Energy Spectrum and Positive Excess of Muons in Cosmic-Ray Air Showers

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(Received August 2, 1961)

A magnet spectrometer and an array of 15 large scintillator disks have been used to examine the distribution of the muon component of extensive air showers with respect to both energy and distance from the axis. An empirical function which has been found to be consistent with the observations is

$$\rho_\mu(N,r,\geq E)=\frac{14.4r^{-0.75}}{(1+r/320)^{2.5}}\left(\frac{N}{10^6}\right)^{0.75}\left(\frac{51}{E+50}\right)\left(\frac{3}{E+2}\right)^{a(r)},$$

with $a(r)=0.14r^{0.37}$, where ρ_μ gives the average number of muons per square meter with energy exceeding E Bev at r meters from the axis of a shower that contains N charged particles. According to this relation, the mean energy at sea level of the muons in air showers is 7 Bev, and they carry, in a shower of N particles, an energy of $0.9\times10^{15}(N/10^6)^{0.75}$ ev. The positive excess which has been observed in muons examined independently of air showers was not found in the muons detected in the air showers. We conclude that π-meson production dominates K-meson production by a factor of at least 10 in a region of meson energies about 10 Bev.

1. EQUIPMENT

As part of an experimental investigation of cosmic-ray extensive air showers (EAS) near sea level, a study has been made of the muon component, utilizing a large array of plastic scintillator detectors and a vertically oriented magnet spectrometer.

Arrival of EAS was detected by an array of 15 plastic scintillator disks, each of area 0.87 m^2, monitored by a 5-in.-diameter photomultiplier tube (Dumont 6364). The disks were arranged in three groups of 5 each, forming a central cluster and two rough concentric circles of radii 150 m and 500 m. (See Fig. 1.)

Within the central cluster was a vertically oriented magnet spectrometer. A magnetic field of 12 000 gauss was applied over a gap 45 in. long, 20 in. wide, and 8 in. deep. Six horizontal trays of $\frac{5}{8}$-in. Geiger-Müller (GM) counters were spaced above, below, and within the magnet gap. Three trays (called "sagitta" trays) mounted with the counter axes parallel to the magnetic field measured the curvature of the particle trajectory normal to the field; the other three (called "scattering" trays) were oriented with the counter axes perpendicular to the field and determined whether the trajectory appeared straight in a vertical plane parallel to the field.

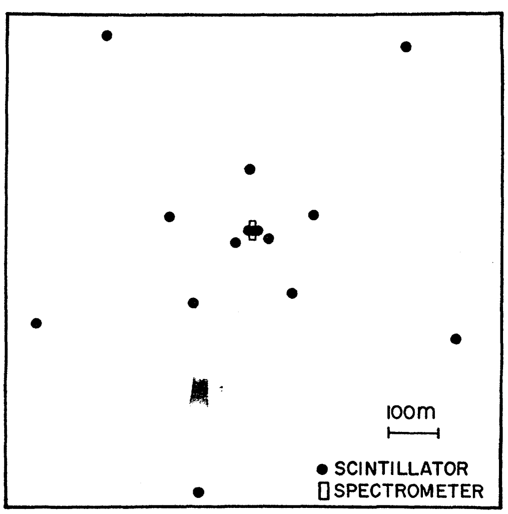

FIG. 1. Location of spectrometer and scintillators in a horizontal plane.

Above the uppermost tray of counters, lead was piled to a thickness of 15 cm in order to shield the counters from the soft component of the showers. Additionally, all trays but the uppermost were shielded by at least two feet of magnet iron, except in a direction along the magnet gap. The spectrometer aperture was ideally 25.4 cm^2-sr for undeflected particles, but was reduced by 33% by counter layer inefficiencies and muon interactions.

Photographic records were obtained at a central station whenever coincidence circuits indicated arrival of a penetrating particle in the spectrometer and at least one other charged particle in one of the scintillator disks, within a resolving time of 6 μsec. These records yield the density of charged particles at each scintillator, the measured sagitta of the penetrating particle trajectory in the spectrometer (in units of the GM counter radius), the arrival times of the particles at each detector, and subsidiary information from the spectrometer to aid in the identification of the penetrating particle as noninteracting (i.e., muon) or interacting. This subsidiary information consists of a measure of the straightness of the trajectory (as determined by the scattering trays) in a plane parallel to the magnetic field, and an indication if more than one counter was struck in any of the six trays of counters. Those events were discarded in which more than one counter was struck in any tray or in which the trajectory was not straight in a plane parallel to the magnetic field.

