than the other two.[11] Accordingly, one must expect uncertainties in the shifts of the order of 20% of the (half) half-widths, if one calculates them from the perturbation expansion with a cut-off impact parameter and an estimate for the strong collision term. One might be tempted to overcome this difficulty by using instead one of the shifts obtained from the dispersion relation, because the widths substituted into this relation agreed rather well with each other. However, also these shifts deviate from each other by up to 20% in the region of interest (Fig. 1), again in terms of the width w_1.

[11] There are certainly procedures that are consistent with the dispersion relation, e.g., that of Vainshtein and Sobel'man (reference 38 of the preceding paper) who would write the complete damping constant $w+id$ as an analytic function of our $A(z)+iB(z)$. However, the choice of this analytic function is not unique, and the results are therefore not necessarily more accurate than those obtained with the procedures used in reference 1, which are more convenient numerically.

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Measurement of Stark Profiles of Neutral and Ionized Helium and Hydrogen Lines from Shock-Heated Plasmas in Electromagnetic T Tubes*

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The slab of plasma behind primary or reflected shock fronts served as thermal light source. Electron densities were determined from absolute continuum intensities. Temperatures in helium followed from intensity ratios of ion and neutral lines, or at lower temperatures and in hydrogen from relative line and continuum intensities. Line profiles were measured by scanning with monochromators from shot to shot. Experimental widths and shifts were compared with values calculated from measured electron densities and temperatures. For hydrogen (H_α,H_β,H_γ), neutral helium (5876, 5016, 4713, 4471, 3889, 3188A) and ionized helium (4686, 3203A), the widths agree within 10% with theory. He II 4686A exhibits a blue-shift, probably due to the polarization of the plasma near the emitting ions. The measured shifts of the neutral helium lines tend to be smaller than the calculated values, and the agreement with calculated shifts is poorer than in case of the widths.

INTRODUCTION

In dense plasmas with temperatures in the 1–10 ev range, the dominant line-broadening mechanism is Stark broadening caused by electric microfields from electrons and ions surrounding the emitting atoms or ions. The resulting Stark profiles depend almost exclusively on the electron (ion) density, and are only weak functions of the temperature. Thus, measured Stark profiles can be used to determine electron densities also in situations where the temperature is only approximately known, or even when the existence of a temperature is questionable.

In precision experiments, the accuracy of such electron density measurements is limited by the uncertainties in the theory of Stark broadening, which have been reduced considerably in the past years. For hydrogen,[1–3] neutral helium,[4] and ionized helium[5] lines the errors introduced by the various approximations in the theory are estimated to correspond to 10 or 20% uncertainties in the electron densities. This should be compared with the limiting precision of 5% for electron densities from absolute continuum intensity measurements in the visible, which is mainly determined by the accuracy of presently available intensity standards. (Microwave methods are not applicable to the dense plasmas considered here.)

Both the Stark broadening and the continuum intensity method will have the cited accuracies only if wave functions are exactly known (hydrogen and ionized helium), or at least with a precision that is commensurate with that of the rest of the theory (neutral helium). For this reason, calculations and measurements were first performed for these cases, and the motivation of this experiment was to check to what extent the theoretically estimated accuracies

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§ This paper also contains some of the material in a thesis submitted by A. W. Ali in partial fulfillment of the requirements for the Degree of Master of Science at the University of Maryland, College Park, Maryland.

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would be achieved in practice. If results were encouraging, one would then extend the Stark broadening calculations to heavier elements using, e.g., the Bates and Damgaard method[6] for the calculation of atomic matrix elements. This will allow the determination of electron densities also in situations where the continuum intensity method is not applicable, because the latter usually depends on the knowledge of wave functions for low-lying excited states to which free-bound transitions occur, whereas in line broadening only wave functions of highly excited states enter critically.

Stark profiles of the early members of the Balmer series of hydrogen have been measured previously in water-stabilized arcs,[7–9] and of helium lines in a pulsed arc[10] or in an explosive driven shock tube.[11] In all these experiments comparison could only be made with the less precise Holtsmark[12] (for hydrogen and ionized helium) or Lindholm-Foley[13] theories or in one case[11] with quantum mechanical calculations involving the Born approximation.[14] Furthermore, the experimental accuracies would hardly permit a significant comparison with the subsequent calculations (references 1 to 5 and 39, 40). Since then the precision of arc experiments has been improved considerably, and measurements of hydrogen line profiles[15,16] and neutral helium line profiles[17] were reported. They all indicate that the accuracy of the line-broadening theory is indeed as good as expected, if not better. That the line-broadening theory predicts line shapes in a consistent way is also shown by new experiments with conventional shock tubes.[18] But an independent check is very desirable in view of the complexities in the analysis of any spectroscopic experiment.

