Possible Theoretical Interpretations of the Excited Hyperons*

J. FRANKLIN, R. C. KING,† AND S. F. TUAN
Department of Physics, Brown University, Providence, Rhode Island
(Received June 8, 1961)

The recently discovered excited hyperons are discussed in terms of four alternative interpretations. These appear to be the only possibilities which are not inconsistent with the current experimental and theoretical situation. Each possibility is analyzed in terms of theoretical models and their experimental consequences summarized. Experiments which would serve to determine which interpretation is correct are discussed.

INTRODUCTION

THE experimental evidence currently available suggests, in addition to the firmly established $I=1$ excited hyperon state Y_1^* of mass 1385 Mev and as yet undetermined spin and parity,[1,2] the possible existence of an isosinglet $\pi-\Sigma$ resonance state Y_0^* ($I=0$) of mass ≈ 1405 Mev and width $\Gamma\sim 20$ Mev.[3] Further, there are some indications from the production reaction $\pi^-+p\rightarrow\Sigma+\pi+K$ at 1.97 Bev/c[4] of an excited state Z^* (π-Σ resonance)[5] of undetermined isospin at the higher mass value ≈ 1560 Mev. In this note we wish to analyze these resonances in terms of theoretical models which can accommodate them. Unfortunately, the lack of reliable determinations for the spin and parity of the excited states in question [as well as information concerning (Λ,Σ) relative parity] make these interpretations ambiguous.[6] However, they do serve to emphasize the experiments which will most readily clarify our current understanding.

The Y_1^* and Y_0^* have been given a variety of interpretations falling into two main classes. Class I would explain excited hyperons as generated dynamically by the strong pion-hyperon interaction and class II attempts to relate excited hyperons to $\bar{K}-N$ forces and explains Y^* as a $\bar{K}-N$ virtual bound state. In what follows, an attempt is made to determine to what extent the Y^*'s can be explained by the strong $\pi-Y$ interactions alone and to what extent the influences of $\bar{K}-N$ channels are required. This will be done in the framework of some simple models of pion-hyperon interaction, notably static dispersion relations and an extended version of the bound state model recently proposed for the Σ hyperon.[7] The influence of the $\bar{K}-N$ system will be treated in the scattering length approximation of Dalitz and Tuan.[8,9]

The various theoretical possibilities can be categorized into four alternative explanations of the current experimental situation. These alternatives are discussed in the following section and their experimental consequences summarized in Table I. We have included here for completeness some which have already been discussed in the literature.

THEORETICAL INTERPRETATIONS

Case A. Even (Λ, Σ) parity; $f_\Sigma\approx f_\Lambda\approx f_N$

This case is essentially global symmetry with enough globally unsymmetric forces to agree with experiment. It has been discussed by Lee and Yang[5] using group theoretical methods and by Amati *et al.*[10] using a method related to the static Chew-Low model. The theory predicts Y_1^* in essential agreement with what is known

TABLE I. Alternative cases for excited hyperons.

Case	(Λ,Σ) parity	Y_0^*	Y_1^*	$\dfrac{Y_1^*\rightarrow\pi+\Sigma}{Y_1^*\rightarrow\pi+\Lambda}$	Dalitz-Tuan solution	Z^* isospin
A	even	$S_{\frac{1}{2}}$	$P_{\frac{3}{2}}$	$\sim 13\%$	$(b-)$	2
B	(1) even	$P_{\frac{3}{2}}$	$P_{\frac{3}{2}}$	0	$(+)$	2
	(2) even	$S_{\frac{1}{2}}$	$P_{\frac{3}{2}}$	$\ll 10\%$	$(b-)$	2
C	odd	$P_{\frac{1}{2}}$	$S_{\frac{1}{2}}$	$\gtrsim 9\%$	$(a-)$	(?)
D	odd	$S_{\frac{1}{2}}(P_{\frac{1}{2}})$	$S_{\frac{1}{2}}$	0	$(a-)$	0 or 1