The data from 2700 hr of running were used to make a histogram showing the number of times at least one EAS detector disk and the spectrometer were struck within the coincidence resolving time of 6 μsec as a function of the actual time separation of the counts as determined from the photographic record. Events were grouped in bins of 0.2-μsec width. This plot shows a strong peak of 0.8-μsec width around simultaneous

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TABLE I. Coincidences with the different scintillator disks.

Disk	Distance from spectrometer (m)	Number of coincidences observed	Corrected number of coincidences	Predicted number of coincidences
1	43	465	362 ±21.6	405
2	3	721	477 ±26.8	505
3	4.5	629	446 ±25.0	496
4	13	578	462 ±24.0	460
5	36	507	403 ±22.4	410
6	165	291	189 ±17.1	140
7	125	307	205 ±17.5	202
8	125	321	219 ±17.9	202
9	145	261	159 ±16.1	168
10	185	211	109 ±14.5	135
11–15 (av)	488	127.8	25.8± 5.0	26

arrival and a flat background rate for all other bins. The size of the background is in agreement with the expected rate of chance coincidences. In the following analysis only data from the peak are used, and a fraction equal to the expected background in these bins is subtracted.

2. LATERAL DISTRIBUTION

Several experiments[1–6] have measured features of the lateral distribution of muons in EAS. Generally, each experiment has required some minimum energy of the muons for detection, and has obtained the spatial distribution of muons of energy above this threshold for some range of air-shower sizes and some range of distances from the air-shower core. Between pairs of experiments there is very little overlap of these ranges so that the experimental results are often not directly comparable. The experiment of the MIT group[1] was similar to the one reported here in low-energy cutoff, EAS detectors, and range of distances from the core, although not in the range of shower sizes. An analytic function proposed by Greisen[7] to fit the MIT data describes the lateral distribution of muons with $E\geq 1$ Bev and includes a term accounting for the effect of shower size. This function also gives a good fit to the data obtained here, except for a difference in over-all normalization of about 20%, which is within the experimental uncertainties. (Our data indicate fewer muons than were observed by the MIT group.) With the normalization suggested by our data, the lateral distribution for muons of energy ≥ 1 Bev in a shower of N scintillator particles is

$$\rho_\mu(N,r)=\frac{14.4r^{-0.75}}{(1+r/320)^{2.5}}\left(\frac{N}{10^6}\right)^{0.75}\text{ muons/m}^2. \quad (1)$$

Since most of the coincidences observed at Cornell involved only one, or at most a few, scintillator disks, we could not determine the shower sizes and core locations accurately and so deduce a muon lateral distribution directly. Instead, with an *a priori* assumption of the muon lateral distribution, we could compute the expected coincidence rates between the spectrometer and each of the scintillator disks. Comparison with the number of coincidences actually recorded then indicated whether the *a priori* assumption was reasonable. Table I gives for each scintillator disk (except for disks 11–15, which make up the group most distant from the spectrometer, and which are averaged) the raw observed number of coincidences with the spectrometer, the corrected observed number, and the number predicted. The corrected observed number is obtained from the raw observed number by subtracting the expected number of chance coincidences (102 for each scintillator) and the calculated contribution due to knockon electrons produced in the air by the muons. (See Appendix I. This correction is appreciable only for the 3 scintillators nearest the spectrometer.) The computed number is obtained from a calculation given in Appendix II, using for the muon lateral distribution the function defined in Eq. (1) above.

The sensitivity of the apparatus to showers of different numbers of electrons, with cores landing at different distances from the spectrometer, is indicated in Figs. 2 and 3.

