Parity Nonconservation Induced by the Universal Fermi Interactions into the Pion-Nucleon Vertex*

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The problem whether the interpretation of strong interactions as high-energy effects of the universal Fermi interactions (UFI) is consistent with experimental evidence on parity conservation in low-energy nuclear physics is investigated. The parity-nonconserving part of the one-nucleon-off-shell $\pi-N$ vertex, which originates in the UFI (which we consider smeared out with a heavy vector boson of such a mass that they bind an extreme relativistic nucleon-antinucleon pair into a pion) of the nucleons, is evaluated using dispersion methods and is found to have a relative magnitude of order 10^{-5} when compared with the parity-conserving part. This yields a parity-nonconserving $\pi-N$ scattering amplitude of the same relative order of magnitude, a result which does not contradict the existing experimental data.

1. INTRODUCTION

SINCE at high energies the local universal Fermi interactions (UFI) become strong, it has been proposed[1] that they might bind a $N-\bar{N}$ pair into a pion and account for the strong $\pi-N$ interaction. The UFI being parity nonconserving, one would expect that they induce a parity nonconservation also into the strong interactions. On the other hand, there is good experimental evidence for parity conservation[2] (the relative strength of the parity-nonconserving amplitude is of order $\lesssim 10^{-4}$) in low-energy nuclear physics. It is therefore necessary to investigate whether the parity nonconservation in strong interactions originating from UFI is compatible with the above-mentioned experimental evidence.

From the theoretical point of view the problem of parity nonconservation in strong interactions has been discussed by many authors.[3] It has been proved that because of charge symmetry for the $\pi-N$ vertex with the two nucleons on the mass shell, CP invariance implies P and C invariance separately. Nevertheless this is not the case if one of the nucleons is kept off-shell. Since in $\pi-N$ scattering such a vertex plays an important role, it is interesting to get an idea about the order of magnitude of its parity-nonconserving part. This we shall do using dispersion theory. We shall write a dispersion relation for the $\pi-N$ vertex with one nucleon off-shell and shall consider in the unitarity relation the lowest-mass parity-conserving (π,N) and the lowest mass parity-nonconserving (B,N) states (B being the intermediate boson of the UFI). For the mass M of the intermediate boson we shall insert the value deduced in reference 1 from the condition that the UFI be strong enough to bind an extreme relativistic $N-\bar{N}$ pair into a pion. It is this choice of the value of M that makes the whole calculation nontrivial, since in this case the UFI coupling constant $g_F M^2$ is of the order of unity and its effects could be comparable with those of strong interactions; another choice of M (say of the order of magnitude of the nucleon mass) would imply the usual weak interaction, which is evidently negligible relative to the strong $\pi-N$ interaction. It is this strong energy dependence of the UFI upon which the philosophy of the whole program initiated in references 3 and 1 rests.

Our result is the following: The parity-nonconserving admixture in the one-nucleon off-shell $\pi-N$ vertex induced by the (B,N) intermediate state is at most of the order $10^{-2}\, m/M \sim 10^{-5}$ (m = nucleon mass) in amplitude. This is consistent with the existent experimental evidence[2] and thus shows that the model advanced in reference 1 cannot be discarded by parity conservation arguments in low-energy nuclear physics.

2. ONE NUCLEON OFF SHELL π-N VERTEX

We shall consider the vertex

$$\begin{aligned}\langle 0|f(0)|p,s;l,\lambda\rangle &= -ig\tau_\rho i\gamma q[\gamma_5 F_1(x)+iF_2(x)]i\gamma p\frac{u(p,s)}{(p_0/m)^{\frac{1}{2}}}\frac{\epsilon_\rho(\lambda)}{(2l_0)^{\frac{1}{2}}}\\ &= -ig\tau_\rho\left\{\frac{x-i\gamma q}{2x}[\Gamma_1(x)\gamma_5+i\Gamma_2(x)]+\frac{x+i\gamma q}{2x}[\Gamma_1(-x)\gamma_5+i\Gamma_2(-x)]\right\}\frac{u(p,s)}{(p_0/m)^{\frac{1}{2}}}\frac{\epsilon_\rho(\lambda)}{(2l_0)^{\frac{1}{2}}}. \qquad (1)\end{aligned}$$