Such an independent test should be provided by measuring Stark profiles of spectral lines emitted from shock-heated plasmas. If the shock fronts are plane and can be observed tangentially to this plane, and if boundary layers are negligible, one has the advantage of an essentially homogeneous plasma along the line of sight, whereas in the arc experiments, inconveniently, side-on observations must be unfolded for the radial distributions and end-on observations suffer from unknown inhomogeneities near the electrodes. In addition, electromagnetic shock tubes offer greater flexibility in the plasma conditions, especially at the higher temperatures needed for the excitation of ionized helium lines. These advantages of the shock tube are somewhat offset by the need for good time-resolution.

One might question the establishment of local thermal equilibrium in the shock-heated plasma. But experiments with electromagnetic T tubes filled to an initial pressure of about 1 mm Hg of hydrogen or helium[19–21] all suggest that quasi-stationary local thermal equilibrium states do exist behind the shock fronts. Furthermore, the analysis of the measurements described in this paper does not depend on the validity of the assumption of local thermal equilibrium in any critical way, because both line broadening and absolute continuum intensities have only a very weak temperature dependence.

Since there is therefore no obvious reason why shock tube measurements of Stark profiles should be less reliable than arc experiments, an extensive study of Stark profiles from shock-heated plasmas was undertaken.

SHOCK TUBE APPARATUS

The electromagnetic shock tubes employed in this investigation were first of the T type with hemispherical (nickel or tungsten) electrodes at the ends of the short arm, and later simply straight tubes with electrodes protruding radially through two holes near one end of the tube (Pyrex or quartz). In both designs, use was made of the magnetic repulsion of the current between the electrodes by the return current which went through a "back strap" near the end of the tube.[22,23] The diameter of the shock tubes was approximately 2 cm, and an aluminum reflector was placed about 6 cm from the electrodes. (Most measurements were made on the reflected shock wave.)

To scan the line profiles, a large number of shots is needed. This requires good reproducibility in the operation and good accessibility of the tube for cleaning purposes. Considerable improvement in this direction was made by omitting the arm of the *T* and by using *O*-ring vacuum seals everywhere,[24] instead of vacuum wax. Thorough pumping after each shot, accurately refilling the system to a preselected pressure, and constancy in the voltage of the power supply used to charge

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[16] W. Wiese, D. R. Paquette, and J. E. Solarski, Proceedings of the Fifth International Conference on Ionization Phenomena in Gases, 1961 (to be published by North-Holland Publishing Company; Amsterdam).
[17] W. B. Johnson, Bull. Am. Phys. Soc. **6**, 283 (1961).
[18] N. N. Sobolev *et al.* (to be published). See also N. N. Sobolev *et al.*, Optics and Spectroscopy **10**, (1961).
[19] E. A. McLean, C. E. Faneuff, A. C. Kolb, and H. R. Griem, Phys. Fluids **3**, 843 (1960).
[20] W. Wiese, H. F. Berg, and H. R. Griem, Phys. Rev. **120**, 1079 (1960).
[21] W. Wiese, H. F. Berg, and H. R. Griem, Phys. Fluids **4**, 250 (1961).
[22] A. C. Kolb, Phys. Rev. **107**, 345 (1957); *Magnetohydrodynamics*, edited by Rolf K. M. Landshoff (Stanford University Press, Stanford, California, 1957), p. 76.
[23] Further descriptions of *T* tubes can be found in references 19 and 20.
[24] R. Lincke (to be published).

the capacitor bank are basic requirements for reproducibility. More critically the reproducibility is determined by the quality of the transmission line and the spark gap switch which controls the discharge.

The power requirements for these shock tubes are quite modest: one 0.1-μf capacitor at 35 kv for the hydrogen work and two similar capacitors for the helium work. Great care was taken to minimize the self-induction in the parallel plate transmission lines so that ringing frequencies of 2 or 1 Mc/sec were achieved. This insured good efficiency and avoided much of the contamination by electrode materials encountered in slower systems. Also, these high-frequency oscillations damped out almost completely before the shock front reached the point of observation, which helped to reduce electromagnetic shielding problems.

The discharge was switched with a spark gap (in air) triggered by releasing high-pressure argon through a small hole in one of the electrodes. This system proved to be entirely satisfactory as long as the edges of the hole were kept sharp. There was no need to use several spark gaps in parallel, and the relatively long jitter times inherent in our method of triggering were, therefore, of no consequence.