One may well question the uniqueness of the solution represented by Eq. (1), since the determination of the muon lateral distribution in this experiment is coupled to assumptions, based on previous experiments, about the shape of the electron distribution and about the

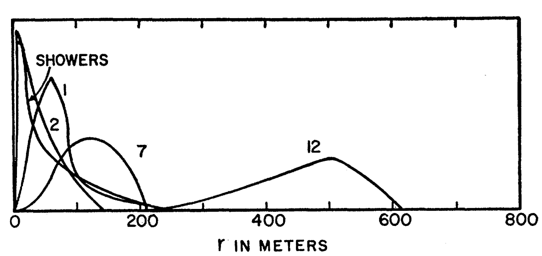

FIG. 2. Distribution of recorded coincidences between scintillator particles and muons, with respect to the distance from the muon spectrometer to the axis of the air shower. The curves were computed on the basis of the muon lateral distribution given by Eq. (1) and the electron lateral distribution given in Appendix I The curves marked "2" and "1" refer to scintillator particles recorded in the nearest and farthest counters of the central group: "7" refers to a typical counter of the ring at 150 m; and "12" to a typical counter in the outer ring. The curve marked "showers" refers to coincidences involving more than one scintillator.

[1] G. Clark, J. Earl, W. Kraushaar, J. Linsley, B. Rossi, and F. Scherb, Nuovo cimento **8**, 623S (1958).
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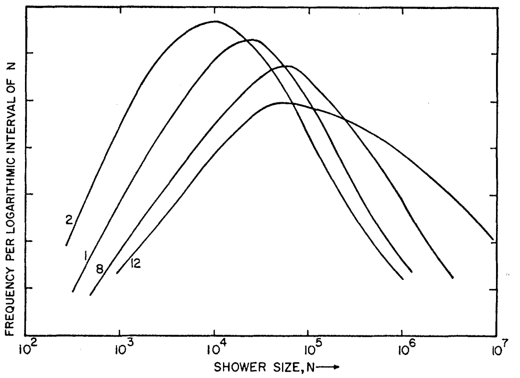

FIG. 3. Computed distribution of recorded coincidences with respect to shower size. The ordinate is (to arbitrary linear scale) the contribution to the counting rate per unit of $\log N$, for coincidences between a meson in the spectrometer and one or more particles in the scintillator indicated on each curve.

relation between the number of muons and the total number of charged particles in the showers. If these assumptions are correct, Eq. (1) gives the muon density correctly within about 30% over the range from about 20 to 500 m from the cores of the showers. If the number is more nearly proportional to the total number of charged particles, the correct lateral distribution is a little steeper than that indicated by Eq. (1); but a change of 0.1 (the approximate uncertainty) in the exponent of N requires for compensation only an approximately equal change in the exponent of r. The sensitivity of the conclusions to the precise lateral distribution of electrons is not very great, since this distribution is steeper than that of the muons. Both of these sources of uncertainty are comparable in importance with the statistical errors. These conclusions have been confirmed by repetitions of the calculations outlined in Appendix II under a variety of assumptions about the lateral distributions and the exponent of N.

3. ENERGY DISTRIBUTION

Recorded coincidences between scintillator disks and the spectrometer could be conveniently broken into 4 groups. Group A consists of events in which more than one scintillator disk is struck, whereas groups B, C, and D consist of events in which a single scintillator disk is struck in the local cluster of 5, the 150-m circle of 5, or the 500-m circle of 5 disks, respectively. For each group we could calculate the relative probability, as a function of distance from the EAS core to the spectrometer, that an EAS would produce a coincidence. We found that these probabilities peaked fairly sharply for each group at some characteristic distance, which was not very sensitive to the assumed muon lateral distribution. (Figure 2 shows graphically the results of the calculation using the muon lateral distribution given above.) We could then identify each group with its characteristic distance and so investigate the muon energy spectrum as a function of distance from the EAS core.

Integral energy distributions of the muons in each group were drawn, and a function $\rho_\mu(N,r,\geq E)$ was sought which fit these data and which reduced to the previously adopted $\rho_\mu(N,r)$ when $E=1$ Bev. The function chosen was

$$\rho_\mu(N,r,\geq E)=\rho_\mu(N,r)\left[\left(\frac{3}{E+2}\right)^{a(r)}\left(\frac{51}{E+50}\right)\right], \quad (2)$$

with $a(r)=0.14r^{0.375}$.