* This work was supported by the U. S. Atomic Energy Commission.
† University Fellow.
[1] M. Alston, L. Alvarez, R. Eberhard, M. Good, W. Graziano, H. Ticho, and S. Wojcicki, Phys. Rev. Letters **5**, 520 (1960). M. Alston and M. Ferro-Luzzi, Revs. Modern Phys. **33**, 416 (1961). O. Dahl, N. Horwitz, D. Miller, J. Murray, and P. White, Phys. Rev. Letters **6**, 142 (1961). H. Martin, L. Leipuner, W. Chinowsky, F. Shively, and R. Adair, *ibid.* **6**, 283 (1961). J. P. Berge, P. Bastien, O. Dahl, M. Ferro-Luzzi, J. Kirz, D. H. Miller, J. J. Murray, A. H. Rosenfeld, R. D. Tripp, and M. Watson, *ibid.* **6**, 557 (1961). M. M. Block, E. B. Brucker, R. Gessaroli, T. Kikuchi, A. Kovacs, C. M. Meltzer, R. Kraemer, M. Nussbaum, A. Pevsner, P. Schlein, R. Strand, H. O. Cohen, E. M. Harth, J. Leitner, L. Lendinara, L. Monari, and G. Puppi, Nuovo cimento (to be published, 1961).
[2] R. H. Dalitz and D. H. Miller, Phys. Rev. Letters **6**, 562 (1961).
[3] M. H. Alston, L. W. Alvarez, P. Eberhard, M. L. Good, W. Graziano, H. K. Ticho, and S. G. Wojcicki, Phys. Rev. Letters **6**, 698 (1961).
[4] A. R. Erwin, R. H. March, W. D. Walker, and J. Ballam, Bull. Am. Phys. Soc. **6**, 311 (1961). Also A. R. Erwin (private communication).
[5] T. D. Lee and C. N. Yang, Phys. Rev. **122**, 1954 (1961). Lee and Yang predicted Z^* on the basis of global symmetry as an $I=2$, $\pi-\Sigma$ resonance; the excited state of reference 4 at 1560 Mev could, on the other hand, be consistent with an isospin assignment $I=0$, 1, or 2. We shall use the connotation Z^* for this resonance without prejudice as to its actual isospin state.
[6] We make here the reasonable assumption $J\leq\frac{3}{2}$ for the excited states.
[7] Y. Nambu and J. J. Sakurai, Phys. Rev. Letters **6**, 377 (1961). S. Barshay, *ibid.* **1**, 97 (1958), Nuclear Phys. **13**, 435 (1959). S. Barshay and M. Schwartz, Phys. Rev. Letters **4**, 618 (1960). S. Barshay and H. Pendleton, *ibid.* **6**, 421 (1961).
[8] R. H. Dalitz and S. F. Tuan, Phys. Rev. Letters **5**, 425 (1959); Ann. Phys. **8**, 100 (1959), **10**, 307 (1960).
[9] R. H. Dalitz, Revs. Modern Phys. **33**, 471 (1961).
[10] D. Amati, A. Stanghellini, and B. Vitale, Nuovo cimento **13**, 1143 (1959); Phys. Rev. Letters **5**, 524 (1960).

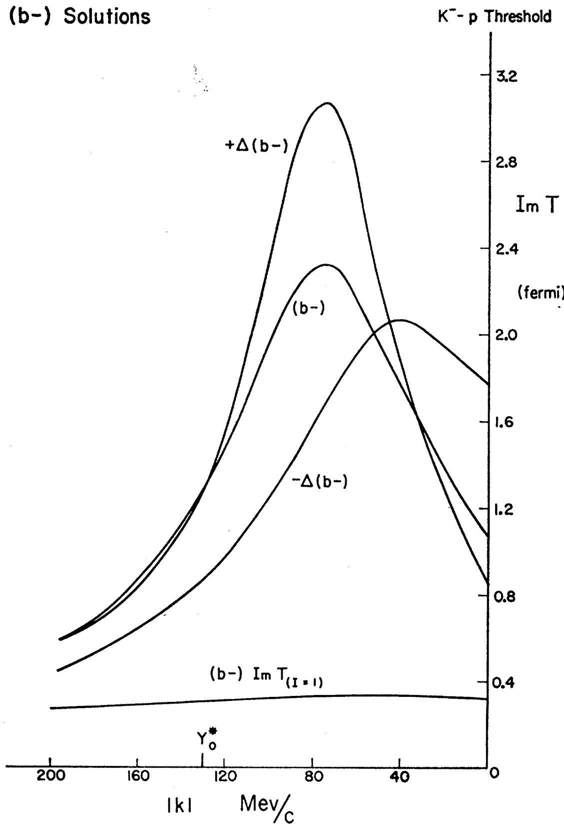

FIG. 1. Im T as a function of center-of-mass momentum $|k|$ for the region of unphysical $K^- - p$ energies. The curves labeled $(b-)$, $+\Delta(b-)$, and $-\Delta(b-)$ correspond to the mean, maximum, and minimum resonance values of Im T, respectively. Im $T_{I=1}$ for the solution $(b-)$ is also shown as a function of $|k|$. Y_0^* marks the position of the "resonance" observed at 1405 Mev.