Preliminary (photographic) spectra showed that silicon and oxygen produce the strongest impurity lines. Since these are wall materials, no major improvement in this situation could be expected by using spectroscopically pure filling gases and a refined vacuum system. All the measurements reported here were accordingly made without diffusion pumps or cold traps. This simplified the operation of the experiment and considerably improved the reliability of the apparatus, without noticeably reducing the precision of the final results. One will not be able to reduce the impurity radiation by using a better vacuum system, unless at the same time the interaction between the plasma and the walls is inhibited, e.g., by a magnetic field parallel to the shock tube axis.[25] Employment of an electrodeless discharge as a driver, as in some conical shock tubes,[26] will not help much, because spectral lines from electrode materials do not appear further down the shock tube anyway.

The planarity of the shock fronts and the homogeneity of the layer of plasma behind it were determined in a separate experiment. The experimental arrangement consisted of two photomultipliers monitoring the continuum intensity emerging from different sections of the shock front symmetrically around the tube axis but in the same horizontal plane.

SPECTROSCOPIC APPARATUS

An ideal system for line profile measurements in time-dependent plasmas would provide both time and wavelength resolution. This has been accomplished to a certain extent by splitting the wavelength range covered by a line into a few bands which are then fed into separate photomultipliers.[27] A few channels are sufficient when line shapes are known and when only half-widths are desired. Such techniques will be useful for the diagnostics of, e.g., plasmas produced in diaphragm-type shock tubes, which inherently have a rather small repetition rate. But, for an experiment set up for the investigation of line broadening proper, a much finer subdivision of the wavelength band would be required, which is clearly impractical.

The obvious solution to this problem would be to use a stigmatic photographic spectrograph with a pin-hole slit and a rotating film drum, or, conversely, a photographic spectrograph equipped with a Kerr-cell or mechanical[28] shutter. The first system yields time resolution at a given position in the shock tube, the second produces a snapshot with spatial resolution, provided the instrument is stigmatic. The latter method was actually tried at first, but the internal consistency of the measured profiles was not satisfactory.[29] These difficulties are most likely due to photographic effects connected with short exposure times (a few tenths of a μsec) and the low light level, which is barely sufficient to expose even the most sensitive films. The instrument used for this study had an aperture of $f/6$, and considerable improvement in photographic measurements should be forthcoming when spectrographs with very high optical speeds become available, possibly approaching an effective $f/2$ relative aperature.[30]

If, however, a reproducible pulsed light source is available, it is no longer necessary to measure the whole line profile simultaneously. In the present experiment,[29] the line profiles were scanned from shot to shot in 1 to 3 A steps with monochromators. (Jarrell Ash 50-cm Seya-Namioka or Bausch & Lomb 25-cm instruments were used.) The resolution of these monochromators (as much as 1A) is sufficient, because the observed lines were all considerably broader than that. A second monochromator monitored the continuum intensity on each shot. Great care was taken that both instruments viewed the same section of the shock tube.

Photoelectric recording was used throughout, and both line and continuum signals were displayed on the face of the same dual beam oscilloscope (Tektronix 551) to insure time-correlation. The whole continuum channel was pulse-calibrated *in situ* by imaging the positive crater of a carbon arc[31] onto the appropriate section of the shock tube in the direction of the optical axis. The additional absorption in the imaging lens and shock tube wall was taken into account, and the cali-

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[31] J. Euler, Ann. Physik **11**, 203 (1953).

bration pulses were produced with a fast mechanical shutter.

The advantage of this intensity calibration method is that the amplitude of the calibration pulses is of the same order as that of the shock continuum pulses, whereas a tungsten ribbon lamp only yields much lower intensities.[32] The estimated error in the intensity calibration is about 5%. A comparable error due to uncertainties in the absolute intensity standards must be added. The total error in the absolute intensity measurements should therefore be approximately 10%.

For the line profile measurements, no intensity calibration was required, because both the efficiency of the monochromator and the response of the photomultiplier are sufficiently flat over the wavelength range covered by a spectral line. The wavelength drive had to be calibrated to measure the line shifts which was done by determining the positions of lines emitted from Geissler tubes with a precision of slightly better than 1A.

This procedure was combined with the determination of the widths and shapes of the apparatus function by scanning very slowly over these lines and displaying the signal on an oscilloscope, using the time-axis as wavelength scale. Here the precision was 0.2 to 0.5A.

All three calibrations were checked before and after measurement runs, and in no instance were significant drifts observed, not even in the intensity calibration. This is probably due to the use of very stable power supplies (0.1% at 1200 v), and to the avoidance of photomultiplier fatigue from overexposure by, e.g., a dc calibration source.