The factor $51/(E+50)$ was necessary in order to obtain a simple form for $a(r)$, and furthermore was suggested by the π-μ decay probability at the atmospheric depth where most of the muons are produced. Figure 4 shows the data for each group, with smooth curves drawn according to Eq. (2).

As a further check on the validity of Eq. (2), we evaluated $\epsilon_\mu(N,\geq E)$, the total number of muons with energy exceeding E in a shower of N charged particles. This is given by

$$\epsilon_\mu(N,\geq E)=2\pi\int_0^\infty r\rho_\mu(N,r,\geq E)dr. \quad (3)$$

The integration was performed numerically for values of $E=1$, 10, 100, and 1000 Bev, and the results were compared with a previous estimate of the energy spectrum,[7] made on the basis of measurements of the number of muons in EAS at various depths underground. As shown in Fig. 5, agreement with this independent determination of $\epsilon_\mu(N,\geq E)$ is quite good. We find ϵ_μ to

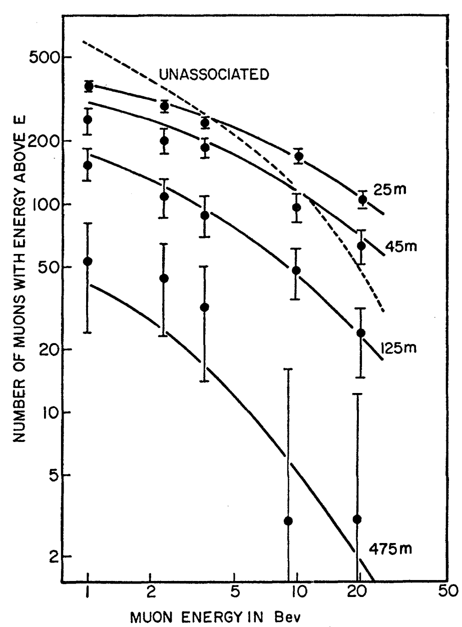

FIG. 4. Integral muon energy spectrum for particles not associated with observable air showers (dashed curve) and for shower-associated muons at various mean distances from the shower axis. The solid curves were drawn according to Eq. (2) given in the text.

be well approximated by

$$\epsilon_\mu(N, \geq E) = (N/10^6)^{0.75}\left[1.3\times10^5\left(\frac{2}{E+2}\right)^{1.29}\right]. \quad (4)$$

According to Eqs. (2) or (4), the mean energy of the muons in EAS is 7 Bev, and the muons at sea level in an EAS of N particles carry altogether 0.9×10^{15} $(N/10^6)^{0.75}$ ev. It should be pointed out that this energy is large compared with that carried by any other component of the EAS near sea level, and hence one could regard the phenomenon as a muon shower accompanied by other particles. This is a graphic indication that most showers observed at sea level are well past the maximum in development of the electronic and nuclear components. [The average energy in the electronic component and that in the nuclear component are both found to be about $0.2\times10^{15}(N/10^6)$ ev.[7]]

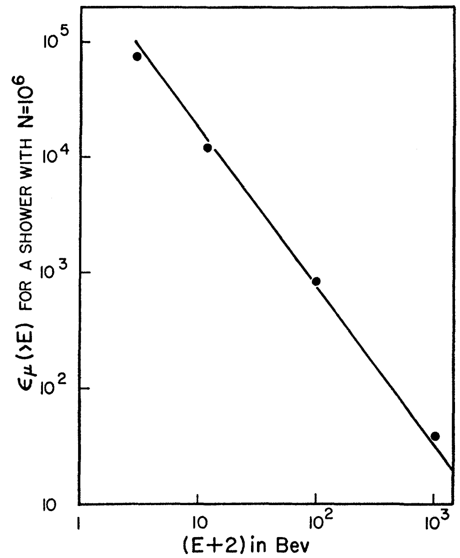

FIG. 5. Total number of muons with energy exceeding E in a shower of 10^6 particles, plotted vs $(E+2)$. The circles were computed from the data of the present article; the solid curve was obtained from reference 7.