from current experiments[11] and a $\pi - \Sigma$ resonance, Z^*, $(I=2, J=\frac{3}{2})$ at energy ≈ 1540 Mev and half-width $\Gamma/2$ of about 100 Mev. The latter presumably could be related to the excited state reported by Erwin *et al.*[4,5] if the experimental width and isospin assignment should be found to be compatible with theory. Global symmetry is inadequate, however, in that it lacks any prediction for Y_0^*. A convenient solution out of the impasse can be obtained by requiring that Y_0^* be generated by the strong S-wave association of the $K^- - p$ channel through the $(b-)$ solution of Dalitz and Tuan.[12] The $(b-)$ solution[9]:

$$A_0 = -1.85 \pm 0.15 + i(1.10_{-0.3}^{+0.9}),$$
$$A_1 = -0.10 \pm 0.20 + i(0.65 \pm 0.15) \qquad (1)$$

is plotted in Fig. 1 for the imaginary part of the $\bar{K} - p$ scattering amplitude T in the unphysical region below $K^- - p$ threshold. This produces a resonance in the $I=0$, $\pi - \Sigma$ channel[8] (which could correspond to the Y_0^* excited state). The curve Im $T_{I=1}$ shows no indication that this type of solution can generate resonant effects in the $I=1$ channel in the energy region of interest above the $\pi - \Sigma$ threshold (Im $I_{I=1}$ of Fig. 1).

Several attractive features about the $(b-)$ solution are:

(i) The $I=0$ resonant energy calculated is 18 ± 3 Mev below the $K^- - p$ threshold, i.e., at mass value 1414 (± 3) Mev; the full width Γ for the best (mean) value of the $(b-)$ set is 23 Mev. Both facts are not inconsistent with preliminary experimental data on Y_0^*.[3] It is interesting to note also that the introduction of energy dependence into the $(b-)$ scattering length from exchange of an $I=0$, ω^0 particle[13] (intermediate vector boson) between $\bar{K}$ mesons and nucleon along the lines suggested by Dalitz,[9] gives a new resonance position at 1406 Mev (from 1414 Mev) for the best value of A_0. The scattering length A_0 then shows a variation from $A_0 = (-1.85 + i1.10)$f at the $K^- - p$ threshold to $A_0 = (-1.68 + i0.31)$f at lab momentum 175 Mev/c; this suggests that the energy dependence for a_0 is relatively slight.[14]

(ii) The rather strong argument of Schult and Capps[15] that the branching ratios for the $K^- + d \rightarrow \pi + Y + N$ reactions can best be explained by the presence of an isospin zero $(J=\frac{1}{2})$ $\pi - \Sigma$ resonance a few Mev below the $K^- + p$ threshold.

(iii) In the context of the $(b-)$ solution, the vector theory of strong interactions[9,16] supplies a convenient dynamical framework for understanding the qualitative features of low-energy $K - N$ and $\bar{K} - N$ interactions V_I $(I=0, 1)$ in terms of the exchange of vector bosons with contributions Y_e $(I=1, J=1)$—the pion-pion resonance[17] and X_0 $(I=0, J=1)$—the ω^0 particle. We

[11] The experimental situation is compatible with $P_{\frac{3}{2}}$ assignment for Y_1^* (see reference 2) if at lower production energies (850 Mev/c K^- momenta and below), the reaction $K^- + p \rightarrow Y_1^* + \pi$ is S-wave production from a $D_{\frac{3}{2}}$ $K^- - p$ initial state.

[12] The validity of such a compromise model which ignores $\bar{K} - N$ effects in the $I=1$ channel and requires them for $I=0$ rests on the (questionable) assumption that there would be little coupling between states with different quantum numbers. Y_0^* as a $K^- - p$ quasi bound state would be similar to $\pi^+ + p \rightarrow \Sigma^+ + k^+$, $k^+ + n \rightarrow k^0 + p$ anomalies which violate global symmetry, but are such that the global unsymmetry is quite limited in scope though not in magnitude (see reference 5).

