Attempt to Determine the n-n Scattering Length from the Reaction D$(n,p)2n$

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The high-energy part of the proton spectrum at 4° from the reaction D$(n,p)2n$ at 14.4 Mev has been analyzed using the final-state interaction formalism, taking into account only the $n-n$ interaction in the final state. The primary interaction has been calculated in a kind of Born approximation. It has been shown that the shape of the high-energy part of the proton spectrum is dependent mostly on the $n-n$ scattering length, a_{nn}, and that by varying a_{nn}, a good fit to the shape of this part of the spectrum is obtained. From this analysis, a value of $a_{nn}=(-22\pm2)$f has been obtained. However, this value of a_{nn} is to some extent dependent on the accuracy of the approximations used. A more refined theoretical treatment would therefore be desirable.

1. INTRODUCTION

THERE has been much discussion on the difference between the $n-p$ and $p-p$ singlet scattering lengths (a_{np} and a_{pp}) and whether the purely nuclear part of the $n-p$ and $p-p$ interactions are equal.[1,2] Needless to say, this problem is of fundamental importance because of the hypothesis of charge independence of nuclear forces. The values of a_{np} and a_{pp} as derived from experiments are very accurate, but a_{pp} has to be corrected for the Coulomb interaction. This correction depends somewhat on the assumed shape of the nuclear potential, but the corrected values for different nuclear potentials, a_{pp}', still differ appreciably from a_{np}:

$$a_{pp}'\sim-17 \text{ f}, \quad (a_{pp}=-7.69 \text{ f}),$$
$$a_{np}=-23.7 \text{ f}.$$

Schwinger[3] attributed this difference to the magnetic interaction between nucleons and obtained a good agreement between a_{np} and a_{pp}' using the Yukawa potential. Salpeter[4] showed that the agreement is destroyed for all kinds of potentials if one uses the repulsive core. Besides, it has been pointed out by Salpeter[4] and later Riazuddin[5] that one should take into account the spread of the charge and magnetic moment of the nucleon, as has been indicated by recent electron-nucleon scattering data.[6] The calculations have shown that the electromagnetic correction is then too small to account for the discrepancy between a_{np} and a_{pp}'. However, Riazuddin[5] showed that the difference between a_{np} and a_{pp}' may be explained by taking into account the $\pi^0-\pi^{\pm}$ mass difference and using coupling constants ($g_{\pi^0}{}^2$ and $g_{\pi^{\pm}}{}^2$) differing by 1.5%. A knowledge of the $n-n$ scattering length would certainly help towards the understanding of this problem.

The measurement of a_{nn} has been a problem of long standing. Since it is at present practically impossible to measure a_{nn} in the same way as a_{np} and a_{pp}, that is by $n-n$ scattering, one is obliged to look for other less direct methods. It has been pointed out by Watson[7] and others[8,9] that the interactions between the final products of a reaction leading to three particles in the final state can influence strongly the energy and angular distribution of the products. Moreover, in the case of final-state interaction of two neutrons, the distribution of energy of their relative motion in the region of small relative momenta is dependent essentially on one parameter: the $n-n$ scattering length. This may be seen in the following way: The matrix element for the reaction may be written in the form[7]

$$M\sim\int\varphi_{2n}^*(r)F(r)r^2dr, \tag{1}$$

where $\varphi_{2n}(r)$ is the singlet S-wave part of the wave function of the two neutrons in the final state, and $F(r)$ denotes all other factors whose dependence on the relative momentum of the two neutrons is weak. In the zero-range approximation, which works well in this case, we can write for the two-neutron wave function:

$$\varphi_{2n}(r)\sim\sin k''r/k''r+f(k'')e^{ik''r}/r, \tag{2}$$

where k'' is the relative wave number of the two neutrons and $f(k'')$ is the $n-n$ scattering amplitude. We shall be interested in the form of the matrix element for very small k''. Since $F(r)$ is large essentially only within the volume of the primary interaction of the incoming neutron with the deuteron, r is effectively limited to a few R_d, where R_d is the radius of the deuteron. The condition $k''\ll10^{13}$ cm^{-1} is then equivalent to $k''r\ll1$ so

