

Current Buildup in Neon Gas Discharges at Small Overvoltages

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This work deals with the manner in which current increases in a slightly overvolted neon discharge. The increase is exponential with a time constant θ . It is assumed that the discharge is sustained entirely by secondary emission from the cathode and that this emission is caused by Ne^+ ions and neon atoms excited to a metastable state. The model contains no adjustable parameters. Although most of the secondary emission is caused by ions, the metastables are dominant in controlling the buildup of current. The theory which is developed is valid for overvoltages ΔV small enough so that θ is large compared to the diffusion time of the metastables. Under these conditions the product $\theta\Delta V$ is constant. The rate of buildup has been observed as a function of overvoltage in a plane parallel discharge tube with movable electrodes. Over the range $1.5 < p_0d < 11$ cm-mm Hg the buildup is observed to occur on the average 15% faster than calculated.

I. INTRODUCTION

THE rate at which current increases in an overvolted discharge is intimately related to the mechanisms which are responsible for the conduction of the current. This rate of increase determines the formative breakdown delay which has been studied both theoretically and experimentally. We cite specifically the work of Llewellyn-Jones,¹ Kachickas and Fisher,² and Davidson.³ In addition to presenting the results of research on the breakdown delay problem these authors refer to other relevant work. Previous experimental studies have been done under conditions such that the diffusion of atoms excited to a metastable state does not play an important role. In the present work we discuss a theory which includes the effect of metastables for the particular case of a Townsend discharge in neon at low pressures for overvoltages less than one volt. In such a discharge the buildup is limited by the diffusion of metastable atoms to the cathode. We have measured the time constant of the buildup as a function of voltage and electrode separation in a tube filled with neon to a pressure of 40 mm Hg. The observations agree with the theory without introducing any arbitrary parameters.

II. TOWNSEND DISCHARGES AT SMALL OVERVOLTAGES

A Townsend discharge is characterized by the absence of non-linear effects. This generally implies currents less than about one microampere because space charge and other cumulative effects are important at higher currents. We shall consider a discharge that is sustained by the return to the cathode of products which release secondary electrons. From a phenomenological point of view we describe such a discharge in terms of a "return function," $L(V, t)$, defined as follows: An electron which leaves the cathode at time $t=0$,

when the voltage applied to the discharge is V , causes on the average $L(V, t)dt$ secondary electrons to leave the cathode during the interval $(t, t+dt)$. The dependence of the electron current $i(t)$ at the cathode upon earlier currents is given by:

$$i(t) = i_{00} + \int_{-\infty}^t L(V, t-T)i(T)dT, \quad (1)$$

where i_{00} denotes the electron current at the cathode caused by agents external to the discharge.

Let $K(V) \equiv \int_0^\infty L(V, t)dt$ denote the total number of secondary electrons which a typical electron from the cathode subsequently causes to leave the cathode. When i_{00} and V are independent of time the solution of Eq. (1) with $i(0) = i_{00}$ is

$$i(t) = i_{00} \left[\frac{1 - K(V) \exp(-t/\theta)}{1 - K(V)} \right], \quad (2)$$

where the time constant θ is determined by

$$\begin{aligned} 1 &= \int_0^\infty L(V, t) \exp(-t/\theta) dt \\ &= K(V) \int_0^\infty R(V, t) \exp(-t/\theta) dt. \end{aligned} \quad (3)$$

The function $R(V, t)$ is defined as $L(V, t)/K(V)$.

The object of this paper is to compare the relationship between θ and V given by Eq. (3) with observations made near the breakdown voltage V_b . The usual definition of V_b is $K(V_b) = 1$ which is the requirement for a self-sustaining discharge. The difference between the applied voltage and V_b is the overvoltage ΔV . In the limit of small ΔV the relationship between θ and V given by Eq. (3) turns out to be

$$\theta\Delta V = \int_0^\infty tR(V_b, t)dt / (dK/dV)_b. \quad (4)$$

The subscript b means that a quantity is evaluated for the conditions obtaining at breakdown. Equation (4)

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¹ F. Llewellyn-Jones, *Ionization and Breakdown in Gases* (Methuen and Company, Ltd., London, England, 1957).