[13] A. Abashian, N. E. Booth, and K. M. Crowe, Phys. Rev. Letters **5**, 258 (1960). We ignore here the contribution to energy dependence from the vector boson representing a pair of $J=1-$, $I=1$ resonating pions at c.m. energy 5.2 m_π suggested by recent experiments (see reference 17). Such contributions from "distant singularities," even if substantial, are unlikely to affect greatly the energy dependence of the scattering length analysis [see J. Franklin and S. F. Tuan, Nuovo cimento **20**, 1024 (1961)].

[14] We should like to point out that this calculation of energy dependence is to be taken with some reserve, since we make the approximation that b_0 is small relative to a_0 in the calculation—an assumption less satisfactory for the $(b-)$ solution than the corresponding situation for $(a-)$ $(b_1/a_1 \sim -\frac{1}{4})$.

[15] R. L. Schult and R. H. Capps, Phys. Rev. **122**, 1629 (1961).

[16] J. J. Sakurai, Ann. Phys. **11**, 1 (1960).

[17] W. D. Walker, H. R. Fechter, R. H. March, D. Lyon, P. Satterblom, and A. R. Erwin, Bull. Am. Phys. Soc. **6**, 311 (1960). J. A. Anderson, V. X. Bang, P. G. Burke, D. D. Carmony, and N. Schmitz, Phys. Rev. Letters **6**, 365 (1961) and H. Courant (private communication).

have

$$V_1(K-N)=X_0+Y_e, \quad V_1(\bar{K}-N)=-X_0+Y_e,$$
$$V_0(K-N)=X_0-3Y_e, \quad V_0(\bar{K}-N)=-X_0-3Y_e. \quad (2)$$

The interactions $V_1(K-N)$ and $V_0(\bar{K}-N)$ are known to be strongly repulsive and strongly attractive (with the interpretation of the Y_0^* resonance as an $I=0$, $\bar{K}-N$ bound state), respectively, whereas $V_0(K-N)$ is weakly repulsive. This picture can be fitted with $X_0>3Y_e$ and requires in addition that $V_1(\bar{K}-N)$ be attractive but sufficiently weak to be unable to form a bound state.

At the moment it is not known whether the $(-)$ type solution is correct from the study of Coulomb-nuclear interference and K^--p elastic scattering at low energy.[18,19]

Case B.[20] Even (Λ, Σ) parity; $f_\Sigma \ll f_\Lambda \approx f_N$

This case emphasizes the fact that the agreement of case A with experiments conducted to date does not necessarily require global symmetry $(f_\Sigma \approx f_\Lambda)$. In fact, $f_\Sigma \ll f_\Lambda$ has the attractive consequence that the Y_0^* as well as the Y_1^* and Z^* can be produced from $\pi-Y$ interactions alone. This is then the only case which would not require a $\bar{K}-N$ bound state.

Amati *et al.*[10] have pointed out that $f_\Sigma \ll f_\Lambda$ would lead to an $I=0$ resonance as well as the Y_1^* and Z^*. However, the location they predict for the $I=0$ resonance is coincident with the Z^*, considerably above the Y_0^* position. A more detailed analysis[21] of the $\pi-Y$ scattering equations with $f_\Sigma \sim 0$ indicates that their estimate is not correct and the $I=0$ resonance could indeed be at a low enough energy to be identified with the Y_0^*.

Experimentally, this case differs from case A in predicting $P_{\frac{3}{2}}$ decay for the Y_0^*, no $\pi-\Sigma$ decay of the Y_1^*, $(+)$ type Dalitz-Tuan $\bar{K}-N$ solutions (constructive Coulomb-nuclear interference in K^--p scattering), and the half-width prediction $\Gamma/2 \sim 10$, 25, 70 Mev for the Y_0^*, Y_1^*, and Z^*, respectively.

There is some recent evidence for an $I=0$, $J=\frac{3}{2}$, $\pi-\Sigma$ resonance at 1525 Mev.[22] If this state were associated with the $I=0$ resonance discussed here, it could be taken as evidence for small f_Σ, but the Y_0^* would then have to be explained as a $\bar{K}-N$ bound state as in case A. A small but nonvanishing f_Σ would have the effect of raising the $I=0$ resonance position above that for $f_\Sigma=0$ so that it could be at 1525 Mev.