Here p, s (l,λ) are the momentum and spin-isospin of the nucleon (π meson); $f(x)=(i\gamma\partial+m)\psi(x)$, ψ being the nucleon field operator; $q=p+l$; $x^2=-q^2$ [we use a $(-1, 1, 1, 1)$ metric]; Γ_1 (Γ_2) is the form factor corresponding to the parity-conserving (-nonconserving) part of the vertex. The first form of the matrix element used

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[1] K. Baumann, P. G. O. Freund, and W. Thirring, Nuovo cimento **18**, 906 (1960) and B. Jouvet (private communication to Professor Thirring).

[2] D. H. Wilkinson, Phys. Rev. **109**, 1610 (1958); F. Boehm and U. Hauser, Bull. Am. Phys. Soc. 4, 460 (1959).

[3] W. Thirring, Nuclear Phys. **10**, 97 (1959); **14**, 565 (1960). V. G. Soloviev, *ibid.* **6**, 618 (1958). G. Morpurgo and B. F. Touschek (unpublished). S. Fubini and D. Walecka, Phys. Rev. **116**, 194 (1959). G. Barton, Nuovo cimento **19**, 512 (1961).

in (1) follows directly from invariance considerations under the Lorentz group and PC conservation, while the second form (which we shall use in the following) is just a convenient rewriting of the first one.[4] For Γ_i ($i=1,2$) we assume the correctness of the following unsubtracted dispersion relations:

$$\Gamma_i(x)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\mathrm{Im}\Gamma_i(x')dx'}{x'-x-i\epsilon}. \tag{2}$$

Using standard techniques one can obtain expressions for the absorptive parts of Γ_i as sums over intermediate states of which we select the lowest-mass parity-conserving (πN) (Fig. 1) and parity-nonconserving (BN) (Fig. 2) states.

The contribution from Fig. 1 to the absorptive part of Γ_i is, as one can easily see,[4]

$$\mathrm{Im}\Gamma_i^{\pi}(x)=h_i^*(x)\Gamma_i(x)\quad(i=1,2), \tag{3}$$

where

$$h_1(x)=\sin\alpha_{11}(x)e^{i\alpha_{11}(x)},\qquad h_2(x)=\sin\alpha_1(x)e^{i\alpha_1(x)}, \tag{4}$$

α_1 and α_{11} being the $T=\frac{1}{2}$, $S_{\frac{1}{2}}$, and $T=\frac{1}{2}$, $P_{\frac{1}{2}}$ phase-shifts, respectively, of the $\pi-N$ scattering.

Inserting (3) and (4) into (2) one obtains two independent homogeneous integral equations of the Omnès[5] type. These can be solved in the normal way and for us it is important to observe that they admit a solution[4] of the form

$$\Gamma_1(x)=G_1(x),\qquad \Gamma_2(x)=0, \tag{5}$$

with

$$G_1(x)=\exp[Q_1(x)-Q_1(m)],\qquad Q_1(x)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\alpha_{11}(x')}{x'-x-i\epsilon}dx'. \tag{6}$$

The factor $\exp[-Q_1(m)]$ in $G_1(x)$ is due to the fact that by definition $\Gamma_1(m)$ is normalized to unity. The solution (5) is strictly parity conserving and it was to be expected that the completely parity-conserving graph (Fig. 1) should yield a parity-conserving solution. The unphysical integration region ($x<0$) in (6) can be avoided using the relation[6]

$$\alpha_{11}(-x)=\alpha_1(x). \tag{7}$$

Now let us see the parity nonconservation induced by the graph shown in Fig. 2. The contribution of this graph to the absorptive parts of Γ_i we compute using first-order perturbation theory for the two NB vertices and the nucleon propagator and inserting $G_1(x)$ for the πN vertex (marked in Fig. 2 by a circle) since considering the parity-nonconserving part of this πN vertex would lead to higher order effects. Hence the contribution of the graph shown in Fig. 2 will not contain Γ_i and will thus lead to an inhomogenity in the Omnès equations. Writing down these contributions in the above-mentioned way using a $V-A$ BN interaction and performing the integrations in the c.m. system, one obtains after lengthy but straightforward computations