² G. A. Kachickas and L. H. Fisher, *Phys. Rev.* **91**, 775 (1953).

³ P. M. Davidson, *Proc. Roy. Soc. (London)* **A249**, 237 (1959).

TABLE I. Various parameters pertaining to a discharge between plane parallel molybdenum electrodes in neon at 40 mm Hg.

Row												
1.	$p_0 d$ (mm Hg-cm)	1.25	1.50	1.75	2	2.5	3	4	5	7	9	11
2.	V_b (volts)	148.5	146.0	145.0	145.1	147.6	151.9	162	173	196	220	239
3.	E/p_0 (volts/cm-mm Hg)	119	97.5	82.9	72.6	59.0	50.6	40.5	34.6	28.0	24.4	21.7
4.	$\exp[\eta(V_b - V_0)]$	5.32	5.40	5.50	5.60	5.82	6.08	6.52	6.96	7.69	8.59	8.95
5.	$p_0 T_+$ (mm Hg- μ sec)	7.8	10	13	17	23	30	47	67	103	150	208
6.	G_b	0.617	0.667	0.713	0.749	0.794	0.812	0.803	0.740	0.619	0.525	0.432
7.	$p_0 T_m$ (mm Hg-msec)	1.1	1.6	2.1	2.8	4.3	6.3	11	17	34	56	84
8.	$100 \times (\partial G / \partial V)_b$ (volts) $^{-1}$	0.79	1.02	1.04	1.12	1.19	1.13	1.10	1.00	0.57	0.43	0.29
9.	$100 \times (\partial G / \partial \xi)_b$ (cm)	0.406	0.612	0.856	1.12	1.70	2.28	3.27	3.79	4.04	3.76	3.21
10.	γ_b	0.203	0.198	0.192	0.187	0.178	0.170	0.158	0.149	0.137	0.123	0.119
11.	derivatives ^a (volts) $^{-1}$	0.0803	0.0977	0.102	0.108	0.116	0.125	0.137	0.150	0.161	0.166	0.164

^a This term is the denominator of Eq. (8).

asserts that delayed secondary emission occurring during $(t, t+dt)$ contributes to θ in proportion to the length of the delay t and to the rate of release of secondaries $R(V_b, t)$. The rate of buildup $1/\theta$ is proportional to $(dK/dV)_b \Delta V$ which is the excess of $K(V)$ over its breakdown value.

It is interesting to note that Eqs. (2) and (4) agree with other formulations of the problem. For example, the right side of Eq. (4) is positive whenever increasing the applied voltage increases the number of discharge products, so that a positive overvoltage ($\Delta V > 0$) implies $\theta > 0$ or a discharge increasing with time. Conversely $\theta < 0$ when $\Delta V < 0$ so that under these conditions $i(t)$ reaches the steady state value $i_{00}/[1 - K(V)]$.

We have shown that under fairly general conditions a discharge which is maintained by secondary emission at the cathode builds up according to Eq. (2). After several time constants have elapsed this buildup is exponential. For small overvoltages the product $\theta \Delta V$ is constant. The value of this constant is given by Eq. (4).

III. A PARTICULAR DISCHARGE

In this section we discuss a model for a discharge between plane parallel molybdenum electrodes in neon. On the basis of this model we evaluate $\theta \Delta V$ in terms of independently observed parameters. In the next section we shall compare these results with direct observations of the buildup of such a discharge.