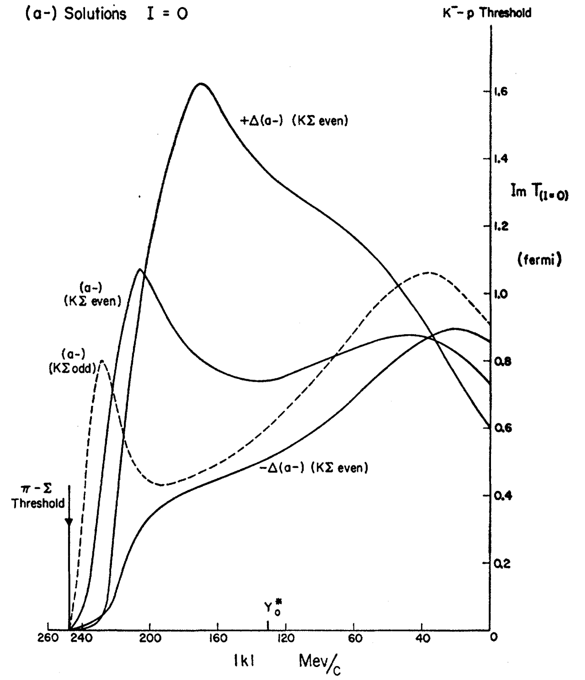

FIG. 2. Im $T_{I=0}$ as a function of center-of-mass momentum $|k|$ for region of unphysical K^--p energies. Solid (broken) curves correspond to the parity assignment $K-\Sigma$ even (odd). All curves are normalized to $q=q_0$ and $E=E_0$ at a K^--p laboratory momentum of 172 Mev/c; q and E are, respectively, c.m. momentum and total energy of the $\pi-\Sigma$ system, while q_0 and E_0 are their values at K^--p lab momentum 172 Mev/c. Curves labeled $+\Delta(a-)$ and $-\Delta(a-)$ are those leading to maximum and minimum resonance values of Im $T_{I=0}$. Y_0^* marks the position of the "resonance" observed at 1405 Mev.

It should be pointed out, however, that application of $\pi-Y$ dispersion relations to $I=0$ or 1 resonances above the $\bar{K}-N$ threshold is unreliable. This is because $\bar{K}-N$ states would have to be included in the unitarity condition and would change the structure of the equations *at the resonance energy*. The 1525-Mev resonance would then have to be considered in terms of coupled (through unitarity) $\pi-Y$ and $\bar{K}-N$ equations and could occur due to the influence of the $\bar{K}-N$ system even if f_Σ were not small. Establishing $J=\frac{3}{2}$ for the Y_0^*, however, would be a definite indication that $f_\Sigma \sim 0$, since the $\pi-Y$ dispersion relations should be reliable at predicting resonances below the $\bar{K}-N$ threshold. If the experimental upper limit on the branching ratio $(Y_1^* \rightarrow \Sigma \text{ vs } \Lambda)$ could be pushed considerably below the present 10% upper limit, this would be independent evidence for $f_\Sigma \ll f_\Lambda$.

Case C. Odd (Λ, Σ) parity; (a-) solution of Dalitz-Tuan

It is well known that the $(a-)$ solution predicts a Y_1^* with a reasonable width $(\Gamma/2=21\pm4$ Mev) and

[18] R. C. King, R. E. Lanou, and S. F. Tuan, Phys. Rev. Letters 6, 500 (1961); Brown University High-Energy Physics Internal Report No. 101, 1961 (unpublished).

[19] Detailed reanalysis of Coulomb-nuclear interference data is currently under study by the Alvarez group at Berkeley. R. Ross and W. Humphrey (private communication).

[20] The probable existence of a resonance in low-energy $\Sigma-N$ singlet scattering (R. H. Dalitz, reference 9) does not necessarily rule out the possibility that $f_\Sigma \ll f_\Lambda$ because most of the contribution to the singlet force comes from fourth-order (two-pion exchange) diagrams and could be large even for small f_Σ.

[21] J. Franklin (to be published).