[1] See, for example, L. Hulthén and M. Sugawara, *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 5, and H. A. Bethe and F. de Hoffman, *Mesons and Fields* (Row, Peterson and Company, Evanston, Illinois, 1956), Vol. 2, and references therein.
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that we can write:

$$M\sim\int[r+f(k'')]F(r)rdr =[\bar{r}+f(k'')]\int F(r)rdr, \quad (3)$$

where

$$\bar{r}=\int rF(r)rdr/\int F(r)rdr.$$

Here $\bar{r}$ can be taken as a quantity which measures the extension of the function $F(r)r$. It has, in general, a value close to that of the radius of the deuteron, i.e., $\bar{r}\sim 4$ f. Since the $n-n$ scattering amplitude approaches the value of ~ 20 f as $k''\rightarrow 0$, it will be the main factor in determining the energy dependence of M. We can get a rough idea of this dependence by neglecting $\bar{r}$ in comparison with $f(k'')$ and taking the form of $f(k'')$ for small k'':

$$f(k'')=(1/k'')e^{i\delta(k'')}\sin\delta(k''),$$
$$k''\cot\delta(k'')=-1/a_{nn}\quad\text{for}\quad k''\quad\text{small}.$$

The cross section is then

$$d\sigma\sim|M|^2\rho(k'')\sim|f(k'')|^2\rho(k'') \sim[a_{nn}^2/(1+k''^2a_{nn}^2)]\rho(k''), \quad (4)$$

where $\rho(k'')$ is the energy density of the final states. It should be realized that this is only a rough approximation for $\bar{r}\sim 4$ f and $a_{nn}\sim 20$ f, and the effect of $\bar{r}$ on the cross section is not negligible. As an interesting consequence of the effect of $\bar{r}$, it may be seen that the cross section is not the same for $+a_{nn}$ and $-a_{nn}$. However, as a_{nn} becomes very large, the two cross sections become the same.

Any reaction involving in the final state two neutrons plus another particle can therefore provide information on the $n-n$ scattering length if the energy distribution of relative motion of the two neutrons can be measured. In most cases, this distribution is measured indirectly by measuring the energy distribution of the third particle. There have been several attempts to get information on a_{nn} in this way. Phillips and Crowe[10] have measured the γ spectrum from $\pi^-+D\rightarrow 2n+\gamma$. They were able to establish $a_{nn}=-15.9$ f, corresponding to a dineutron unbound by approximately $\epsilon=\hbar^2/ma_{nn}^2=160$ kev with limits $a_{nn}=-8.5$ f and $a_{nn}=-\infty$, where m is the nucleon mass. They also concluded that a bound dineutron of more than 50-kev binding energy is less than 0.1% probable. Brolley *et al.*[11] studied the reaction $T(d,He^3)2n$, and the spectrum of He^3 has been analyzed by Komarov and Popova[12] who used $\epsilon=70$ kev ($a_{nn}\sim 24$ f) to fit the data. However, due to the lack of knowledge of the exact triton and He^3 wave functions, and to the considerable uncertainties of the data, it was not possible to determine a_{nn} with any certainty. Finally, one should mention that there have been considerable efforts in the past in the search for a bound state of the dineutron. The results were all negative.[13]

We present in this paper an analysis of the proton spectrum from the reaction $D(n,p)2n$ in an attempt to determine the $n-n$ scattering length. This reaction has the advantage of involving only single nucleons in the final state, which facilitates the theoretical treatment considerably.

2. CALCULATIONS

In the proton spectrum at 4° from the reaction $D(n,p)2n$ at[14] 14.4 Mev (see Fig. 1), there are two pronounced peaks. The peak near the maximum energy, $E_{\max}$, corresponds kinematically to the process in which the two neutrons are recoiling backwards in the c.m. system with small relative momentum. The lower energy peak near 5.6 Mev corresponds kinematically to the proton and one neutron moving forward with small relative momentum in the c.m. system and the other neutron recoiling backwards. The interaction between two particles with small relative momentum in the final state must be considered if these peaks are to be explained.