In order to appreciate the model it is helpful to compare the contributions of ions with that of metastables in Eq. (4). This involves estimating the respective number of secondaries released by each mechanism and the delay in their release. We list in Table I various parameters pertaining to the discharge under consideration. Row 1 contains values of the product of gas pressure and electrode spacing $p_0 d$. Row 2 contains observed values of the breakdown voltage V_b . The quotient $E/p_0 = V_b/p_0 d$ is tabulated in row 3. The average number of electrons which reach the anode per electron leaving the cathode is listed in row 4. Values of the ionization coefficient η and the voltage V_0 are taken

from Kruithof and Penning.⁴ Row 5 lists values of $p_0 T_+$, the product of pressure and transit time for Ne^+ ions from anode to cathode. The transit times are derived from the work of Hornbeck.⁵ The average number of metastables which diffuse to the cathode per electron leaving is called G_b . This quantity is listed in row 6. The method of evaluating G_b will be discussed below. Row 7 lists values of $p_0 T_m = p_0 d^2/\pi^2 D$, the product of the pressure and the fundamental diffusion time for metastables. Values of the diffusion coefficient D are obtained from the work of Phelps and Molnar.⁶

A comparison between G_b and $\exp[\eta(V_b - V_0)] - 1$ shows that approximately ten times as many ions as metastables reach the cathode. According to Hagstrum⁷ the secondary emission coefficient is the same for both particles so that the metastables release approximately ten percent of the secondaries. A comparison between $p_0 T_m$ and $p_0 T_+$ shows that the metastable diffusion time is a few hundred times as long as the ion transit time. From the considerations following Eq. (4) we thus conclude that near breakdown the predominant contribution to $\theta \Delta V$ is that of the metastables. Ions are expected to contribute a few percent.

In addition to Ne^+ ions and metastables there are three other sources of secondary emission at the cathode, namely Ne_2^+ ions, imprisoned photons, and non-imprisoned or prompt photons. The gross effect of the Ne_2^+ ions is included with that of the Ne^+ ions insofar as Kruithof and Penning's measurements have lumped the two species together. However, the transit time and secondary emission yield of Ne_2^+ are both less than those of Ne^+ so that a Ne_2^+ ion contributes less to $\theta \Delta V$ than a Ne^+ ion. Prompt photons contribute almost nothing to $\theta \Delta V$ because they return to the cathode so rapidly. A consideration of imprisoned photons would probably increase the predicted $\theta \Delta V$.

In what follows we assume that the discharge is entirely sustained by the return of metastables and ions to the cathode. We approximate the contribution to

⁴ A. A. Kruithof and F. M. Penning, *Physica* 4, 430 (1937).

⁵ J. A. Hornbeck, *Phys. Rev.* 84, 615 (1951).

⁶ A. V. Phelps and J. P. Molnar, *Phys. Rev.* 89, 1202 (1953).

⁷ H. D. Hagstrum, *Phys. Rev.* 96, 336 (1954).

$\theta\Delta V$ made by the ions by assuming that all ions reach the cathode after a delay T_+ . We ignore the influence of photons and Ne_2^+ ions. The equation, analogous to Eq. (1), describing this particular model is

$$i(t) = i_{00} + \gamma_+ i_+(t) + eS\gamma_m D[\partial M(x,t)/\partial x]_{x=0}. \quad (5)$$

The variable x is the distance from the cathode. The ion current at the cathode is $i_+(t)$. The secondary emission coefficient is γ_+ for ions and γ_m for metastables. The derivative of the metastable density with respect to position evaluated at the cathode is $[\partial M(x,t)/\partial x]_{x=0}$; e is the electronic charge; S is the area of the discharge; and D is the diffusion coefficient for metastables.

In order to evaluate $[\partial M(x,t)/\partial x]_{x=0}$ we must examine the behavior of the metastables. Their production is described by a coefficient μ which denotes the average number of metastables produced by an electron per unit distance in the direction of the field. The equation

$$\frac{\partial M(x,t)}{\partial t} = \frac{\mu i(x,t)}{eS} + D \frac{\partial^2 M(x,t)}{\partial x^2} - A p M(x,t) - B p^2 M(x,t), \quad (6)$$