[22] M. Ferro-Luzzi, R. Tripp, and M. Watson, UCRL Memo. University of California Radiation Laboratory Report No. 310, 1961 (unpublished).

can be consistent with a fairly low (Σ/Λ) ratio at resonance ($>9\%$)[9] for odd (Λ,Σ) parity. Even (Λ,Σ) parity seems ruled out in this case because of the difficulty in obtaining such a low (Σ/Λ) ratio. In Fig. 2 we have plotted Im $T_{I=0}$ for the $(a-)$ solution including the range of errors given by Dalitz,[9] the curves suggest that this solution can accommodate a possible $I=0$ resonance as well, especially for $+\Delta(a-)$.[23] The possibility thus exists that both resonance $Y_1^*(S_{\frac{1}{2}})$ and $Y_0^*(P_{\frac{1}{2}})$ could be explained as bound $\bar{K}-N$ states indicated by the $(a-)$ solution.[24] The positions calculated [for $+\Delta(a-)$] are 1370 Mev and 1390 Mev for Y_1^* and Y_0^*, respectively; these locations are somewhat lower than the experimental values, but there may well be disturbances to the theoretical expectations, arising from $(\bar{K}N-\pi Y)$ couplings. For example, Duimio and Wolters[25] find a Y_1^* likely, using a static model for odd (Λ,Σ) parity. Their resonance has the same quantum numbers as the $Y_1^*(S_{\frac{1}{2}})$ which could thus be considered as coming from the combined effects of the $\pi-Y$ and $\bar{K}-N$ systems.

Qualitatively, therefore, $(a-)$ can be made to explain Y_1^* and Y_0^* in terms of a tighter bound $I=1$ state and a less tightly bound $I=0$ state, needed for K^--deuterium data.[15] Unfortunately, such a set of bound states does not seem to give an obvious and simple interpretation of the character of the observed $(K-N)$ and $(\bar{K}-N)$ potentials in terms of vector bosons (discussed in case A).

Case D. Odd (Λ, Σ) parity— the pion binding model

Several authors have suggested a model of pion-hyperon interactions which considers the Σ as a $\pi-\Lambda$ bound S state.[7] In particular, Nambu and Sakurai were able to obtain a reasonable estimate of the scalar $(\pi\Sigma\Lambda)$ coupling constant on this basis. They further found that the $\pi-\Lambda$ structure would be a loose one, much as in the deuteron. Such a model would necessarily include further states due to the possible binding of additional S-wave pions following Bose statistics (there would be no exclusion principle for the pions). In the absence of $\pi-\pi$ forces, each pion would be expected to add 80 Mev $(M_\Sigma - M_\Lambda)$ and this would lead to the spectrum of states in Table II.[26]

The states listed are those for which the isospin wave function is completely symmetric with respect to all

TABLE II. Spectrum of low-energy states in the pion binding model.

		No $\pi-\pi$ forces		With $\pi-\pi$ forces	
State	Model	Isospin	$M_\Sigma - M_\Lambda$ (Mev)	Isospin	$M_\Sigma - M_\Lambda$ (Mev)
Λ		0	0	0	0
Σ	$(\Lambda\pi)$	1	80	1	80
Λ^*	$(\Lambda\pi\pi)$	0, 2	160	0	$\lesssim 130$
Y_1^*	$(\Lambda\pi\pi\pi)$	1, 3	240	1	270
Y_0^*	$(\Lambda\pi\pi\pi\pi)$	0, 2, 4	320	0	290 (380)

pions. This permits each pion to be in the same S state with respect to the Λ.

If the bound-state model is to make sense, it would have to be modified to agree with the non-observation of doubly charged hyperons, stable under strong decay. This could be done if the assumption were made that the $\pi-\pi$ force were repulsive in the $I=2$ and attractive in $I=0$ state. Although this is in apparent conflict with studies of τ meson decay, it is not necessarily inconsistent with $\pi-\pi$ scattering theory if the $I=1$ $\pi-\pi$ resonance is taken into account.[27]

In any event, the Σ bound state model would be inconsistent if the $I=2$ pion forces were not sufficiently repulsive, so this assumption will be made in what follows.

The spectrum then becomes that listed in the last two columns of Table II. The states with $I>1$ are removed by the $I=2$ repulsion. An $I=0$ stable Λ^* would remain just above (<50 Mev) the Λ mass. This particle is similar to the Λ^* suggested by several authors,[9,28] except that it has the same parity as the Λ. Its experimental consequences would be the same as this previously discussed Λ^*.