4. POSITIVE EXCESS

The spectrometer supplied the sense as well as the magnitude of trajectory curvature. Hence data were obtained on the relative numbers of positively and negatively charged muons. If we let the number of positive muons be μ^+ and the number of negative muons be μ^-, then we can define η, the positive excess, by

$$\eta = (\mu^+ - \mu^-)/(\mu^+ + \mu^-).$$

Previous measurements[8] on muons independent of air shower observations give $\eta=0.11$. This nonzero value can be explained either in terms of the excess of protons over neutrons in the primary cosmic rays, or in terms of a charge asymmetry in meson creation (owing to charge asymmetry in K-meson and hyperon production), or by some combination of these two effects. Our measurements may be summarized as follows: For 1647 events which fell in the 6-μsec coincidence resolving time but not in the 0.8-μsec peak, $\eta=0.088\pm0.025$, consistent with the previous measurements mentioned above. From the events occurring in the 0.8-μsec peak and involving single scintillators, chance coincidence backgrounds were subtracted on the basis of this experimental asymmetry. Values of η were then computed separately for each scintillator disk (since the proportion of background events varied widely from one disk to another), and a weighted average of the results was computed, yielding $\eta=0.006\pm0.077$. The 559 coincidences ($273\mu^+$,$286\mu^-$) involving more than one scintillator disk required no chance coincidence correction, and yielded $\eta=-0.023\pm0.044$. The combined result for the charge asymmetry of mesons in air showers is $\eta=-0.016\pm0.037$, which is consistent with zero but not with the value of η for mesons unassociated with EAS.

At energies for which data are available, K^+ production dominates over K^- production[9]; and this asymmetry tends to enhance μ^+ because the production of mesons through hyperon decay is relatively very inefficient in energy transfer. We must then conclude that in the energy range studied here, K mesons are produced an order of magnitude less frequently than pions. The muons contributing most heavily to this result have a spectrum shown by the uppermost solid curve in Fig. 4, with a median energy of about 9 Bev. A corollary of this observation is that the muon charge asymmetry derived independently of air showers is due to the proton excess in the primary cosmic rays and not appreciably to charge asymmetry in strange-particle processes.

APPENDIX I

Coincidences Caused by Knockon Electrons

We evaluate the probability that a muon will be accompanied at ground level by a knockon electron with energy ≥10 Mev, which enters a scintillator disk and is detected.

The probability that a muon of energy E will, in dx gm-cm^{-2} of material, produce a knockon of energy E' in dE' has been given by Bhabha[10] as follows:

$$\varphi(E,E')dE'dx = \frac{0.3Z}{A\beta^2}m_ec^2\frac{dE'}{(E')^2}\left[1-\frac{\beta^2E'}{E_m'}+\frac{1}{2}\left(\frac{E'}{E+mc^2}\right)\right]dx,$$

[8] J. Pine, R. Davisson, and K. Greisen, Nuovo cimento **14**, 1181 (1959).

[9] V. Cocconi, T. Fazzini, G. Fidecaro, M. Legros, N. Lipman, and A. Merrison, Phys. Rev. Letters **5**, 19 (1960).

[10] H. J. Bhabha, Proc. Roy. Soc. (London) **A164**, 257 (1938).

where

$$E'=2m_ec^2\frac{p^2c^2\cos^2\theta}{[m_ec^2+(p^2c^2+m^2c^4)^{\frac{1}{2}}]^2-p^2c^2\cos^2\theta}$$

θ is the laboratory angle between muon and knockon, and E_m' is attained when $\theta=0$.

The electrons of concern are in the energy range 10 to 100 Mev and are generated by muons typically of several Bev. Under these conditions the above equations can be simplified to

$$\varphi(E')dE'dx=0.15m_ec^2[dE'/(E')^2]dx,$$

and

$$E'=2m_ec^2\cot^2\theta.$$

Neglecting multiple scattering, the knockons created at a height h above ground and arriving at a distance r from the path of the muon satisfy $\cot\theta=h/r$. By combining the last three relations, we obtain

$$\varphi(E')drdh=0.15\rho rdrdh/h^2.$$

For ρ, the density of air in the region of interest, we assume a constant value of 0.13 g/cm^2 m. The number of knockons per unit area is then

$$n(r)=3\times10^{-3}\int_{h_{\min}}^{h_{\max}}\frac{dh}{h^2}\approx\frac{3\times10^{-3}}{h_{\min}}.$$