relates the rate of change of the density of metastables

$$G = \frac{eSD[\partial M(x,t)/\partial x]_{x=0}}{i(t)} = \frac{eSD[dM(x)/dx]_{x=0} \exp(t/\theta)}{i(0) \exp(t/\theta)} \\ = -\frac{\mu}{\alpha} \frac{\xi}{\alpha} \left[\frac{1}{(1 - \xi^2/\alpha^2) \sinh \xi d} \right] \left[e^{\eta V(1 - V_0/V)} - \cosh[\xi d(1 - V_0/V)] - \frac{\alpha}{\xi} \sinh[\xi d(1 - V_0/V)] \right], \quad (7)$$

where

$$\xi^2 = (Ap + Bp^2 + 1/\theta)/D.$$

The quantity ξ is a measure of the relative importance of diffusion, volume loss, and accumulation of metastables as the current increases. The function G and its derivatives with respect to ξ and V have been evaluated

$$\theta\Delta V = \frac{-\frac{1}{2\xi_b D} \left[\frac{\partial G}{\partial \xi} \right]_b + T_+ \{ \exp[\eta_b V_b(1 - V_0/V_b)] - 1 \}}{\left[\frac{\partial G}{\partial V} \right]_b + \frac{1}{\gamma_b^2} \frac{d\gamma}{dV} + \left\{ \frac{d}{dV} [\eta V(1 - V_0/V)]_b \exp[\eta_b V_b(1 - V_0/V_b)] \right\}}. \quad (8)$$

For the purpose of evaluating the terms of Eq. (8), we use values of the secondary emission coefficient γ_b defined by Eq. (9).

$$1/\gamma_b = G_b + \exp[\eta_b(V_b - V_0)] - 1. \quad (9)$$

These values are consistent with our model of the discharge, recalling that $\gamma_m = \gamma_+$. The values of γ_b given by Eq. (9) are listed as row 10 of Table I and shown on

$\partial M(x,t)/\partial t$, to the rates of production $\mu i(x,t)/eS$, of diffusion $D\partial^2 M/\partial x^2$, and of destruction $-ApM - Bp^2M$. The coefficients A , B , and D have been determined by Phelps and Molnar.⁶ Values of μ are available from Rogowski's⁸ reduction of the data of Kruithof and Penning.⁴ In Eq. (6) we assume that M vanishes at both electrodes situated at $x=0$ and at $x=d$. This neglects the reflection of metastables by the electrodes. Following Kruithof and Penning we assume that neither ions nor metastables are produced between the cathode and $x_0 = V_0 d/V$. For $x > x_0$, $i(x,t)$ increases exponentially: $i(x,t) = i(0,0) \exp[\alpha(x - x_0)] \exp(t/\theta)$. The parameter α is the average number of electrons produced by an electron per unit distance in the direction of the field. Insofar as the density builds up very slowly compared to all processes in the discharge, $M(x,t)$ is separable. We shall calculate the time constant of the exponentially increasing solution $M(x,t) = M(x) \exp(t/\theta)$. This could be done by extending the work of Molnar⁹ to express the integral in Eq. (4) as an infinite series. Instead we obtain the result by solving Eq. (6) for $M(x)$, which enables us to determine the ratio of the metastable particle current density at the cathode to the electron particle current density at the cathode. This ratio is denoted by G and is given by the following equation:

for the conditions which are found to obtain at breakdown. The results are tabulated in Table I as rows 6, 8, and 9, respectively. In order to obtain an expression for $\theta\Delta V$ analogous to the more general Eq. (4), we expand the terms on the right side of Eq. (5) around their values at breakdown. The procedure is discussed in the Appendix, where we find that

Fig. 1 for comparison with Varney's¹⁰ measurements of γ for neon ions on a molybdenum surface in the presence of neon. Values of G_b are given by Eq. (7). We shall see in the next section that measurement of the rate of buildup as a function of applied voltage gives by ex-

⁸ W. Rogowski, Z. Physik **115**, 257 (1940).

⁹ J. P. Molnar, Phys. Rev. **83**, 933 (1951).