The calculations have been performed using a kind of Born approximation in which the primary interactions between the incident neutron and nucleons in the deuteron are treated as the perturbing potential. In the final state, however, a square-well interaction between the two neutrons is taken into account exactly. The interaction between the proton and either neutron in the final state has been neglected since we sought only to fit the high-energy part of the spectrum where information about $n-n$ interaction may be obtained. The effect of the $n-p$ final-state interaction in this region where the two neutrons have small relative momenta is expected to be small, for the proton must have large relative momentum with respect to either neutron and the time during which $n-p$ interaction comes into play is very short. This procedure, which takes into account only the interaction of one pair of particles in the final state at a time, has been applied previously to a number of similar problems.[2,15]

Let us denote the neutron and proton in the deuteron, and the incident neutron by numbers 1, 2, and 3, respectively. Then the respective positions of these particles are specified by vectors $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$. Let E be

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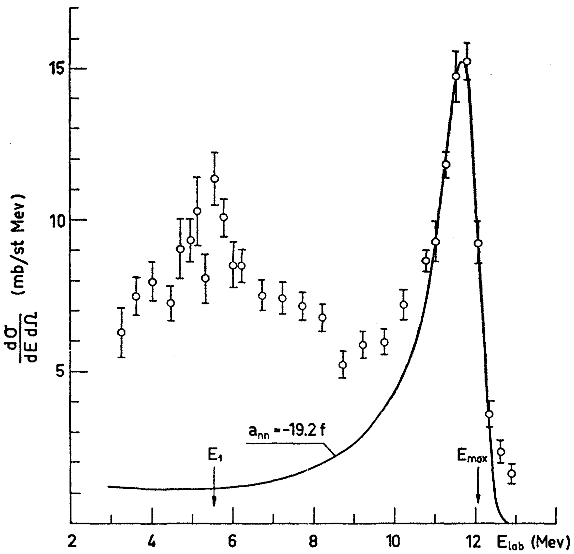

FIG. 1. The experimental data for $D(n,p)2n$ at 14.4 Mev and at 4° in the laboratory system compared with the calculated spectrum using $a_{nn}=-19.2$ f after smearing and normalization. E_{max} indicates the maximum proton energy for $Q=-2.225$ Mev. This corresponds to the case when the two neutrons are moving together with zero relative energy backwards in the center-of-mass system. E_1 indicates the proton energy in the case where a neutron and the proton are going together with zero relative energy in the forward direction in the center-of-mass system.

the total kinetic energy in the c.m. system before reaction, E' the total kinetic energy in the c.m. system after reaction, and E'' the relative energy of the system of two neutrons. Then energy conservation gives

$$E'+E''=E-\epsilon_d, \tag{5}$$

where ϵ_d is the binding energy of the deuteron. In terms of wave numbers, (5) reads:

$$k'^2+\tfrac{4}{3}k''^2=k^2-\tfrac{4}{3}\alpha^2,$$

where

$$\alpha^2=(m/\hbar^2)\epsilon_d,$$
$$k^2=(4m/3\hbar^2)E,$$
$$k'^2=(4m/3\hbar^2)E'=\tfrac{3}{2}(4m/3\hbar^2)E_p,$$
$$k''^2=(m/\hbar^2)E''.$$