The upper limit, which depends on the maximum transferrable energy, is immaterial. The lower limit is determined by the minimum energy of recorded electrons, 10 Mev. We assume the electrons lose energy at a uniform rate of 0.33 Mev/m along their path: then if E_0 is the energy at ground level,

$$\begin{aligned}E'=2m_ec^2\cot^2\theta&=E_0+(h/3)\sec\theta\\&\geq10+(h/3)\sec\theta,\end{aligned}$$

from which (with distances expressed in meters)

$$h_{\min}\approx\tfrac{1}{6}r[r+(360+r^2)^{\frac{1}{2}}].$$

This calculation of $n(r)$ is valid only for small r and h, when the assumptions of uniform air density and constant electron energy loss are fairly accurate; at large r (beyond 100 m) the predicted values of $n(r)$ are too large. However, the distribution is so steep about the muon track that the number of knockon coincidences is significant only for the three scintillators lying within 20 m of the muon spectrometer. In 2700 hr of running time, in which 7.5×10^5 muons were recorded, the numbers of coincidences computed by the above formula are 174, 108, 25, 4, and 3 for scintillators 2, 3, 4, 5, and 1, respectively.

Multiple scattering of the electrons in the air broadens the above distribution and reduces these numbers somewhat. An exact calculation is very involved, but in first approximation the electrons which without scattering would be contained in a circle of radius r are instead contained in a circle of radius $(r^2+r_s^2)^{\frac{1}{2}}$, where

$$r^2=h^2\tan^2\theta=\frac{2m_ec^2h^2}{E_0+h/3},$$

$$r_s^2=\frac{1}{3}\frac{(21)^2h^2}{E_0(E_0+h/3)}\frac{h}{300}.$$

The density reduction at r is therefore approximately a factor

$$\frac{r^2}{r^2+r_s^2}=(1+h/2E_0)^{-1}=1-\frac{3r^2}{6h+r^2}.$$

This may be inserted in the integrand of the equation for $n(r)$. Again dropping terms of order $h_{\min}/h_{\max}$, the revised result is

$$\begin{aligned}n(r)&=\frac{3\times10^{-3}}{h_{\min}}\left[-2+\frac{18h_{\min}}{r^2}\ln\left(1+\frac{r^2}{6h_{\min}}\right)\right]\\&=\frac{3\times10^{-3}}{h_{\min}}\left(1-\frac{3}{2}\epsilon+\frac{3}{3}\epsilon^2-\frac{3}{4}\epsilon^3+\cdots\right),\end{aligned}$$

with

$$\epsilon=\frac{r^2}{6h_{\min}}=\frac{r}{r+(r^2+360)^{\frac{1}{2}}}.$$

Accordingly, the revised numbers of knockon coincidences at scintillators 2, 3, 4, 5, and 1 are 142, 81, 14, 2, and 1, respectively.

APPENDIX II

Predicted Number of Coincidences with a Scintillator Disk

Consider an air shower containing N scintillator particles landing at a position (r,θ) where the polar coordinates are in a horizontal plane centered at the spectrometer. Let $g_e(r)$, $g_n(r)$, and $g_m(r)$ represent the lateral distributions of scintillator particles, nuclear-active particles, and muons, respectively. If then we let A_e, A_n, and A_m be the sensitive area of a scintillator disk, and of the spectrometer for nuclear-active particles and mesons, respectively, and if we describe the differential shower size frequency spectrum as $f(N)dN$, we may write C^j, the rate at which coincidences are expected with the jth scintillator disk, as

$$\begin{aligned}C^j=\int_0^\infty\int_0^{2\pi}\int_0^\infty &f(N)h(N)A_mg_m(r)\exp[-h(N)A_mg_m(r)]\\&\times\exp[-NA_ng_n(r)]\{1-\exp[-NA_eg_e(r_j)]\}rdrd\theta dN,\end{aligned}$$

where the function $h(N)$ describes the dependence of the number of muons on the shower size and where $Ng_e(r_j)$ is the number of scintillator particles per unit area at the jth scintillator. (r_j is a function of r and θ.)

The triple integral was performed numerically, with limits of integration $3\times10^2 \leq N \leq 3\times10^7$ and $0\leq r \leq 700$ m.