¹⁰ R. N. Varney, Phys. Rev. **93**, 1156 (1954).

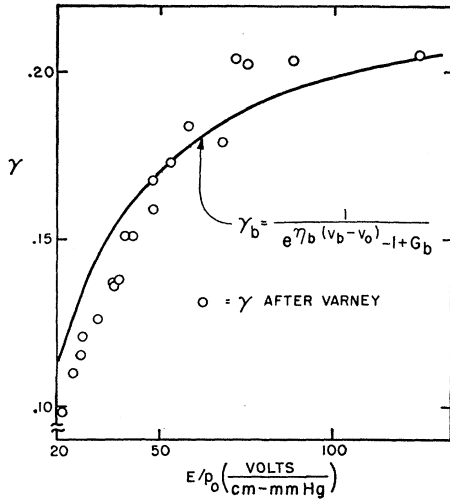


FIG. 1. A comparison between the values of γ_b used in this work and the values of γ determined by Varney for Ne^+ ions on molybdenum.

trapolation a precise indication of the breakdown voltage V_b . Knowing V_b , one can determine pd and $E/p = V_b/pd$ from an independently observed relationship between V_b and d . The dependence upon E/p of all of the parameters necessary to evaluate $\theta\Delta V$ from Eq. (8) is available in the literature previously cited.

The terms on the right side of Eq. (8) are plotted as Fig. 2 in order to illustrate their relative importance. The denominator of Eq. (8) is also tabulated as row 11 of Table I. Note that the contribution to θ caused by the ions becomes quite appreciable at higher pd 's. The fact that the contribution of the metastables declines

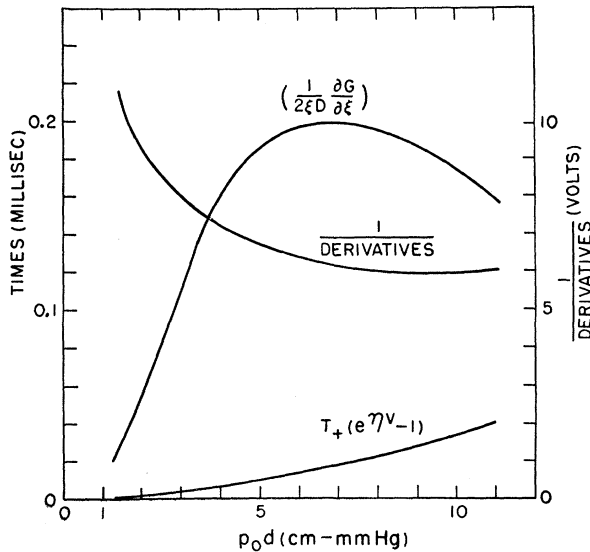


FIG. 2. The individual terms on the right side of Eq. (8) are plotted. The contribution of the positive ions to the buildup time constant is $T_+[\exp(\eta V) - 1]$. The contribution of the metastables is $(1/2\xi D)(\partial G/\partial \xi)$. The "1/derivatives" term incorporates the dependence of the number of secondary electrons upon voltage.

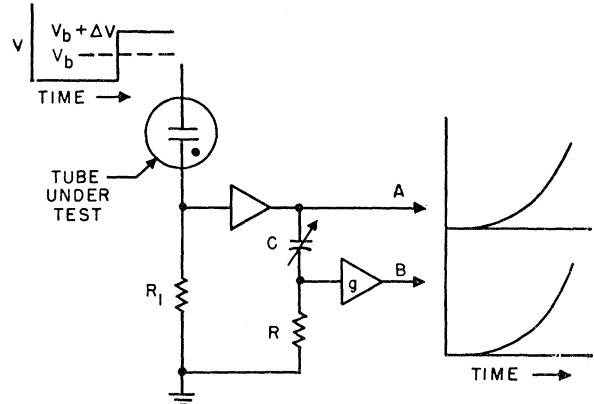


FIG. 3. The scheme used to measure the buildup time constant θ is shown. The value of the condenser C is adjusted until the outputs A and B coincide. When this occurs $\theta = (g-1)RC$, where g is the voltage gain of the indicated amplifier.

above about $pd=5$ is surprising because the diffusion time increases rapidly with d . However, fewer metastables actually reach the cathode at the higher pd 's because of their increased exposure to volume destruction.

