (a) is for a classical electron distribution, and case (b) is for a degenerate distribution.

If one tries to develop the correction to the classical theory of free electron absorption in terms of powers of $\hbar\omega/kT$ or $\hbar\omega/E_F$ for the cases of a classical or a degenerate distribution, respectively, it is found that the linear terms in this correction are missing. This is because of the mutual compensation of the two types of error involved in the classical theory. By numerically integrating the exact quantum theoretical expression for free carrier absorption and comparing it with the classical result as a function of $\hbar\omega$ it can be shown that the classical theory predicts approximately the correct value for the absorption constant even for rather large value of $\hbar\omega$ (see Fig. 2). For a classical carrier distribution the correction that must be applied to the classical absorption constant is less than 8% out to $\hbar\omega = 15kT$. For a degenerate distribution the error is slightly greater, but does not exceed 14% out to $\hbar\omega = 8E_F$.

The accidental accuracy of the classical theory for isotropic scattering is, as we have said above, due to the cancellation of the errors involved in the classical theory. The isotropic scattering assumed corresponds fairly well to acoustic mode scattering. If we were to treat another scattering mechanism for which $W_{\mathbf{k}\mathbf{k}'}$ depends upon \mathbf{k} and \mathbf{k}' , such as impurity scattering,

there would be additional inaccuracies in the classical theory of free carrier absorption which would make the quantum treatment necessary for $\hbar\omega \sim kT$ or E_F depending on the electron distribution. For a dominantly small-angle scattering mechanism one would expect the absorption constant to be much smaller than classically predicted, since the addition of the photon energy to the carrier would place a lower limit on $\mathbf{k} - \mathbf{k}'$.

HOT ELECTRON EFFECTS

Before closing we would like to make a few remarks concerning the possibility of observing hot electron effects on free electron absorption, in an attempt to understand why such experiments⁹ have not met with success. We shall assume that the electron distribution can be roughly characterized by a Boltzmann distribution with an effective temperature T^* and we will discuss the cases for which $\hbar\omega \gg kT^*$ and $\hbar\omega \ll kT^*$. For $\hbar\omega \gg kT^*$ the energy $E_{\mathbf{k}'}$ of the final state will approximately be $\hbar\omega + kT^*$ and will not be very sensitive to variations in T^* . Similarly a variation in T^* will not affect greatly the density of final states $\rho(E + \hbar\omega)$ or the factor¹⁰ $\mathbf{v} - \mathbf{v}' \simeq \mathbf{v}$.¹⁰ As long as $\hbar\omega \gg kT^*$, $f(E) - f(E + \hbar\omega) \simeq f(E)$ and the integral of $f(E)$ over $d\mathbf{k}$ merely sums over the carriers, whose number is assumed to be constant. For large photon energies, therefore, the free electron absorption constant should not depend strongly on the electron temperature.

For $\hbar\omega \ll kT^*$ the dependence of free electron absorption on T^* can be deduced from Eq. (25). In this expression the average value of v^2 will vary as T^* and will just cancel the dependence on $1/T^*$ of $\partial f / \partial E$. The free carrier absorption should therefore vary with T^* as the energy dependence of $1/\tau$, the only remaining factor. For acoustic mode scattering and nonpolar optical mode scattering, $K \propto (T^*)^{1/2}$. For impurity scattering $K \propto (T^*)^{-1/2}$. Experimentally, however, one may have to use light of rather long wavelength to observe any dependence of absorption on electron temperature.

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⁹ A. F. Gibson and J. W. Granville (unpublished), communicated by J. B. Gunn.

¹⁰ For the case of polar scattering, the $v - v'$ dependence of the optical matrix elements is removed by the $(\mathbf{k} - \mathbf{k}')^{-1}$ dependence of the scattering matrix elements leaving only the weak dependence of the absorption on the final density of states.