The effect that the pion-pion forces would have to contribute to lead to the spectrum in Table II can be determined as follows: The $I=2$ $\pi-\pi$ force would have to add at least 60 Mev per pion pair in order that the $I=2$ $(\Lambda\pi\pi)$ state be unbound with respect to $\pi\Sigma$ decay, while the combined effect of the $I=0$ and $I=2$ $\pi-\pi$ forces in the $(\Lambda\pi\pi\pi)$ $I=1$ state would have to add 30 Mev (10 Mev per $\pi-\pi$ pair) to fit the Y_1^* position. The completely symmetric $I=1$ isospin wave function for 3 pions can be written $\psi_1(3\pi)=\frac{1}{3}[5\varphi_{0,1}(3\pi)+2\varphi_{2,1}(3\pi)]$, where $\varphi_{0,1}(3\pi)$ $[\varphi_{2,1}(3\pi)]$ is the isospin wave function obtained by first adding two pions to get $I=0$ (2) and then combining this wave function with the additional pion to get $I=1$. Each pion pair in the completely symmetric state would thus be in the ratio $(5/9)$:$(4/9)$ of $I=0$ to $I=2$ states. This would require the $I=0$ $\pi-\pi$ force to reduce the energy by 30 Mev (this figure is based on the 60-Mev $I=2$ repulsion) per $\pi-\pi$ pair to give the desired result of 10 Mev added per pair. This

[23] The increase of Im T close to the $K^- - p$ threshold is due to kinematic factors, but a "resonance" effect (in the sense of a nearly vanishing denominator) can be seen at lower energy [≈ 1390 Mev for the $+\Delta(a-)$ solution].

[24] It has been suggested [M. Gell-Mann (private communication)] that scattering amplitude sets intermediate between the $(a-)$ and $(b-)$ sets may be needed to account satisfactorily for Y_0^* and Y_1^* as bound state resonances. Detailed fitting of low-energy scattering parameters are currently underway (cf. reference 19).

[25] F. Duimio and G. Wolters, Nuovo cimento **20**, 359 (1961).

[26] Higher states would also be possible but are not considered here.

[27] B. R. Desai, Phys. Rev. Letters **6**, 497 (1961). Reference 15 and 16 of this letter contain references to earlier τ decay results.

[28] J. Franklin and S. F. Tuan, Nuovo cimento **20**, 1024 (1961).

leads to the Λ^* energy being less then 130 Mev above the Λ and thus definitely bound.

Calculated on this same basis the Y_0^* would come out at 380 Mev above the Λ. This is because the symmetric $I=0$ $(\Lambda\pi\pi\pi\pi)$ state is formed by just adding a pion to $\psi_1(3\pi)$ to get the four-pion $I=0$ state which is completely symmetric. Thus one would again expect 10 Mev to be added to the energy per pion pair and the four-pion state has six pion pairs. If this state is to be identified with the Y_0^*, attractive four-body pion forces would be required to reduce its energy to the Y_0^* position.

The above considerations require the $(a-)$ Dalitz-Tuan solution $[-\Delta(a-)]$ corresponding to a resonance in the $S_{\frac{1}{2}}$ $\pi-\Lambda$ system (Y_1^*) and no $I=0$, $\bar{K}-N$ bound state.[29] If the $(\Lambda+4\pi)$ state is not the Y_0^* but some higher state, then the $+\Delta(a-)$ solution would be required to produce the Y_0^* as discussed in case C.

It should be noted that some other consequences of this bound-state model are that $f_\Sigma=0$ (neither the Λ nor the pion could emit a pion) and $Y_1^*\rightarrow\Sigma+\pi$ is forbidden if the decay is through the pion-pion interaction. Also any higher resonances, such as the Z^*, would have to be $I=0$ or 1, $J=\frac{1}{2}$. Since the energy levels have all been fit to experiment, the only real test of this model would be the existence of the scalar Λ^* and the $S_{\frac{1}{2}}$ decay of Y_1^* and Y_0^*.

