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Left-Right Asymmetry in Pair Annihilation for Transversally Polarized Positrons

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A left-right asymmetry of the photons in the electron-positron annihilation cross section is found for the case when only the incident positron is polarized. This asymmetry results from a radiative correction. In the calculation the target electron is unpolarized and it is assumed that the polarizations of the resulting photons are not measured. The maximum asymmetry found is several parts in ten thousand.

I. INTRODUCTION

SEVERAL[1–3] left-right asymmetries in scattering have recently been predicted where the interaction is purely electromagnetic and only one particle polarization is known. These asymmetries come from a radiative correction. In this paper we find a similar effect in electron-positron pair annihilation. We consider the case where a polarized positron strikes an unpolarized electron. It is assumed that the polarizations of the resulting two photons are not measured. The possible interactions with an atom to which the electron may be bound are not taken into account. We consider that we are looking in the direction of the positron velocity and that the photon plane is horizontal. Left or right, then, refers to the direction of emission of a photon. A photon emitted to the right will be said to have a positive angle, while one to the left has a negative angle. The cross section, calculated here, changes sign on changing the signs of the photon angles or on reversing the direction of positron spin.

II. CALCULATION OF THE CROSS SECTION

We have calculated the spin-sensitive correction to the annihilation cross section by the standard Feynman method.[4] The calculation is essentially a modification of that of Harris and Brown[5] (hereafter called HB), who found the lowest order radiative correction to the cross section for the case of no polarization. We adopt their notation by choosing q_1, q_2, p_1, and p_2 as the four-momenta of the two photons, the electron, and the positron, respectively. In the rest frame of the electron the necessary modification to treat the case of the polarized positron consists of inserting into the Dirac traces, next to the positron energy projection operator, the spin projection operator, $\frac{1}{2}(1+\mathbf{s}\cdot\boldsymbol{\sigma})$. Here $\mathbf{s}$ is a unit vector in the direction of the positron spin, and $\boldsymbol{\sigma}$ has as its three components the four-by-four Pauli spin matrices. The simple form of this spin projection operator for the moving positron results from the equal masses of the electron and positron and the fact that only transverse polarization of the positron gives a contribution. In the rest frame of the positron, the spin projection operator would be $\frac{1}{2}(1-\mathbf{s}\cdot\boldsymbol{\sigma})$. In terms of Dirac γ matrices,[6]

$$\mathbf{s}\cdot\boldsymbol{\sigma}=\tfrac{1}{2}i\epsilon_{lmn}s_l\gamma_m\gamma_n,$$

where ϵ_{lmn} is the antisymmetric three-index symbol and repeated Latin indices are to be summed from 1 to 3.

In the lowest order calculations, the only imaginary part is the pure imaginary $\mathbf{s}\cdot\boldsymbol{\sigma}$. Therefore, the traces involving this factor either vanish or cancel each other, since otherwise they would contribute an imaginary part to the cross section. Thus, the lowest order in which $\mathbf{s}\cdot\boldsymbol{\sigma}$ gives a contribution results from the interference between the matrix elements proportional to e^2 and e^4. In the resulting e^6 order trace, only the imaginary parts of integrals over the virtual photon momenta combined with the imaginary $\mathbf{s}\cdot\boldsymbol{\sigma}$ can give nonvanishing terms. This is in contrast to the unpolarized case treated in HB where only the real parts of the integrals contribute. These integrals are given in HB and by Brown and Feynman.[7] The imaginary parts were checked by the method of residues.[2] The trick of changing q_1 to $-q_1$ and p_2 to $-p_2$ to obtain the annihilation cross section from the Compton cross section already calculated[3] cannot be used here since the imaginary parts of the integrals are greatly changed by this substitution. Thus, while several radiative correction diagrams contributed in the Comp-

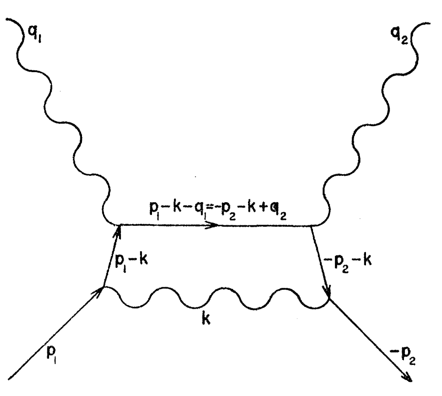

FIG. 1. The Feynman diagram which contributes to the asymmetry.

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[1] A. O. Barut and C. Fronsdal, Phys. Rev. **120**, 1871 (1960).
[2] C. Fronsdal and B. Jaksic, Phys. Rev. **121**, 916 (1961).
[3] S. C. Miller and R. M. Wilcox, Phys. Rev. **124**, 637 (1961).
[4] R. P. Feynman, Phys. Rev. **76**, 749, 769 (1949).
[5] I. Harris and L. M. Brown, Phys. Rev. **105**, 1656 (1957).
[6] The metric used is $g_{00}=1$, $g_{11}=g_{22}=g_{33}=-1$. Also units are chosen such that $\hbar=c=1$.
[7] L. M. Brown and R. P. Feynman, Phys. Rev. **85**, 231 (1952).

ton case, the only diagrams contributing here are the one shown in Fig. 1 and the one obtained from it by interchanging q_1 and q_2.