The function $f(N)dN$ was obtained by differentiating an integral spectrum given by Greisen[7] and then increasing the normalization by 16% to account for the net effect of the altitude above sea level and the material above each scintillator.

The functional form for $g_e(r)$ was that used by the air shower group at Cornell, and is

$$g_e(r)=4.11Nr^{-0.75}(r+82)^{-3.25}(610+r) \text{ particles/m}^2.$$

Values for $g_n(r)$ were taken from the work at Sydney.[2]

A_m was obtained as the mean spectrometer sensitive area by calculating from geometry the sensitive area as a function of zenith angle and integrating this over zenith angle, weighting with the zenith angle distribution of shower arrivals, also obtained by the Cornell group. It turns out that C^j is very nearly linear in A_m so that negligible error is introduced by using a mean value. A_n was estimated by comparing the number of events in which interactions occurred in the spectrometer with the number in which no interaction was observed, for showers whose cores landed close enough to the spectrometer to make the density of muons and nuclear-active particles roughly equal.

ACKNOWLEDGMENTS

The authors wish to thank Dr. John Delvaille and Dr. Francis Kendziorski who designed and operated the air shower array, and Dr. William Pak who shared in the operation of the spectrometer.

PHYSICAL REVIEW VOLUME 124, NUMBER 6 DECEMBER 15, 1961

π^-p Interactions at 224 Mev*

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(Received August 11, 1961)

Interactions of 224-Mev negative pions with protons were investigated using a 15-cm hydrogen bubble chamber in a 13-kgauss field. Seventeen hundred elastic scatterings were analyzed yielding a cross section of 16.0±0.8 mb for this process. No evidence for powers of $\cos\theta$ higher than two was observed in the angular distribution. The charge-exchange cross section, based on 1200 events was 34.4±1.9 mb. The results of a random-search phase-shift analysis, using these data in conjunction with earlier π^+-p elastic scattering results and recoil proton polarization measurements (π^--p), are reported. A search for pion production yielded three events of the type $\pi^-+p \rightarrow \pi^-+\pi^++n$ corresponding to a cross section of ~30 μb. No events of the type $\pi^-+p\rightarrow\pi^-+\pi^0+p$ were observed.

INTRODUCTION

THE main features of pion-proton scattering at energies less than about 300 Mev are in good accord with the results of the Chew-Low-Wick static theory and with the forward-scattering dispersion relations. Experimental and theoretical knowledge concerning the finer aspects of the π-p interaction, such as s-wave phase shifts, the small p-wave phase shifts, d-wave phase shifts, and inelastic scattering processes, is in a much less certain state.

The present experiment was undertaken to obtain further data on some of the latter phenomena. The experimental method involved the observation of several thousand interactions of 225-Mev negative pions in a hydrogen bubble chamber. It was anticipated that this technique would permit a statistical accuracy comparable to that of existing counter data, but would not be susceptible to systematic errors of the same types as with counter techniques. Furthermore, rare processes, such as inelastic scattering with the production of an additional pion, can be readily identified with the bubble chamber technique.

In analyzing the data of this experiment, the following reactions were considered:

(1)	$\pi^-+p\rightarrow\pi^-+p,$	Elastic
(2)	$\rightarrow\pi^0+n,$	Charge exchange
(3)	$\rightarrow\gamma+n,$	Radiative absorption
(4)	$\rightarrow\pi^-+p+\gamma,$	Bremsstrahlung
(5)	$\rightarrow\pi^-+\pi^++n,$	π^+ production
(6)	$\rightarrow\pi^-+\pi^0+p,$	π^0 production
(7)	$\rightarrow\pi^0+\pi^0+n.$	

* This work supported in part by the U. S. Atomic Energy Commission. A thesis based on this work has been submitted to the Carnegie Institute of Technology by J. Deahl in partial fulfillment of the requirements for the degree of Doctor of Philosophy. Preliminary accounts of the work have been given: J. Deahl, M. Derrick, J. Fetkovich, T. Fields, and G. B. Yodh, Bull. Am. Phys. Soc. **5**, 71 (1960); J. Deahl, M. Derrick, J. Fetkovich, T. Fields, and G. B. Yodh, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, New York, 1960), p. 185.

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