The cross section is given by

$$d\sigma=(2\pi/\hbar)(2m/3\hbar k)|M|^2\rho(k''), \tag{6}$$

where $\rho(k'')$ is the energy density of final states,

$$\rho(k'')=(1/2\pi)^6k'^2dk'd\Omega'k''^2dk''d\Omega''.$$

In the case where interactions between all pairs of nucleons in the final state are considered, and taking into account only S states of relative motion,

$$M^2=\tfrac{2}{3}\times\tfrac{1}{4}\sum_{i=1}^{4}\sum_{f=1}^{8}|\langle(1-P_{13}Q_{13})\psi_f\chi_fV\varphi_i\chi_i\rangle|^2$$
$$+\tfrac{1}{3}\times\tfrac{1}{2}\sum_{i=5}^{6}\sum_{f=1}^{8}|\langle(1-P_{13}Q_{13})\psi_f\chi_fV\varphi_i\chi_i\rangle|^2. \tag{7}$$

P_{13} and Q_{13} are the space and spin exchange operators for particles 1 and 3. $(1-P_{13}Q_{13})$ appearing here takes into account antisymmetrization of both the initial and final wave functions with respect to the two neutrons. ψ_i and ψ_f are the initial and final space wave functions, respectively, and χ_i and χ_f the initial and final spin wave functions, respectively. φ_i is the solution of the Schrödinger equation for

$$H_0=T_1+T_2+T_3+V_{12}.$$

The perturbing potential is

$$V=V_{13}+V_{23}.$$

The T's are the kinetic energy operators and V_{ij} the interaction potential between particles i and j. The subscripts 1 to 4 denote the quartet states belonging to the total angular momentum of the whole system and spin of the two-particle system $(\frac{3}{2},1)$, respectively. The subscripts 5 and 6 denote the doublet states of $(\frac{1}{2},1)$ while 7 and 8 denote the doublet states of $(\frac{1}{2},0)$.

It may be shown on taking the appropriate normalized spin functions and applying the P_{13} and Q_{13} operators that (7) becomes

$$|M|^2=\tfrac{2}{3}|M_1|^2+\tfrac{1}{3}|M_2|^2+\tfrac{1}{3}|M_3|^2, \tag{8}$$

where M_1, M_2, and M_3 are the matrix elements corresponding to $(\frac{3}{2},1)$, $(\frac{1}{2},1)$, and $(\frac{1}{2},0)$ states, respectively. We are considering only the final-state interaction between the two neutrons and since it comes only in $|M_3|$,

$$|M|^2=\tfrac{1}{3}|M_3|^2,$$

where

$$M_3=\frac{\sqrt{3}}{2}\int\varphi_{2n}^*(|\mathbf{r}_1-\mathbf{r}_3|)e^{-i\mathbf{k}'\cdot[\mathbf{r}_2-\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_3)]}$$
$$\times[V_{13}(|\mathbf{r}_1-\mathbf{r}_3|)+V_{23}(|\mathbf{r}_2-\mathbf{r}_3|)]$$
$$\times\varphi_d(|\mathbf{r}_1-\mathbf{r}_2|)e^{i\mathbf{k}\cdot[\mathbf{r}_3-\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)]}d\tau. \tag{9}$$

$\varphi_d(|\mathbf{r}_1-\mathbf{r}_2|)$ is the Hulthén wave function for the ground state of the deuteron,

$$\varphi_d(|\mathbf{r}_1-\mathbf{r}_2|=N\frac{e^{-\alpha|\mathbf{r}_1-\mathbf{r}_2|}-e^{-\beta|\mathbf{r}_1-\mathbf{r}_2|}}{|\mathbf{r}_1-\mathbf{r}_2|},$$

where $N=[\alpha\beta(\beta+\alpha)/2\pi(\beta-\alpha)^2]^{\frac{1}{2}}$, and $\beta=7\alpha$. The final-state wave function for the two neutrons has been taken as a square-well S-wave function,

$$\varphi_{2n}=\frac{A\sin(K''|\mathbf{r}_1-\mathbf{r}_3|)}{|\mathbf{r}_1-\mathbf{r}_3|}\quad\text{for}\quad|\mathbf{r}_1-\mathbf{r}_3|\leqslant b,$$
$$=(4\pi)^{\frac{1}{2}}\left[\frac{\sin(k''|\mathbf{r}_1-\mathbf{r}_3|)}{k''|\mathbf{r}_1-\mathbf{r}_3|}+f\frac{e^{ik''|\mathbf{r}_1-\mathbf{r}_3|}}{|\mathbf{r}_1-\mathbf{r}_3|}\right]$$
$$\text{for}\quad|\mathbf{r}_1-\mathbf{r}_3|>b,$$