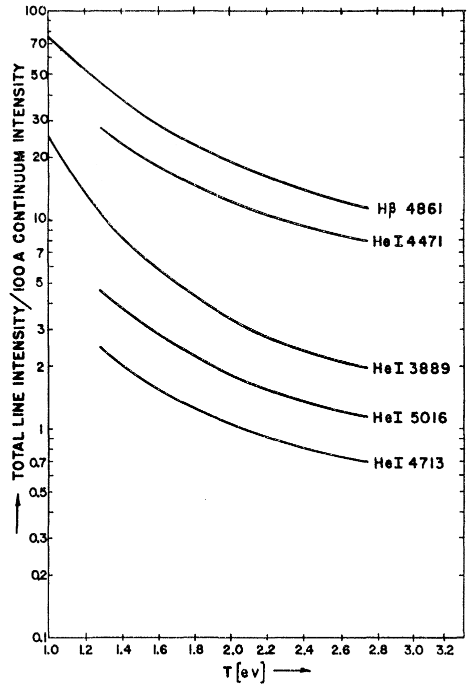

FIG. 1. Ratios of total line intensities and continuum intensities in adjacent wavelength bands of 100A widths.

ELECTRON DENSITY AND TEMPERATURE DETERMINATION

The existence of local thermal equilibrium is not doubtful for the high electron densities ($N_e \approx 10^{17}$ cm^{-3}) and relatively low temperatures ($T=1$ to 5 ev) in this experiment. Furthermore, the hydrogen and helium wave functions are so well known that theoretical errors should be entirely negligible in the application of the methods described in this section or, in case of neutral helium, at least should not be larger than the experimental errors.

In hydrogen and also at temperatures below 30 000°K in helium, temperatures were determined from the ratio of total line intensities and the continuum intensity in an adjacent wavelength band. This ratio is practically independent of the electron density,[33] and depends rather strongly on the temperature (Fig. 1). An experimental error of 10% in the intensity ratio will correspond to an error of about 5% in the temperature for hydrogen and 10% for neutral helium, where the calculated oscillator strengths[34] may typically be in error by not more than 10%. (This is suggested by comparison with measured lifetimes[35] and measured line intensity ratios.[36])

At higher temperatures, in helium second ionization sets in, and the temperature can be obtained from the relative intensities of an ionized and a neutral helium line (Fig. 2). The ratio depends linearly on the electron density, but is such a steep function of temperature that errors of 10% in the measured ratio and in the electron density still permit a 2% accuracy in the temperature measurement. In this case, errors in the oscillator strength of the neutral helium line (HeI 5876A) should be negligible, because it corresponds to a transition between two hydrogen-like levels. This is indicated by the good agreement between various calculated values.[34]

The electron density was obtained from the ratio of the continuum signals produced by the shock-heated plasma and the carbon arc. The ratio was calculated as a function of temperature for various electron densities in a wavelength interval centered at 5200 A (Fig. 3). Below 30 000°K, i.e., temperature in a region where second ionization is not important, there is at this wavelength no difference between hydrogen and helium,[29]

[32] In an independent experiment, the carbon arc was compared with a tungsten ribbon filament lamp calibrated by the National Bureau of Standards. Both calibrations were consistent within the experimental error of ±5%.

[33] H. R. Griem, Proceedings of the Fifth International Conference on Ionization Phenomena in Gases, 1961 (to be published by North-Holland Publishing Company, Amsterdam).

[34] E. Trefftz, A. Schlüter, K. H. Dettmar, and K. Jörgens, Z. Astrophys. 44, 1 (1957).

[35] S. Heron, R. W. P. McWhirter, and E. H. Roderick, Proc. Roy. Soc. (London) A234, 565 (1956).

[36] E. A. McLean, *Proceedings of the Fourth Symposium on Temperature, Its Measurement and Control in Science and Industry* (Reinhold Publishing Company, New York, 1961).

because the deviations of the contributing helium levels from hydrogen are extremely small. (For the conditions of this experiment, the negative hydrogen continuum is negligible.) Above that temperature, the curves in Fig. 3 only apply to helium.

Gaunt factors[37] where taken into account in these calculations, and theoretical uncertainties should therefore be negligible for hydrogen and also for helium (because of the hydrogenic structure of the higher levels). The estimated error of 10% in the intensity (see preceding section) will thus cause a 5% error in the electron density at lower temperatures and a 10% error at higher temperatures. The uncertainties in the temperature do not add significant amounts to these errors (i.e., the assumption of local thermal equilibrium is not at all critical), even though one has to use an iteration procedure for the determination of electron density and temperature in case of high temperatures in helium.

It should be noted that these are estimated errors, and that improvements in absolute intensity standards are needed before they can be reduced significantly. Another possibility would be to use interferometric techniques, which were already applied to diaphragm-type shock tubes.[38] But for electromagnetic shock tubes extremely intense light sources are required which permit sufficient time resolution.

