son with the effective-range formula:

$$\frac{4}{3\mu^2}\frac{p^3}{W-m}\cot\delta_{33}=\frac{4}{3\mu^2}\frac{W}{(E+m)(W-m)}\operatorname{Re}\frac{1}{h_{22}}$$
$$=\frac{1}{f^2}\frac{(W_r-W)}{W_r-m}, \tag{6.28}$$

where W_r is the position of the 33 resonance.

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Conservation of Hypercharge*

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It is shown that the law of conservation of hypercharge follows from a certain permutation symmetry of the Lagrangian provided that trilinear interactions are assumed. The permutation involves "primitive" isospin doublet fields, which are linear combinations of fields corresponding to the observed isospin doublets N, Ξ, K, K^G. No permutation symmetry for the observed particles is implied. If the assumption of trilinear interactions is not made, a multiplicative law of hypercharge conservation "modulo 4" is obtained. An argument is presented to show that this is inconsistent with experiment.

I. INTRODUCTION

According to the present description of elementary particles, the strong interactions obey several conservation laws, and hence are invariant under certain "internal" transformations. These are the 3-dimensional isospin group, and two phase groups, which may be taken to be the baryon gauge group and the hypercharge gauge group. Since the known particles satisfy the Gell-Mann–Nishijima relation,

$$Q=I_3+\tfrac{1}{2}U, \tag{1}$$

the conservation of electric charge follows automatically from these for such particles.[1] It is of some interest to see whether the existence of such invariances may be derived from a different assumption, which may be applicable beyond the strong interactions. In this note, we show that a multiplicative law of hypercharge conservation, of the type considered by D'Espagnat and Prentki, and by Racah,[2] is implied by invariance of the strong interactions under a certain permutation of the fields involved. The permutation symmetry is similar to the one studied previously in connection with the muon-electron system,[3,4] and the argument given below is a simple extension of previous arguments to the strong interactions.

It is known that if one restricts interactions to trilinear ones, then the multiplicative law of hypercharge conservation implies the full hypercharge gauge group, which is equivalent to an additive hypercharge conservation law. Such an additive law is required by experiment, for reasons we indicate at the end of this note, and therefore it is necessary to impose some additional restrictions such as trilinear couplings in order to obtain the hypercharge gauge group. We will therefore consider only such couplings in this paper. Such restrictions have been considered before in other connections.

II. DERIVATION OF HYPERCHARGE CONSERVATION

We assume that the strong interactions involve the following groups of particles.

(1) Two baryon isotopic doublets B_1, B_2:

$$B_1=\begin{pmatrix}B_1{}^1\\B_1{}^2\end{pmatrix},\quad B_2=\begin{pmatrix}B_2{}^1\\B_2{}^2\end{pmatrix}. \tag{2}$$

We group these into a quadruplet B by taking

$$B=\begin{pmatrix}B_1\\B_2\end{pmatrix},$$

a doublet in a new space which we call hypercharge space.

(2) A meson isotopic doublet M, and a doublet M^G,

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1 This need not be the case for particles not obeying Eq. (1). In particular, it is possible to construct interactions for such particles which conserve hypercharge, isospin, and baryon number, but not electric charge.
2 B. D'Espagnat and J. Prentki, Nuclear Phys. 1, 33 (1956); G. Racah, *ibid.* 1, 302 (1956).
3 N. Cabibbo and R. Gatto, Phys. Rev. Letters 5, 114 (1960).
4 G. Feinberg, P. K. Kabir, and S. Weinberg, Phys. Rev. Letters 3, 527 (1959).

formed from its charge conjugate:

$$M=\begin{pmatrix}M^1\\M^2\end{pmatrix},\quad M^G=\begin{pmatrix}M^{2\dagger}\\-M^{1\dagger}\end{pmatrix}. \tag{3}$$

We also group these into a doublet,

$$D=\begin{pmatrix}M\\M^G\end{pmatrix},$$

in hypercharge space. These doublets are not to be identified yet with known particles, and their elements do not have well-defined charge, or hypercharge.

(3) The usual Σ and π isotopic triplets and Λ singlet.

Let us now ask how these fields can appear in a Lagrangian invariant under isotopic rotations, and the baryon gauge group, but not the hypercharge or charge gauge group. It is easy to see that the assumed invariances place no restriction on the form of interactions in hypercharge space, although they require that the interactions are scalars in isotopic space. In particular the following terms may appear.

(1) "Hypercharge invariant" terms:

(a) mass terms—$\bar{B}_1B_1$, $\bar{B}_2B_2$, $\bar{M}M$, etc.,
(b) pion interactions—$\bar{B}_1\tau_iB_1\pi_i$, $\bar{B}_2\tau_iB_2\pi_i$, etc., (τ_i is a matrix in isospin space),
(c) M interactions—$\bar{B}_1M\Lambda$, $\bar{B}_2M^G\Lambda$, etc.