$$\mathrm{Im}\Gamma_{1,2}^{B}(x)=G^2\frac{\pi}{(2\pi)^3}\frac{\Theta[x^2-(m+M)^2]}{\mu^2-(x\mp m)^2}\frac{1}{(m^2+a^2)^{\frac{1}{2}}+(M^2+a^2)^{\frac{1}{2}}}\frac{x}{b}\times\int_{V_-}^{V_+}\frac{dV}{m^2-V^2}\left\{\left[m(M^2+a^2)^{\frac{1}{2}}-m(m^2+b^2)^{\frac{1}{2}}\mp\frac{M^2-m^2-V^2}{2}\right]E(V)+[m(m^2+a^2)^{\frac{1}{2}}\mp x(m^2+b^2)^{\frac{1}{2}}\mp\tfrac{1}{2}(V^2-m^2-M^2)]F(V)\right\}, \tag{8}$$

where

$$\begin{aligned}
a&=\{[(x^2+m^2-M^2)^2/4x^2]-m^2\}^{\frac{1}{2}};\\
b&=\{[(x^2+m^2-\mu^2)^2/4x^2]-m^2\}^{\frac{1}{2}};\\
V_{\pm}&=[\pm2ab+m^2+M^2-2(m^2+b^2)^{\frac{1}{2}}(M^2+a^2)^{\frac{1}{2}}]^{\frac{1}{2}};\\
E(V)&=mV[G_1^*(V)+G_1^*(-V)]+m^2[G_1^*(V)-G_1^*(-V)];\\
F(V)&=mV[G_1^*(V)+G_1^*(-V)]+V^2[G_1^*(V)-G_1^*(-V)];
\end{aligned} \tag{9}$$

M, m, μ are the B, N, π masses; G is the BN coupling constant. The integral equation for Γ_2 now reads

$$\Gamma_2(x)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{h_2^*(x')\Gamma_2(x')dx'}{x'-x-i\epsilon}+f_2(x), \tag{10}$$

with

$$f_2(x)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\mathrm{Im}\Gamma_2^{B}(x')dx'}{x'-x-i\epsilon}. \tag{11}$$

We are interested in that solution of the inhomogeneous equation (10) which in the homogeneous case $[f_2(x)\equiv0]$ yields the parity-conserving solution (5). This solution

[4] The parity-conserving part of the one-nucleon off-shell $\pi-N$ vertex has been investigated by A. Bincer, Phys. Rev. **118**, 855 (1960).

[5] R. Omnès, Nuovo cimento **8**, 316 (1958).

[6] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

of Eq. (10) is[5]

$$\Gamma_2(x)=f_2(x)\cos\alpha_1(x)+(1/\pi)\exp[\rho(x)+i\alpha_1(x)]$$
$$\times \mathrm{P}\int_{-\infty}^{\infty}\frac{f_2(x')\sin\alpha_1(x')\exp[-\rho(x')]}{x'-x-i\epsilon}dx', \quad (12)$$

where

$$\rho(x)=\frac{1}{\pi}\mathrm{P}\int_{-\infty}^{\infty}\frac{\alpha_1(x')}{x'-x}dx', \quad (13)$$

and again at the computation of the integrals use is to be made of the relation (7). Equations (8)–(13) allow the computation of $\Gamma_2(x)$ as soon as one knows M, m, μ, G, $\alpha_1(x)$, and $\alpha_{11}(x)$.