Microwave methods are ruled out by the high electron densities in this experiment, and therefore, the measurement of bound-free and free-free continuum intensities indeed appears to be the only practicable method for a determination of electron densities in dense plasmas, if one does not want to use the Stark broadening method.

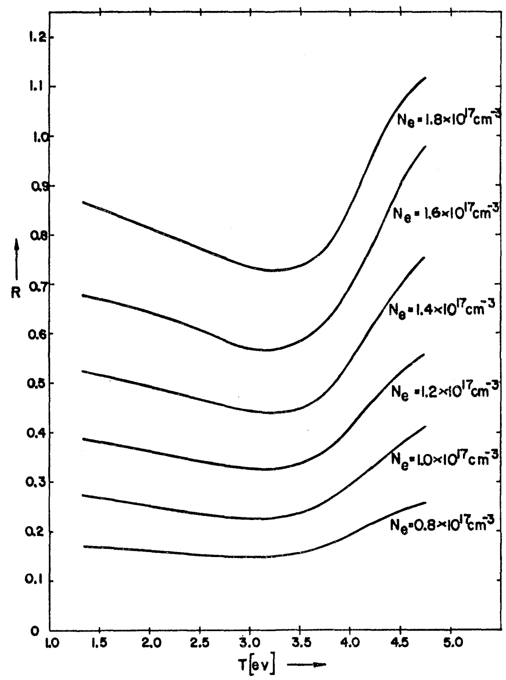

FIG. 3. The ratio of continuum and carbon arc intensities at 5200 A as function of temperature for various electron densities, assuming an emitting plasma of 1.7-cm depth.

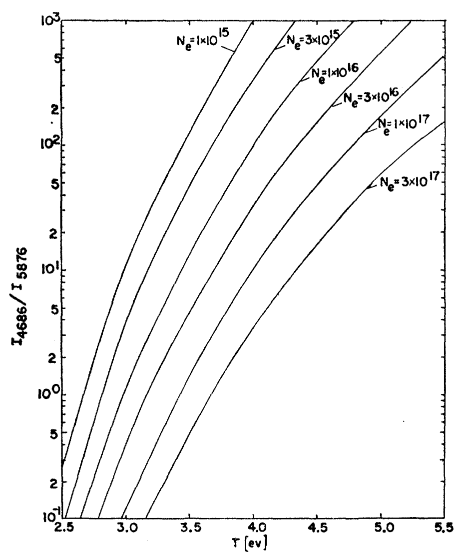

FIG. 2. Intensity ratios of HeII 4686A and HeI 5876A as functions of temperature for various electron densities.

STARK PROFILE MEASUREMENTS

In helium, the whole spectrum was scanned from 3000A to 6000A, and in hydrogen, from 4000A to 5000A and around H_α. For each wavelength setting, several shots were made, using small wavelength intervals on the lines. In the continuum band, absolute intensity measurements were made to check for impurity lines. The pulses were analyzed at times corresponding to some preselected continuum level. Independent runs yielded spectra which could be superimposed within 10%, and each measured profile was obtained from 100 or more analyzed shots. In this way, statistical errors were reduced to a few percent, and smooth curves giving a best fit to the averages of the measured points could be drawn. (These curves were used to find half-widths and shifts of the maxima.)

An important source of error is introduced by the ambiguities in the determination of the continuum below the lines. This is especially difficult at the high temperatures in helium, because then impurity lines appear almost everywhere, and no precise measurements on the line wings are possible. It is estimated that uncertainties in the true continuum level cause errors of up to 5% in the half-widths. Furthermore, the impurity lines render impossible the actual measurement of the asymmetries occurring on the line wings.

[37] W. J. Karzas and R. Latter, Atomic Energy Commission Report AECU-3703 (rev. 1958) (unpublished); and The Rand Corporation Report RM-2091-AEC, 1958 (unpublished).

[38] R. Alpher and D. R. White, Phys. Fluids 2, 153 (1959); 2, 162 (1959).

For hydrogen, these difficulties are less serious. Here, the required temperatures are small and impurity lines are very weak. Furthermore, the hydrogen lines are much stronger, compared with the continuum, than the helium lines, and uncertainties in the continuum level below the lines therefore hardly effect the linewidths. They only influence the accuracy in the determination of the asymptotic behavior of the line wings, which is of some theoretical interest.

The measured widths of hydrogen lines will, therefore, have a precision which is solely determined by the statistical error of about 5%; for helium one has to add another 5% stemming from uncertainties in the background. For the narrower helium lines and the H_α line, corrections for the finite resolution of the apparatus amounted to 10% and the contribution from Doppler broadening (which was also subtracted) was 5% in typical cases. But, uncertainties in these corrections were so small that, even for the narrower lines, errors in the measured Stark widths should never exceed 10%.