IV. MEASUREMENT OF THE RATE OF BUILDUP

In Secs. II and III we derived a relationship between overvoltage and the time constant of the exponential buildup of electron current at the cathode. The result is valid for small overvoltages and correspondingly long time constants. In the absence of recombination, the total current through the discharge is proportional to $i(t)$ to the extent that the transit times of charged particles between the electrodes are neglected. Equations (4) and (8) thus apply to the time constant of the buildup of the total discharge current as well as to $i(t)$.

The time constant θ for the buildup of a discharge is measured by the scheme illustrated on Fig. 3. A voltage proportional to the current through the tube is devel-

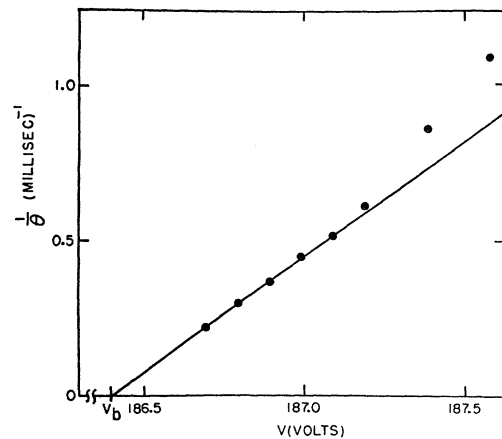


FIG. 4. A typical plot of the rate of buildup $1/\theta$ vs applied voltage. In this case the breakdown voltage is 186.4 v and $\theta\Delta V = 1.37$ msec-v.

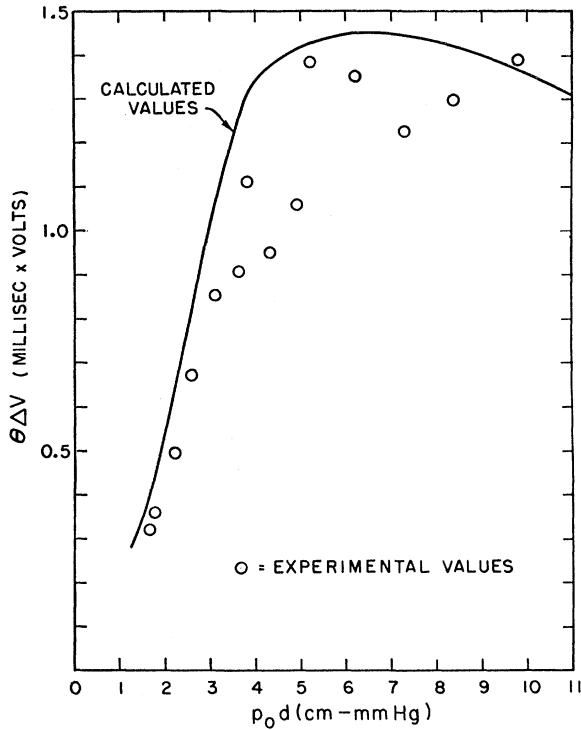


FIG. 5. Experimental values for the product of the buildup time constant θ and the overvoltage ΔV are obtained from plots such as shown on Fig. 4. The calculated values are obtained from Eq. (8) which includes in detail the effects of the diffusion and volume destruction of metastables as well as an approximation to the relatively small contribution of the ions.

oped across R_1 , amplified and displayed as A on a dual trace oscilloscope. The same signal is differentiated by a circuit with variable RC , amplified by an amount g and displayed as B . When RC is adjusted so that the displays A and B coincide, $\theta = (g-1)RC$.