where b is the range of the square-well potential. The

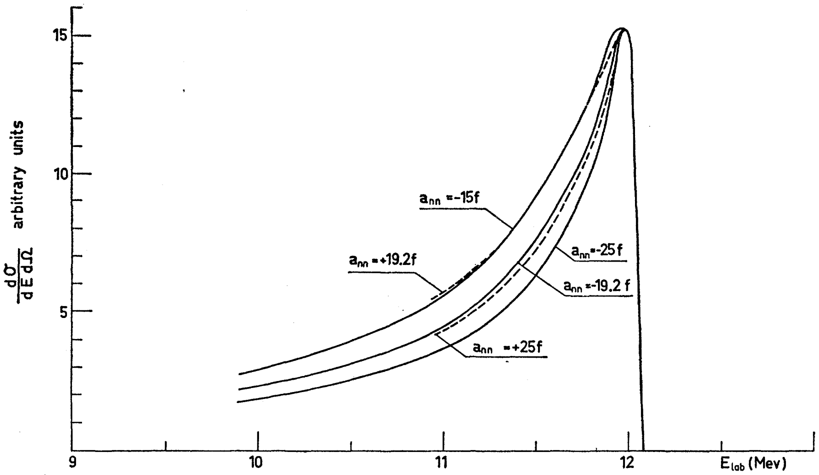

FIG. 2. Five theoretical spectra of protons from the reaction $D(n,p)2n$ at 14.4 Mev at 4° in the laboratory system. These spectra have been calculated using the final-state interaction formalism, taking into account only the interaction between the two neutrons as described in Sec. 2.

use of the square-well wave function is certainly an approximation. The effect of the approximation may be estimated by comparing the values of M_3 calculated with the square-well wave function and with the asymptotic wave function. The resulting change in shape of the curve which may be measured in terms of the ratio of the values of M_3 at 11 and 11.7 Mev has been found to be 3%. In the effective-range approximation,

$$f=\left[-\frac{1}{a_{nn}}+\tfrac{1}{2}r_0k''^2+ik''\right]\left[k''^2+\left(-\frac{1}{a_{nn}}+\tfrac{1}{2}r_0k''^2\right)^2\right]^{-1},$$

where r_0 is the effective range. When f is determined by the parameters a_{nn} and r_0, the outside wave function is given. By matching the outside and inside wave functions at $|\mathbf{r}_1-\mathbf{r}_3|=b$; A and $K''=[m(E''+V_{nn})]^{\frac{1}{2}}/\hbar$ may be determined. Here V_{nn} is the depth of the singlet $n-n$ interaction potential.

For the perturbing potential $(V_{13}+V_{23})$, V_{13} has been taken to be a square well with depth V_{nn} and range b consistent with the $n-n$ interaction potential used in the calculation of φ_{2n}. However, for ease of calculation, V_{23} has been chosen in the form of a δ function:

$$V_{23}=V_{np}\delta(|\mathbf{r}_2-\mathbf{r}_3|).$$

V_{np} has been determined by comparing a calculation of $n-p$ elastic scattering cross section using Born approximation and $V_{np}\delta(r)$ with the experimental cross section at the same incident energy.