The situation for the measurement of the shifts of the maxima is not as favorable. Here, experimental errors of 1A must be expected, which is so large that only the more pronounced shifts could be determined with sufficient accuracy to allow a significant comparison with theory.

An implicit assumption in all these measurements was that the lines were emitted from optically thin and homogeneous layers. The first requirement restricted the conditions somewhat, under which the strongest lines (HeI 5876A and H_α) could be measured, but these conditions, namely high temperatures and electron densities, could easily be achieved in this experiment. (Only H_α had to be corrected for self-absorption, using temperatures and densities obtained from a measurement of the H_β line and the continuum.) This differs from the usual situation in arcs, where the Stark profiles of these lines are difficult to measure for this reason.

The condition that the emitting layers are homogeneous will be met if the shock fronts are planar and if boundary layers are negligible. The planarity of the shock fronts was checked experimentally (see the shock tube apparatus section), and from theoretical estimates[19] it follows that boundary layers cannot grow significantly in the relevant times. It is, therefore, believed that the only real limitations imposed by the electromagnetic shock tube as a light source on precision measurements of spectroscopic quantities are connected with radiation from impurity atoms and ions and small statistical fluctuations. Accordingly, the accuracies of the measured Stark widths are expected to be about 5% for hydrogen lines (except for H_α) and 10% for helium lines and H_α, which is quite adequate for a test of the theory of Stark broadening, since errors in the calculated widths due to uncertainties in the measured electron densities are of the same order.

RESULTS AND COMPARISON WITH THEORY

Measured and calculated Stark profile parameters are listed in Table I for three hydrogen, five neutral helium, and two ionized helium lines, together with independently measured electron densities and temperatures. The indicated error brackets are, in all cases, obtained from the estimated experimental errors. In case of the calculated widths and shifts, the indicated errors correspond to the errors in the measured electron densities, since the uncertainties stemming from errors in the measured temperatures are negligible. Therefore, in the line broadening calculations, the assumption of local thermal equilibrium is not critical.

The calculated widths of the hydrogen lines H_α and H_γ were derived graphically from the complete Stark profiles[39] and that of H_β from the most recently tabulated profiles.[3] All neutral helium line widths and shifts follow from the theory for isolated lines,[4] except for HeI 4471A which was calculated using the hydrogen

TABLE I. Comparison of measured and calculated linewidths and shifts.

Line	Transition	Density [10^{17} cm^{-3}]	Temperature [10^3 °K]	Full half-width [A] measured	Full half-width [A] calculated	Ratio of meas. and calc. widths	Shift of maximum [A] measured	Shift of maximum [A] calculated
H_α	2–3	0.92±0.05	25±2	7.8±1.0	8.1±0.3	0.97±0.15	···	···
H_β	2–4	0.80±0.04	14±1	40.7±2.0	41.3±1.2	0.99±0.08	···	···
H_γ	2–5	0.94±0.05	14±1	49.2±2 5	50.4±1.5	0.98±0.08	···	···
HeI 5876	2^3P-3^3D	1.59±0.15	49±1	5.5±0.6	6.2±0.6	0.89±0.20	+0.7±1.0	−0.6±0.1
HeI 5876	2^3P-3^3D	1.26±0.12	43±1	4.9±0.5	5.2±0.5	0.94±0.20	0.0±1.0	−0.5±0.1
HeI 5016	2^1S-3^1P	1.65±0.08	24±2	13.0±1.3	14.5±0.7	0.90±0.15	−4.8±1.0	−6.0±0.3
HeI 4713	2^3P-4^3S	1.30±0.07	20±2	14.0±1.4	15.2±0.7	0.92±0.15	+6.0±1.0	+8.9±0.4
HeI 4471	2^3P-4^3D	1.30±0.07	20±2	45.0±4.5	46.6±1.5	0.97±0.15	−4.5±1.0	···
HeI 3889	2^3S-3^3P	1.50±0.08	26±2	4.5±0.5	4.2±0.2	1.07±0.15	+1.2±1.0	+1.1±0.1
HeI 3188	2^3S-4^3P	1.50±0.08	29±2	13.4±1.3	12.6±0.7	1.06±0.15	+4.1±1.0	+3.8±0.2
HeII 4686	3–4	1.59±0.15	49±1	4.9±0.5	5.1±0.5	0.96±0.20	−2.2±1.0	−0.9±0.1
HeII 4686	3–4	1.26±0.12	43±1	4.2±0.4	4.4±0.5	0.97±0.20	−1.0±1.0	−0.7±0.1
HeII 3203	3–5	1.22±0.12	47±1	14.0±1.4	13.8±1.4	1.01±0.20	···	···

[39] H. R. Griem, A. C. Kolb, and K. Y. Shen, U. S. Naval Research Laboratory Report NRL-5455, 1960 (unpublished).