(2) "Hypercharge noninvariant" terms:

(a) Off diagonal mass terms—$\bar{B}_1B_2$, $\bar{M}M^G$,
(b) Pion interactions—$\bar{B}_1\tau_iB_2\pi_i$,
(c) M interactions—$\bar{B}_1M^G\Lambda$, $\bar{B}_2M\Lambda$.

Here the phrase "hypercharge noninvariant" means that if the fields B_1 and B_2 were identified with nucleon and cascade particle, and the field M with the K meson, then these terms would violate hypercharge and charge conservation, while the others would not. We may write the general free field and trilinear interaction Lagrangian for these fields as follows:

$$\begin{aligned}L_0&=\bar{B}\mathfrak{M}_1B+\bar{D}\mathfrak{M}_2D+\bar{B}\gamma_\rho\partial_\rho B+\tfrac{1}{4}(\partial_\mu\bar{D}\partial_\mu D),\\ L_{\text{int}}{}^{\pi}&=g\bar{B}\mathfrak{M}_3\tau_iB\pi_i+a\epsilon_{ijk}\bar{\Sigma}_i\Sigma_j\pi_k+b\bar{\Sigma}_i\Lambda\pi_i+\text{H.c.},\\ L_{\text{int}}{}^{M}&=c\bar{B}\mathfrak{M}_4D\Lambda+d\bar{B}\mathfrak{M}_5\tau_iD\Sigma_i+\text{H.c.}\end{aligned} \tag{4}$$

We have omitted Dirac operators in the interactions. $\mathfrak{M}_1\cdots\mathfrak{M}_5$ are 2×2 matrices in hypercharge space, while τ_i acts in isospin space.

For this Lagrangian, there is in general no hypercharge conservation, either for the fields B, M, or for any linear combinations of them. The first of these follows because if the $\mathfrak{M}_i$ are arbitrary 2×2 matrices, then L will not be invariant under the transformations which would represent hypercharge for B, D:

$$\begin{aligned}B&\to e^{i\alpha\sigma_3}B,\\ D&\to e^{i\alpha\sigma_3}D.\end{aligned} \tag{5}$$

The nonconservation of hypercharge for linear combinations of B_1, B_2 or M, M^G follows because such a conservation law would require that the matrices $\mathfrak{M}_1\cdots\mathfrak{M}_5$ could be simultaneously diagonalized by a similarity transformation, which is also not possible for arbitrary matrices.

However, if we follow the reasoning of references 3 and 4, we are led to assume invariance of L under the discrete transformation H, which acts as follows:

$$\begin{aligned}HDH^{-1}&=i\sigma_1D,\\ HBH^{-1}&=i\sigma_1B,\end{aligned} \tag{6}$$

all other fields invariant, where σ_1 acts in hypercharge space.

$$\begin{pmatrix}B_1\to iB_2, & M\to iM^G\\ B_2\to iB_1, & M^G\to iM\end{pmatrix}.$$

One may think of this transformation as requiring symmetry among the primitive isospin doublet fields in the Lagrangian. It may be seen that invariance under H implies that the matrices $\mathfrak{M}_1\cdots\mathfrak{M}_5$ all commute with σ_1:

$$\sigma_1\mathfrak{M}_i\sigma_1=\mathfrak{M}_i. \tag{7}$$

It follows immediately that $\mathfrak{M}_i=A_i+B_i\sigma_1$, where A_i, B_i are numbers. It is therefore possible to find a unitary transformation S which diagonalizes all $\mathfrak{M}_i$ at once, i.e., if

$$B'=SB,\quad D'=SD,$$

where S is a 2×2 matrix in hypercharge space, then $L(B,D,\mathfrak{M}_i)=L(B',D',S\mathfrak{M}_iS^{-1})$, and we choose S so that

$$S\sigma_1S^{-1}=\sigma_3,$$

so that

$$S\mathfrak{M}_iS^{-1}=A_i+B_i\sigma_3,$$
$$[S=(\sigma_1+\sigma_3)/\sqrt{2}].$$

Since only the unit matrix and σ_3 in hypercharge space appear in the Lagrangian for the primed fields, this Lagrangian is invariant under the gauge group

$$\begin{aligned}B'&\to e^{i\alpha\sigma_3}B',\\ D'&\to e^{i\alpha\sigma_3}D'.\end{aligned} \tag{8}$$

The primed fields may now be identified with the observed particles N, Ξ, and K by the rule

$$B'=\begin{pmatrix}N\\\Xi\end{pmatrix},\quad D'=\begin{pmatrix}K\\K^G\end{pmatrix}, \tag{9}$$

so

$$N=(B_1+B_2)/\sqrt{2},\quad \Xi=(B_1-B_2)/\sqrt{2},\ \text{etc.}$$

For these fields, the group (8) may be seen to be the hypercharge gauge group.