TABLE I. Values of parameters chosen in the calculation.

a_{nn} (fermis)	r_0 (fermis)	b (fermis)	V_{nn} (Mev)	V_{np} (Mev)
−15	2.83	2.65	12.78	15.49
−19.2	2.80	2.65	13.15	15.49
−25	2.76	2.65	13.42	15.49
+19.2	2.51	2.65	16.20	15.49
+25	2.53	2.65	15.88	15.49

3. COMPARISON WITH EXPERIMENT

Figure 2 shows five calculated spectra of protons for the reaction $D(n,p)2n$ at 4° in the laboratory system. All curves have been normalized to the same value at the peak. In Table I, we list the values which were chosen for the parameters in the calculation. The value of V_{np} is determined by the experimental $n-p$ elastic scattering cross section. In the effective range theory, only two of the four parameters a_{nn}, r_0, b, and V_{nn} are free. The parameters a_{nn} and r_0 were taken as free and their values were chosen in such a way that the resulting value of b was constant in order to simplify the numerical calculation. It is clear that the influence of r_0 on the scattering amplitude f is small for small relative momentum of the two neutrons, the dominant contribution to f coming from the $-1/a_{nn}$ term in the effective-range expansion. Therefore the resulting spectra depend mostly on a_{nn}. This has also been clearly demonstrated by the calculations of McVoy.[2] The value of $a_{nn}=-19.2$ f used for one of the curves was calculated from the known a_{np} and a_{pp} values using the Schwinger procedure,[4] that is to say, magnetic interaction has been corrected for but no account was taken of the spread of the magnetic moment. This procedure is known to be inaccurate for reasons already mentioned in the Introduction, but this value was taken only as a starting point in the calculation.

In order to compare the theoretical calculation to the experiment, the theoretical curves have to be smeared with the experimental resolution of the counting system. This has been done by making use of the experimental width of the proton peak obtained by $n-p$ elastic scattering from a polythene target. Since the absolute values of the theoretical curves are an order of magnitude too big, they have been separately normalized after smearing to the experimental points. Figures 1 and 3 show the smeared and normalized curves together with the experimental data. It may be noted that although normalization has been necessary,

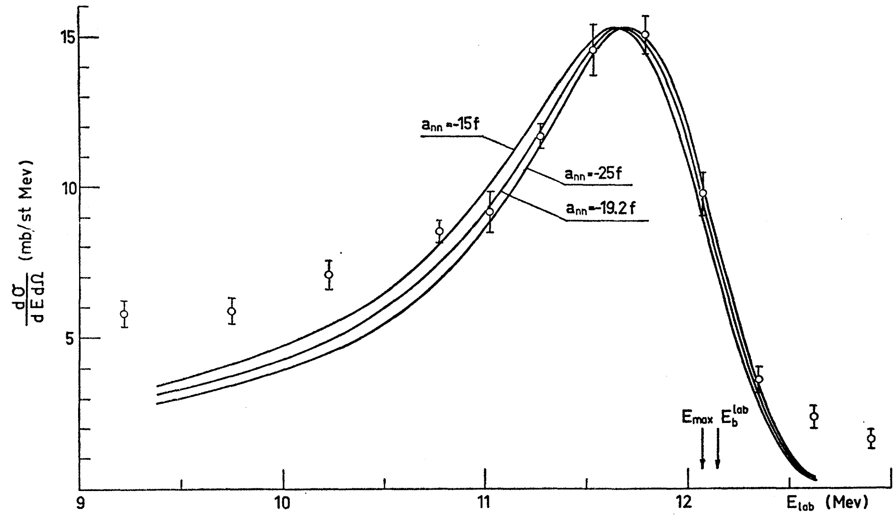

FIG. 3. The experimental data for $D(n,p)2n$ at 14.4 Mev and at 4° in the laboratory system compared with three calculated spectra which have been smeared with the experimental resolution and separately normalized. The three calculated spectra correspond to $a_{nn}=-15$ f, $a_{nn}=-19.2$ f, and $a_{nn}=-25$ f. E_b^{lab} indicates the proton energy in the laboratory system at 4° corresponding to a bound di-neutron of 66-kev binding energy.

no adjustment has been performed on the energy scale.

Only the theoretical curves for a_{nn} negative have been compared with the experimental data. The curves for positive values of a_{nn} have been discarded on the grounds discussed in Sec. 4.