code, because it contains several completely overlapping components. These calculations will be published shortly,[40] together with detailed results for the ionized lines He II 4686A and He II 3203A, which were obtained using the theory for hydrogenic ion lines.[5]

Within the accuracy of the calculations mentioned above, Stark profiles of hydrogen and ionized helium lines are symmetrical and possess no shifts. Actually, slight asymmetries do occur, and also were observed in this experiment. This is most pronounced in H_β, where, at $N_e \approx 10^{17}$ cm^{-3}, the blue maximum of the doubly peaked Stark profile tends to be a few percent higher than the red maximum. This is in agreement with theoretical estimates.[8]

Except for neutral helium lines, shifts of the maxima were only observed for He II 4686A. One might think that this shift would be due to the inhomogeneities in the perturbing electric fields,[41] the effects of which can be estimated using the quadrupole interaction. For the unshifted central Stark component, the impact approximation is then also valid for ions, and the resulting shifts would be completely negligible, because in the average over the directions, the leading term in the perturbation expansion for the shift averages out.[42] (If one ignores this average, i.e., uses an adiabatic formula,[43] and neglects the rotation of the perturbing electric field vectors, a shift of the right order of magnitude but of the wrong sign is obtained.)

A more likely explanation for the observed blue-shift of He II 4686A is the reduction of the Coulomb potential of the nucleus by the plasma polarization in the neighborhood of ions, which results in an average negative space-charge density $\rho(r) = eN_e[2e^2(rkT)^{-1}]$ at a distance r from a singly charged ion. This leads to a blue-shift of the order $\Delta E = 300 \times 4\pi e^3 N_e (kT)^{-1} \langle\alpha|r|\alpha\rangle$[ev], where $\langle\alpha|r|\alpha\rangle = 9a_0$ is the radial matrix element of the upper state of the unshifted component. At $N_e = 1.5 \times 10^{17}$ cm^{-3}, $T = 50\,000°$K, the blue-shift calculated in this way is 0.8A, in fair agreement with the measured values (Table I). For He II 3203A, the calculated blue-shift is about the same. This shift, however, could not be measured because the line is much broader and exhibits two peaks.

Inspection of Table I shows that there is in no instance a significant disagreement between calculated and measured widths. The agreement is always within ~10%, i.e., better than expected from the estimated errors of this experiment. The differences of calculated and measured shifts are larger, as must be expected from theoretical error estimates.[42,44] (That the measured shifts tend to be smaller than the calculated values, might be due to some residual inhomogeneities in the plasma. It may also be the case that some of the calculated shifts are too large because of the transition to linear Stark effect at high ion field strengths or because of Debye shielding.)

Observed and calculated line shapes were in good agreement, e.g., the neutral helium profiles (except for He II 4471A) had dispersion shapes with some asymmetries on the line wings, whereas the hydrogen and ionized helium profiles showed all the characteristic features (central dips in H_β and He II 3203A and strong peaks in H_α and H_γ). The satisfactory agreement of calculated and measured line shapes is demonstrated in Figs. 4, 5, and 6 for H_α, H_β, and H_γ. The calculated profiles correspond to *measured* densities and temperatures, as does the ratio of continuum and line intensities. Each experimental point is the average of three or more measurements, and is not yet corrected for instrument and Doppler broadening.

If the observed hydrogen profiles are fitted on the wings (not shown in Figs. 4–6) by a power law, expo-

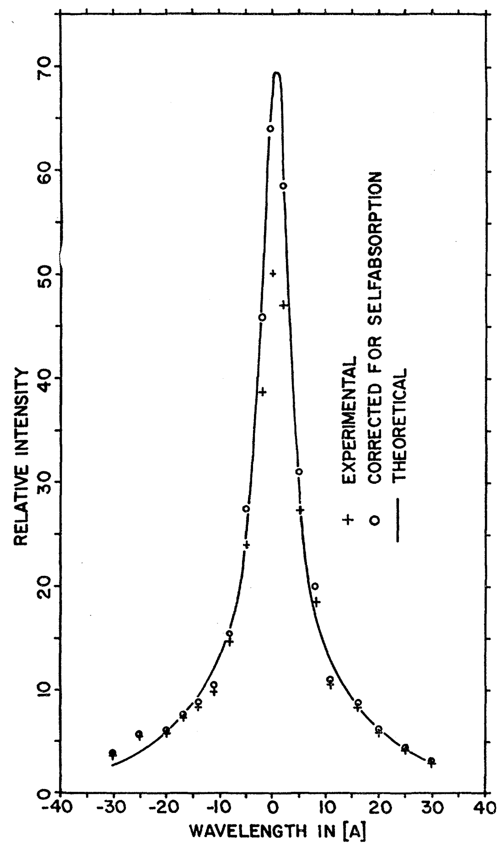

FIG. 4. Comparison of measured and calculated Stark profiles of H_α. $N_e = 0.92 \times 10^{17}$ cm^{-3}, $T = 25\,000°$K.