It is important to realize that the Lagrangian obtained has in general no permutation symmetry for the fields N, Ξ or K, K^G. This is because of the term with σ_3 which remains in the $\mathfrak{M}_i$ after diagonalization. Thus,

for example, the mass term of the baryons will be:

$$(\bar{N} \quad \bar{\Xi})\begin{pmatrix} A_1+B_1 & 0 \\ 0 & A_1-B_1 \end{pmatrix}\begin{pmatrix} N \\ \Xi \end{pmatrix}, \tag{10}$$

so that $m_N = A_1+B_1$, $m_\Xi = A_1-B_1$, and these are unequal unless $B_1=0$. Similarly, the interactions of N and Ξ will in general be different.[5] Even the parities of N and Ξ can be different. In this case however, the parity operator would involve a permutation of the original fields B_1, B_2. The symmetry imposed on the primitive fields B_1, B_2 shows up as a conservation law for the observed fields.

Another point to emphasize is that we have not restricted the interactions by any "minimal" principle. The symmetry H alone is enough to ensure that the $\mathfrak{M}_i$ can be simultaneously diagonalized. Contrary to previous speculations,[3,4] the same result holds for the μ—e system, even if μ and e have anomalous magnetic moments, providing that the primitive fields satisfy a permutation symmetry.

III. FORM OF THE HYPERCHARGE CONSERVATION LAW

The result which has been obtained, that the strong interactions are invariant under the hypercharge gauge group, specifically follows in the case of trilinear interactions. In the general case, the permutation symmetry H implies invariance of the physical fields B', M' under the discrete operation:

$$\begin{aligned} B' &\to i\sigma_3 B', \\ D' &\to i\sigma_3 D'. \end{aligned} \tag{11}$$

It is possible to construct 4 fermion interactions and 4 meson interactions which are invariant under (11), but not for the full group.[6] If this were all the symmetry present, it would make hypercharge a multiplicative quantum number,[7] and allow strong interactions with $\Delta U=4$ ($\Delta S=4$). This possibility can be ruled out experimentally for the known particles by the following argument.

Since the known particles satisfy $Q=I_3+\frac{1}{2}U$, it follows that $\Delta Q=\Delta I_3+\frac{1}{2}\Delta U$ in any interaction. Thus if $\Delta U=4$, then either $\Delta Q\neq 0$ or $\Delta I_3\neq 0$. Experimentally, with great accuracy,[8] processes with $\Delta Q\neq 0$ are absent for known particles. So if $\Delta U=4$ for some strong interactions, then $\Delta I_3=2$. But I_3 is a component of a vector, and if $\Delta I_3=2$, it follows that $|\Delta I|\geqslant 2$. That is, the interactions which change I_3 by 2 units must change the total isotopic spin by at least two units, and so transform in isospin space as a spherical tensor of some rank greater than 1. If this interaction acts twice, then it generates terms with $\Delta I_3=0$ and with $\Delta I=0, 1, 2, 3, 4$, by the Clebsch-Gordan theorem. The terms with $|\Delta I| = 1$ will lead to strong interactions which change isospin, but not strangeness, such as $d+d\to He^4+\pi^0$, which have been looked for, and not found.[9] We therefore conclude that strong interactions with $\Delta S=4$ cannot occur. A similar argument rules out strong interactions which change strangeness, and hence I^2, by any number of units. The crucial step in the argument is that $\mathbf{I}$ adds vectorially, and not linearly. Therefore, hypercharge cannot be a multiplicative quantum number for the known particles.

We conclude that if the conservation of hypercharge is to follow from the permutation symmetry H, it is necessary to make some specific assumption about the form of interactions.

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[5] Since $\bar{K}K=\bar{K}^G K^G$, it follows that K and K^G nevertheless have equal mass, as required by TCP invariance.

[6] An example of an interaction invariant under (11) but not for the full group is $\bar{B}'\sigma_1 B'\bar{B}'\sigma_1 B' = (\bar{N}\Xi+\bar{\Xi}N)(\bar{N}\Xi+\bar{\Xi}N)$.

[7] Compare with the discussion in G. Feinberg and S. Weinberg, Phys. Rev. Letters **6**, 381 (1961).

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[9] Yu. K. Akimov *et al.*, *Proceeding of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 49.