The tube used for these measurements is filled with neon to a pressure of 40 mm Hg. The electrodes are movable molybdenum disks one-half inch in diameter, their largest separation being 0.080 in. A small amount of radium in the tube maintains an initial current i_{00} . The discharge commences to build up after the applied voltage is increased to $V_b + \Delta V$. The breakdown voltage of this tube at the Paschen minimum remained constant within a few tens of millivolts over a period of two years. This suggests that the cathode and gas are relatively stable.

Values of θ are measured as a function of applied voltage while the current increases from approximately 10^{-3} to 10^{-1} microamperes. The results are plotted on Fig. 4. We see that there is indeed a range of V above V_b over which the rate of buildup is proportional to ΔV as predicted by Eq. (4). The constant of proportionality $1/(\theta \Delta V)$ is measured from such curves. The observations are repeated for various values of pd . The values of $\theta \Delta V$ thus obtained are plotted on Fig. 5 for comparison with the values calculated from Eq. (8).

The breakdown voltage for each electrode spacing is also obtained from plots such as Fig. 4. This is done by extrapolating the curve to its intercept with the abscissa axis. As $1/\theta$ becomes arbitrarily small, there is no exponential buildup. This is one characteristic of the breakdown voltage.

V. DISCUSSION

The general features of the experimental results shown on Fig. 5 agree with the calculated values. The experimental points lie below the calculated curve by an average of 15%, which means that the discharge builds up faster than predicted. The result suggests that the effect of the metastables is not quite as large as we predict. In considering the 15% discrepancy in the effect of the metastables, one should recall that the discharge is primarily (about 90%) sustained by ions so that from the point of view of sustaining the discharge the discrepancy amounts to only about 1.5%.

The scatter of the experimental points makes it difficult to decide whether the discrepancy between experiment and theory is caused by the omission of an important mechanism or by some uncertainty in the values of the parameters used. Nevertheless the fact that the model we have postulated explains such a sensitive phenomenon as the rate of buildup leaves little doubt that it is basically correct.

ACKNOWLEDGMENTS

The author is greatly indebted to many of his colleagues for stimulating discussions during the performance of this work and the preparation of the manuscript. The following have been particularly helpful: D. J. Rose, E. I. Gordon, M. A. Townsend, S. J. Buchsbaum, and J. A. Hornbeck.

APPENDIX

Expansion of Equation (5)

In Sec. II of the text we found that under quite general conditions a slightly overvolted Townsend discharge builds up exponentially. We now indicate the method of obtaining the expression analogous to Eq. (4) for the particular model under consideration. According to the definition of G , Eq. (5) becomes:

$$i(t) = \gamma[i_+(t) + Gi(t)], \quad (A1)$$

for $i(t) \gg i_{00}$ and $\gamma_+ = \gamma_m = \gamma$. The number of ions produced in the discharge is given by the usual Townsend factor. We assume that they all return to the cathode after a constant transit time T_+ so that

$$\begin{aligned} i_+(t) &= i(t - T_+) [\exp \eta(V - V_0) - 1] \\ &\equiv i(t - T_+) m_+. \end{aligned} \quad (A2)$$

When the current is increasing exponentially, $i(t) = i(0) \exp(t/\theta)$ so that Eq. (A1) becomes

$$1 = \gamma[m_+ \exp(-T_+/\theta) + G]. \quad (\text{A3})$$

When the electrode spacing is fixed, γ and m_+ are functions of V . The quantity G is a function of V and ξ (which contains θ implicitly). The following are typical of the expansions to first order of these functions about their breakdown values:

$$\begin{aligned} V &= V_b + \Delta V, \\ \gamma &= \gamma_b + (d\gamma/dV)_b \Delta V = \gamma_b + (1/pd)[d\gamma/d(E/p)]_b \Delta V, \\ G &= G_b + (\partial G/\partial V)_b \Delta V + (\partial G/\partial \xi)_b \Delta \xi, \\ \xi &= [(Ap + Bp^2 + 1/\theta)/D]^{\frac{1}{2}} \\ &= \xi_b + (1/2\xi_b D)(1/\theta), \\ \exp(-T_+/\theta) &= 1 - T_+/\theta. \end{aligned}$$

Recall the breakdown criterion $1 = \gamma_b[m_{+b} + G_b]$. When expressions such as the ones we have listed are substituted into Eq. (A3), Eq. (8) of the text follows.