[40] H. R. Griem, A. C. Kolb, and K. Y. Shen, U. S. Naval Research Laboratory Report (to be published).

[41] R. M. Herman, Bull. Am. Phys. Soc. **5**, 234 (1960).

[42] C. S. Shen, Ph.D. thesis, University of Maryland, College Park, Maryland, 1961 (unpublished).

[43] A. Unsöld, *Physik der Sternatomosphären* (Springer-Verlag, Berlin, 1955), p. 329.

[44] H. R. Griem and C. S. Shen, preceding paper [Phys. Rev. **125**, 196 (1962)].

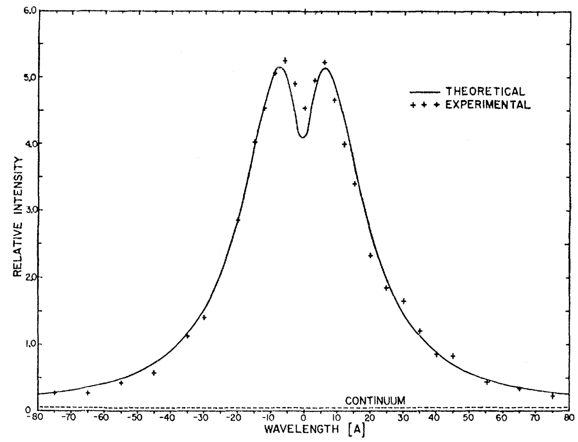

FIG. 5. Comparison of measured and calculated Stark profiles of H_β. $N_e=0.80\times10^{17}$ cm^{-3}, $T=14\,000$°K.

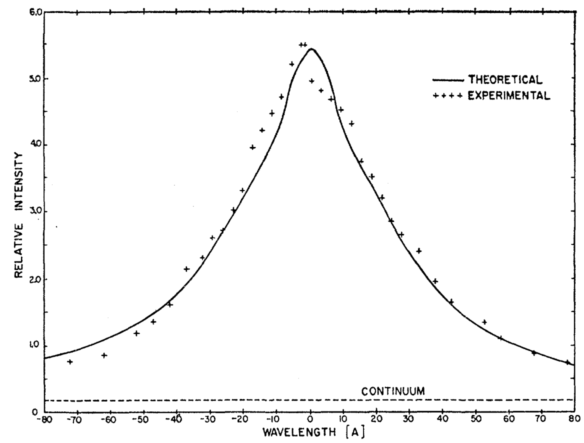

FIG. 6. Comparison of measured and calculated Stark profiles of H_γ. $N_e=0.94\times10^{17}$ cm^{-3}, $T=14\,000$°K.

nents between -2.1 and -2.3 must be used, i.e., values which are intermediate to the quasi-static theory (-2.5) and the impact theory (-2) results, as expected.

SUMMARY

This experiment verified the estimated precision of recent Stark profile calculations (references 1 through 5 and 39, 40), especially with respect to half-widths, and the detailed shapes of the more characteristic lines. The accuracy of the experimental check of the Stark broadening theory is essentially limited to 5% (at low temperatures in hydrogen) or to 10% (at high temperatures in helium) by the errors in the absolute intensity measurement of the bound-free and free-free continuum used to determine the electron density in an independent way, i.e., the differences (Table I) between measured and calculated Stark widths are not significant.

In future experiments, electron density measurements from Stark widths of hydrogen, neutral and ionized helium lines should, therefore, have a limiting precision of at least 10% (hydrogen) to 20% (helium). (More likely than not this precision is rather 5% and 10%, respectively.) The Stark broadening method will accordingly be very valuable for the analysis of plasmas consisting mainly of gases whose atomic properties are less well known, because it will usually be sufficient to add about one percent of hydrogen or helium as electron density probe. In arcs where the total pressure is known, this electron density can then be used to calculate the temperature rather precisely.[15] This also applies to conventional shock tubes,[45] as long as deviations from ideal behavior are not too large.[46] Finally, it should be emphasized that Stark *shift* measurements will yield electron densities with a precision that is, in general, inferior to that of *width* measurements.

[45] L. R. Doherty, Ph.D. thesis, University of Michigan, Ann Arbor, 1961 (unpublished).

[46] A. C. Kolb and H. R. Griem, in Chapter 5 of *Atomic and Molecular Processes*, edited by D. R. Bates (Academic Press Inc., New York 1962).