4. DISCUSSION AND CONCLUSION

It may be seen that there is a distinct difference among the shapes of the three calculated spectra and that by varying the parameter a_{nn}, a good fit to the experimental points may be obtained in the region of energy 11–12.3 Mev. The discrepancy in the absolute value of the experimental and calculated cross sections is most probably due to the use of Born approximation. On the other hand, it has been argued by Watson[7] that the expression (4) in our Sec. I is approximately valid in a more general case. Hence one could deduce the value of a_{nn} just from the shape of the spectrum. Watson's arguments are certainly likely to hold in the case when one can neglect $\bar{r}$ of Eq. (3) in comparison with f, the $n-n$ scattering amplitude. In most practical cases, however, the contribution of $\bar{r}$ is far from negligible implying that the shape of the spectrum is to a certain extent dependent on the shape of $F(r)$ [Eq. (3)], i.e., on the approximation one uses. More theoretical work will be required to show just how much the shape of the spectrum depends on the form of $F(r)$ and also how good the Born approximation is in this respect.

In view of what has been said above, the accuracy with which a_{nn} has been determined from our experiments will probably be considerably worse than what one could tentatively conclude from the statistical analysis.

The value of a_{nn} has been determined by fitting the theoretical curves to the experimental points in the energy region 11–12.3 Mev. By varying both the a_{nn} and the normalization factor, the values of $a_{nn}=(-22\pm 2)$f has been obtained. The error given is purely statistical and does not contain any estimates of the accuracy of the theory.

Below 11 Mev, the experimental points lie far above the theoretical curves. This is to be expected since, at larger relative momenta between the two neutrons, the formalism which takes into account only the final-state $n-n$ interaction is no longer adequate. In the low-energy region, the $n-p$ final-state interaction becomes dominant. Therefore, we took only a limited region corresponding to relative momentum between the two neutrons $k''\leqslant 0.15\times 10^{13}$ cm^{-1} in the determination of a_{nn}. This is where the theory should be most accurate.

It has been seen that only the curves for a_{nn} negative were compared with the experiment, though Fig. 2 shows that a good fit could be obtained also for a positive a_{nn} of around $+25$ f. A positive a_{nn}, however, implies the existence of a bound dineutron. If such a structure existed, one would obtain the same spectrum as the present one with a monochromatic line at the proton c.m. energy $E_b^{\text{c.m.}}=E_{\max}^{\text{c.m.}}+\frac{2}{3}\epsilon$. The ratio of the cross section leading to the bound state $(d\sigma/d\Omega)_b$ and the cross section $d\sigma/d\Omega dE_p$ to an unbound state is given by Migdal[9]:

$$\left(\frac{d\sigma}{d\Omega}\right)_b^{\text{c.m.}} \bigg/ \left(\frac{d\sigma}{d\Omega dE_p}\right)^{\text{c.m.}} = \frac{(6^{\frac{1}{2}})\pi m^{\frac{3}{2}}}{h^3} \times\left(\frac{E''E'}{E_{\max}^{\text{c.m.}}+\frac{2}{3}\epsilon}\right)^{\frac{1}{2}} \frac{|\varphi_{\text{dineutron}}|^2}{|\varphi_{2n}|^2}.$$

From this we obtained $(d\sigma/d\Omega)_b^{\text{lab}}=38$ mb/sr, using $a_{nn}=+25$ f corresponding to a binding energy ϵ of approximately 66 kev. The energy at which the line is expected is indicated in Fig. 3 by E_b^{lab}. Such an intense group could easily be observed in our experiment. Since we have not observed it, we conclude that there is no bound state of the dineutron. Therefore the sign of a_{nn}

is negative. This conclusion may be substantiated by the work of Phillips and Crowe,[10] Cohen, and others.[13]

If the theory gave correctly the absolute value for the cross section one could easily distinguish between a positive and a negative a_{nn} since the absolute cross section is very sensitive to the sign of a_{nn}. In our approximation the cross section at the high-energy peak is by a factor of $\sim$2.5 higher for a negative a_{nn} of around -22 f than for a positive one of the same absolute magnitude. There are no reasons to believe that this ratio would change considerably if one used an exact theory.