3. NUMERICAL RESULTS AND CONCLUSIONS

From (6)–(13) one sees that Γ_2 contains integrals of $\alpha_1(x)$ and $\alpha_{11}(x)$ times certain functions of x between threshold and ∞. The values of these phase shifts are not known in the high-energy region. Nevertheless one can overcome this difficulty by reasoning in the following way. At high energies the phase shifts will have a big imaginary part (due to the predominance of inelastic effects) while their real part decreases. Since at the computation of $f_2(x)$ the integrals of the α's appear through the $G_1{}^*$ factors in $\mathrm{Im}\Gamma_2{}^B$ and here the α integrals appear in the exponent, their imaginary parts produce only phase factors. An analogous reasoning is valid also for the second term in (12) where one has one more integration and a $\sin\alpha_1$ which nevertheless has a modulus ≤ 1 even at high energies. So one could take only the real parts of the α_1 and α_{11} phase shifts and add the moduli of the two terms in (12). One obtains in this way an upper limit for the real value of $\Gamma_2(x)$. Inserting in the formulas of section 2[7]

$$\alpha_{11}(x)=0,$$
$$\alpha_1(x)=(10.1x^2-24.6x+14.5)\times\Theta(1.32-x)\Theta(x-1.08) \quad (14)$$

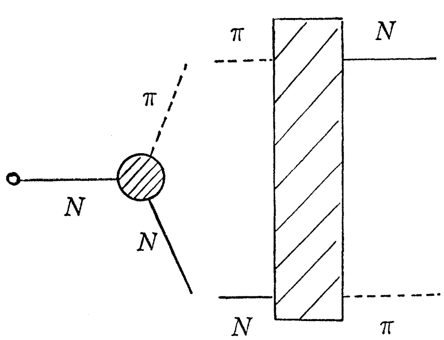

FIG. 1. Graph with lowest-mass parity-conserving intermediate state which contributes to the absorptive parts of Γ_i. External lines marked with a small circle at their ends represent off-shell particles.

[7] In (14), x is dimensionless and numerically equal to the value of $(-q^2)^{\frac{1}{2}}$ in Bev.

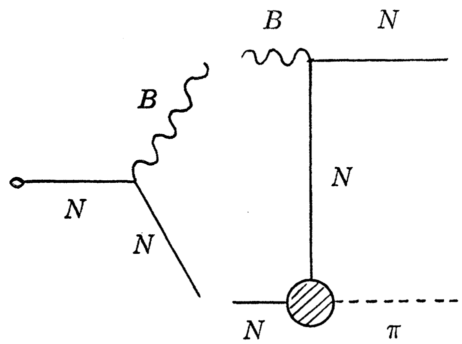

FIG. 2. Graph with lowest-mass parity-nonconserving intermediate state which contributes to the absorptive parts of Γ_i. External lines marked with a small circle at their ends represent off-shell particles.

(the parabola for α being chosen to fit the experimental data[8]), one obtains

$$\mathfrak{F}=\left|\frac{\Gamma_2(x)}{\Gamma_1(x)}\right|_{x\ll M^2}\sim 10^{-2}(m/M)G^2, \quad (15)$$

for the order of parity nonconservation in amplitude near the mass shell (the region interesting in low-energy nuclear physics). Since G^2M^{-2} has to equal the UFI coupling constant $g_F\sim 10^{-5}m^{-2}$, one can see from (15) that $\mathfrak{F}\sim 10^{-7}M/m$ and thus strongly depends on M. Choosing for M the value which is necessary for the UFI to bind an extreme relativistic $N\bar{N}$ pair into a π meson, which by reference 1 is $\sim$400 Bev, one finds $\mathfrak{F}\lesssim 10^{-5}$, which does not contradict experimental evidence.[2] This same order of magnitude for parity nonconservation evidently appears in the Born approximation to $\pi-N$ scattering amplitude due to the $\Gamma_1\Gamma_2$ terms.

It is interesting to remark that a perturbation theory treatment of the graph of Fig. 2 would yield $\mathfrak{F}\sim(1/2\pi^2)(m^2/M^2)G^2$. This result is independent of M and is smaller than (15). This happens because for the particular case of a $V-A$ interaction the graph of Fig. 2 gives a convergent result so that it cannot depend on the cutoff mass. Nevertheless higher order graphs would give divergent and thus M-dependent contributions which might be bigger then the lowest order graph, Fig. 2, and could account for the result (15) obtained by dispersion theory. At any rate first-order perturbation theory cannot be relied upon in this case.

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[8] E. L. Lomon, *1958 Annual International Conference on High-Energy Physics at CERN* (CERN Scientific Information Service, Geneva, 1958), p. 63.