EXPERIMENTS

We conclude by summarizing some of the important experiments that will help differentiate between the various alternatives. The most obvious would be determinations of the spin-parity assignments for Y_0^*, Y_1^*,[30] and Z^*, as well as the isospin of Z^*. The latter can be determined[4] for $\pi^-+p\rightarrow\Sigma+\pi+K$ at 1.97 Bev/c^4 from the strong decay branching ratio $(Z^*\rightarrow\Sigma^0+\pi^0/Z^*\rightarrow\Sigma^++\pi^-)$, which should be 1, 0, or 4 for $I=0$, 1, 2, respectively, with the further check that the decay ratio $[(\Sigma^++\pi^-)/(\Sigma^-+\pi^+)]$ should be 1 for a state of pure isospin. Of course observation of a Z^{*+} or Z^{*++} (this latter can be most conveniently observed from the reaction $\pi^++p\rightarrow Z^{*++}+K^0$ and $\bar{K}^0+p\rightarrow Z^{*++}+\pi^-$, at higher energies than have been investigated to date) would require $I=1$ or 2, respectively.

From the $\bar{K}-N$ point of view, it is essential to establish whether the $(-)$ or $(+)$ type solutions are correct from Coulomb-nuclear interference at low energy.[19] We must point out in this respect that considerations from an optical model description of K^--p scattering have led in all cases [8,31] to the conclusion that only $(+)$ solutions are consistent with the observation that the K^--nucleus interaction is attractive.[32] However, because of the possible formation of a quasi bound state (or even two such bound states in interference) for the $(-)$ type solutions, it is really questionable whether K^--nucleus data at low energy supplies a reliable guide for differentiating between $(+)$ and $(-)$ solutions.[33] It is also to be noted that the $(a+)$ solution [as opposed to $(a-)$] is more likely to show marked effects of energy dependence from "dynamic singularities" due to pion forces.[9,13,34]

Experiments on K_2^0+p scattering and absorption at low enough energy allow quite a strong discrimination between the $(a+)$ and $(a-)$ $\bar{K}-N$ amplitudes[35] as well as a very direct measure of the (Σ/Λ) ratio in the $I=1$ channel; rapid variation of this ratio with energy will favor odd (Λ,Σ) parity[36] because of the kinematic factors involved. On the other hand, the $(b-)$ solution predicts a substantially larger $\Lambda/(\Sigma^0+\Lambda)$ Σ^-/Σ^+ ratio over $(a-)$ for K^--p lab momentum interval 100–200 Mev/c; accurate determination of these quantities at low energy is thus of particular interest [current data[9] for the Σ^-/Σ^+ ratio are in agreement with $(a-)$ but do not rule out $(b-)$].

ACKNOWLEDGMENTS

We have profited from helpful discussions and communications with several physicists, in particular, R. H. Capps, R. H. Dalitz, M. Gell-Mann, M. L. Good, R. C. Hwa, Y. Nambu, A. Pais, J. J. Sakurai, and H. K. Ticho.

[29] The Y_0^* discussed here would have the decay $Y_0^*\rightarrow\Sigma+\pi$ in the $S_{\frac{1}{2}}$ state and would not be related to the low-energy $\bar{K}-N$ system.

[30] Alston *et al.* [Phys. Rev. Letters 5, 557 (1960)], do find some evidence for $J=\frac{3}{2}$ for Y_1^* from an apparent anisotropy in its decay although their Adair analysis is ambiguous. Further anisotropy data at 1.15 Bev/c and higher K^--p energy should clear up the situation.

[31] P. B. Jones, Phys. Rev. Letters **4**, 35 (1960); M. Melkanoff, D. J. Prowse, and D. H. Stork, *ibid.* 4, 183 (1960); R. D. Hill, J. H. Hetherington and D. G. Ravenhall, Phys. Rev. **122**, 267 (1961).

[32] These results have been substantiated by a recent calculation of H. Miyazawa (private communication) applying dispersion theory to the K^--nuclear many body problem. A complete self-consistent calculation of the optical model potential by J. H. Hetherington (private communication from D. G. Ravenhall) also seems to rule out the $(-)$ solutions on the basis of K^--nucleus data.

[33] R. Karplus, L. Kerth, and T. Kycia, Phys. Rev. Letters **2**, 510 (1959).

[34] F. Ferrari, G. Frye, and M. Pusterla, Phys. Rev. Letters **4**, 215 (1960).

[35] N. N. Biswas, Phys. Rev. **118**, 866 (1960).

[36] One such experiment is in progress in the Brookhaven National Laboratory Bubble Chamber Group. Preliminary results are not in contradiction with cases C and D [$(a-)$] with odd (Λ,Σ) parity discussed in the text of this article. W. J. Willis (private communication).