The Dirac traces were calculated by machine using a computer program recently developed by the authors to obtain general Dirac traces.[8] After the integrals over the virtual photon momenta are performed, the spin-dependent part of the differential cross section is found to be

$$d\sigma_s/d\Omega_1 = -\tfrac{1}{4}\alpha r_0^2 \tau\gamma^{-1}(\tau+\gamma)^{-1} X \mathbf{s}\cdot(\mathbf{q}_1\times\mathbf{q}_2), \quad (1)$$

where

$$\begin{aligned} X = (\tau-\gamma)\{&2(\sinh 2x)^{-2} - 3d^{-1}\ln[(\tau+\gamma)/2\tau\gamma] \\ &+2x(d\sinh 2x)^{-1}(5-3\tau-3\gamma)\} + (d\sinh^2 2x)^{-1} \\ &\times[8(\tau^2+\tau\gamma+\gamma^2) - 3(\tau+\gamma)(2+\tau^2+\gamma^2)]\ln(\gamma/\tau) \end{aligned}$$

with

$$\begin{aligned} \tau = (p_1\cdot q_1)/m^2, \quad &\gamma = (p_1\cdot q_2)/m^2, \\ d = 2\tau\gamma - \tau - \gamma, \quad &\sinh 2x = [(\tau+\gamma)(\tau+\gamma-2)]^{\frac{1}{2}}. \end{aligned}$$

Also $d\Omega_1$ is the differential solid angle for q_1, α is the fine structure constant, and r_0 is the classical electron radius. Equation (1), valid in the laboratory system, will apply to a more general Lorentz frame if $\mathbf{s}\cdot(\mathbf{q}_1\times\mathbf{q}_2)$ is replaced by $m^{-1}\det\{s,q_1,q_2,p_1\}$, the four-by-four determinant formed from the vectors s, q_1, q_2, and p_1 with s now the covariant spin vector.[9] Also if the roles of the electron and positron are interchanged, the cross section is unchanged.

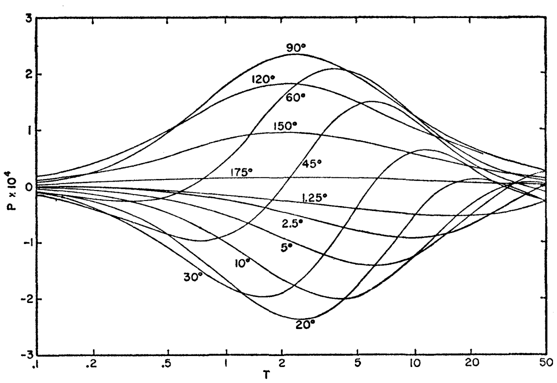

FIG. 2. The ratio of asymmetric to total differential cross section for annihilation is plotted against positron kinetic energy in units of electron rest energy. The angles indicated are those between $\mathbf{q}_1$ and $\mathbf{p}_2$.

At low positron kinetic energies Eq. (1) reduces to the simple form,

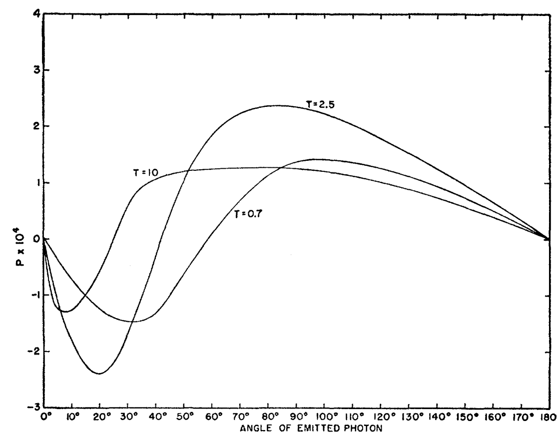

FIG. 3. The ratio of asymmetric to total differential cross section for annihilation is plotted against the angle between $\mathbf{q}_1$ and $\mathbf{p}_2$. The kinetic energy, T, of the positron in units of electron rest energy is indicated on the curves.

$$d\sigma_s/d\Omega_1 = -\alpha r_0^2 v^2 \sin 2\Phi(1+\sin^2\Phi)\mathbf{s}\cdot\mathbf{u}/96, \quad (2)$$

where v is the positron velocity, Φ is the angle between $\mathbf{q}_1$ and $\mathbf{p}_2$, and $\mathbf{u}$ is a unit vector in the direction of $\mathbf{q}_1\times\mathbf{q}_2$.

As pointed out in HB, for a Coulomb correction this cross section should be multiplied by the Sommerfeld factor, $2\pi\xi(1-e^{-2\pi\xi})^{-1}$, with $\xi = e^2/v$. Even with this factor the spin-dependent part of the cross section goes to zero as the kinetic energy goes to zero.

III. NUMERICAL RESULTS

The ratio of the cross section of Eq. (1) to the unpolarized annihilation cross section, $P = (d\sigma_s/d\Omega_1)/(d\sigma/d\Omega_1)$, has been plotted in Figs. 2 and 3. These figures are for the case of maximum asymmetry with $\mathbf{s}$ normal to the photon plane such that $\mathbf{s}\cdot\mathbf{u}=1$. For various fixed angles Fig. 2 gives P versus T, the incident positron kinetic energy in units of the electron rest energy. Figure 3 gives P as a function of the angle $\mathbf{q}_1$ makes with $\mathbf{p}_2$ for three energies. The maximum effect occurs at about 19° and 81° emission angles and at $T=2.6$. If one photon is emitted at one of these two angles, the other photon is emitted at minus the other angle. Therefore, these are equivalent maxima.

ACKNOWLEDGMENTS

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[8] R. M. Wilcox, thesis.

[9] See, e.g., C. Fronsdal and H. Überall, Phys. Rev. **111**, 580 (1958), and other references given there. In our metric, $s^2 = -1$.