It can be said in conclusion that the shape of the proton energy spectrum from the reaction $\mathrm{D}(n,p)2n$ is definitely sensitive on the $n-n$ scattering length. In the experiment described in reference 14, this sensitivity has not been exploited to the full, due to the rather poor energy resolution as can be seen by comparing the smeared curves of Fig. 3 with the unsmeared ones of Fig. 2. If the present calculation could be put on a more rigorous basis, or if a more exact theory could be developed, a measurement of the shape of the proton spectrum could be used for a very accurate determination of a_{nn}.

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Spectra of (p,α) and (p,p') Reactions and the Evaporation Model*†

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The energy spectra of α particles from V, Fe, Co, Ni, and Rb bombarded with 17.5-Mev protons were obtained at a number of angles. Beyond 30°, the spectra are essentially isotropic and have been analyzed in terms of the evaporation model. The level density deduced is of the form $\exp(E_x/T)$ for $E_x=3$ to 12 Mev. The values of T are in the range 1.2–1.5 Mev if the cross sections of the continuum model with $R=(1.4A^{1/3}+2.2)$ fermis are used, and in the range 1.3–1.6 Mev if optical model cross sections are used. This "constant temperature" level density is also consistent with alpha spectra of Ni and Co bombarded 15- and 19-Mev protons. Analysis of published inelastic proton scattering data for elements in the Ti-Zn region, with bombarding energies of 11.3 to 23 Mev, indicates that the (p,p') spectra (in the above excitation interval) are only partially attributable to evaporation. At 11.3 Mev the upper limit of the contribution of evaporation to the proton spectra is estimated to be $\leq 96\%$, while at 23 Mev it is probably much smaller than 22%. The value of T for Fe^{56} deduced from (p,α) and (α,α') reactions is $\sim$1.5 Mev, while inelastic neutron scattering data yield a value of 0.95 Mev. The resolution of this disagreement will require modification of the usual calculations of inverse cross sections.

I. INTRODUCTION

In recent years there have been many investigations of the energy spectra of nuclear reaction products at intermediate bombarding energies. Where the final states are well-separated, the angular distributions can, in general, be described in terms of the optical and direct interaction models. Where the final states are not resolved, corresponding to high excitations of the residual nucleus, analysis of the experimental results has been carried out in terms of the evaporation model of Weisskopf and Ewing.[1,2] The present paper discusses experiments falling into this second category.

The evaporation model describes the spectra of particles emitted from a highly excited compound nucleus in which the available energy is shared by all the nucleons. The compound nucleus has a long lifetime and its decay is assumed to be independent of its mode of formation. In order to describe the spectra of the evaporated particles, it is assumed[1] that the intrinsic probabilities for the formation of all final states are the same. The differential energy spectra are then determined by phase space factors, transmission coefficients for the particles, and the level densities of the residual nuclei.

Theoretical expressions for the spectra are given by Eq. (1) and Eq. (2) in Sec. IV; in all subsequent references to the evaporation model we shall be referring

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† Some of the results of the present investigation have appeared in the following: P. C. Gugelot, Physica **22**, 1019 (1956); R. Sherr, *Proceedings of the University of Pittsburgh Conference on Nuclear Structure, 1957* (University of Pittsburgh and Office of Ordnance Research, U. S. Army, 1957), p. 376; F. P. Brady and R. Sherr, Bull. Am. Phys. Soc. **5**, 249 (1960); F. P. Brady, D. G. Cassel, and R. Sherr, *ibid.* **6**, 48 (1961). Part of this work was submitted by F. P. Brady in partial fulfillment of the requirements for the Ph.D. degree from the Department of Physics, Princeton University.

[1] V. F. Weisskopf and D. H. Ewing, Phys. Rev. **57**, 472 (1940).

[2] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 342.