Theory of the Optical Properties of Quartz in the Infrared

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A theoretical study has been carried out of the eight non-degenerate optical vibrations of α quartz including the problem of their infrared and Raman intensities. The investigation consists of three parts: First, the atomic motions and frequencies are calculated on the basis of a valence force model. It is shown that the 207 cm^{-1} vibration involves atomic motions very similar to those of the α - β transformation. Secondly, a general discussion is given of infrared absorption in complex crystals, which shows that in quartz the intensities are determined by the atomic motions through 12 effective charge parameters. The intensities are calculated in good agreement with experiment on the assumption of a 2-charge model suggested by the valence nature of quartz. It is shown that a 1-charge model, the usual model for an ionic crystal, cannot account for the observed intensities. Finally, the relative Raman intensities are calculated with no

adjustable parameters in good agreement with experiment on the basis of a simple assumption about the atomic polarizabilities. The calculation accounts for the surprising weakness of the 1082-cm^{-1} stretching vibration in the Raman effect. The three parts of the investigation are mutually dependent, since the infrared and Raman intensities depend in an essential way upon the atomic motions corresponding to each frequency. It is shown that a consideration of the Raman intensities as well as the usual comparison of frequencies is required to determine the bending constants of the valence force model. It is inferred from the success of the calculations that the three principal assumptions of the present work, namely the valence force model for the vibrations, the 2-charge model for the infrared intensities, and the simple Raman model, are all applicable for quartz.

I. INTRODUCTION

THIS paper is concerned with the fundamental lattice vibrations of α quartz and the intensities of these vibrations in infrared absorption and Raman scattering. The range of these intensities extends over more than an order of magnitude, so that in some cases a fundamental frequency exhibits a weaker effect than a combination frequency. An adequate explanation of this phenomenon would be very desirable both from the standpoint of practical spectroscopy and of the basic physics of dielectric crystals. The problem of relating the intensities to the corresponding vibrations may be called the *effective charge* problem. The problem is to determine an effective charge and an effective polarizability coefficient¹ for each atom in the unit cell such that all crystal vibrations have the correct infrared and Raman intensities. This problem is essentially

trivial in the diatomic cubic crystals where the effective charge is a single parameter which can be adjusted to give the measured intensity of the single fundamental optical vibration. The problem then remains of explaining the effective charge in terms of the atomic polarizabilities, which may be called the *local field* problem.² In complicated crystals, however, the effective charge problem itself may be formidable. The effective charges become tensors, and the total number of independent components of all these tensors is the number of fundamental lattice vibrations active in the infrared.

It might appear that the effective charge problem in quartz with its 16 fundamental vibrations is hopelessly complicated. Furthermore, the problem cannot be approached until something is known or assumed about the nature of the vibrations. Nevertheless some progress has been made on the problem, notably by Saksena and associates. Saksena and Bhatnagar³ have attempted

¹ The effective charge is sometimes called the dipole derivative, the derivative of the electric moment with respect to the displacement of a certain atom. The polarizability coefficient is the derivative of the polarizability with respect to the displacement of a certain atom. For a discussion of the effective charge as the term is used here see H. Callen, Phys. Rev. **76**, 1394 (1949).

² The local field problem in this sense has been treated by: B. Dick and A. Overhauser, Phys. Rev. **112**, 90 (1958); J. Hanlon and A. Lawson, *ibid.* **113**, 472 (1959); B. Szigeti, Trans. Faraday Soc. **45**, 155 (1949).

³ B. D. Saksena and S. S. Bhatnagar, Proc. Indian Acad. Sci. **A30**, 308 (1949).