by the nucleon pole (see Table I). At 800 Mev, however, this might not be the case, and the more precise experiment suggested in this paper, which would allow an extrapolation beyond 1.5, might show the effect.

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Global Symmetry Resonance in Pion-Hyperon Scattering*

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Dispersion relations for fixed momentum transfer are applied to the problem of the Y_1^*, and it is shown that an s-wave resonance in π-Λ scattering does not appreciably influence the position of the p-wave π-Λ resonance predicted by global symmetry.

The existence of the Y_1^* resonance now seems well established.[1] However, due to the effect of Bose statistics on the final two pions in the reaction

$$K^- + p \rightarrow Y_1^* + \pi \rightarrow \Lambda + \pi^+ + \pi^-,$$

the spin state of the Y_1^* is still an open question.[2] Theoretically, global symmetry[3] suggests that it should be $p_{\frac{3}{2}}$, while the calculation by Dalitz based of the s-wave $\bar{K}$-N scattering lengths shows that $s_{\frac{1}{2}}$ is an equally likely assignment.

The purpose of the present letter is to pursue a remark by Dalitz[4] that, if the resonance should turn out to be $s_{\frac{1}{2}}$, then perhaps the coupling of the $\bar{K}$-N system to the π-Y system might have shifted the resonance predicted by global symmetry to an extreme energy range, or that the coupling might destroy the resonance entirely. In this respect, two mechanisms for destroying global symmetry come to mind. First, there could exist a relatively strong nonglobally symmetric p-wave K interaction (e.g., due to a K meson-nucleon intermediate state); or second, recoil effects might couple a strong s-wave π-Λ interaction into the globally-symmetric p-wave equations. Concerning the first mechanism, there does not appear to be a very strong p-wave interaction in the reaction $\bar{K}+N \rightarrow \pi+\Lambda$.[5] However, since virtual effects may be expected to enter, this possibility is not really precluded. However, there does appear an *a priori* reason why the second mechanism might be significant. Chew *et al.*[6] have shown in π-N scattering that the resonating (3,3) p-wave produces a striking contribution to s-wave equations when recoil is taken into account. It is conceivable, therefore, that a corresponding effect might appear in the globally symmetric p-wave equations if the Dalitz s-wave resonance does in fact exist.

In this note we shall examine the latter possibility without investigating the effects of the first mechanism. A s-wave resonance is assumed for pion-hyperon scattering; it will be shown, however, that such a s-wave resonance has a negligible influence on the position of the $p_{\frac{3}{2}}$ resonance predicted by global symmetry.

As a first attempt, we use unsubtracted dispersion relations with a suitable cutoff. The technique used to couple the s-wave into the p-wave equations is that of Chew *et al.*,[6] and the notation used is identical with theirs. We write the basic dispersion relations for π-Λ scattering as

$$\mathrm{Re}A(\nu_L,\kappa^2) = \frac{P}{\pi}\int_1^\infty d\nu_L' \,\mathrm{Im}A(\nu_L',\kappa^2) \times\left(\frac{1}{\nu_L'-\nu_L}+\frac{1}{\nu_L'+\nu_L-2\kappa^2/M}\right), \quad (1)$$

and

$$\mathrm{Re}B(\nu_L,\kappa^2) = \mathrm{Poles} + \frac{P}{\pi}\int_1^\infty d\nu_L' \,\mathrm{Im}B(\nu_L',\kappa^2) \times\left(\frac{1}{\nu_L'-\nu_L}-\frac{1}{\nu_L'+\nu_L-2\kappa^2/M}\right), \quad (2)$$

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where ν_L is the total meson energy in the lab, κ^2 is one-fourth of the invariant momentum transfer squared, and M is the Λ mass.[7]

According to reference (6), we have approximately

$$-\frac{6}{q^2}\operatorname{Re}f_{P\frac{3}{2}}=\operatorname{Re}\frac{E+M}{2W}\frac{1}{4\pi}\times[A'+\tfrac{1}{2}q^2A''+(W-M)(B'+\tfrac{1}{2}q^2B'')],\quad (3)$$

where the primes denote derivatives with respect to κ^2 evaluated at $\kappa^2=0$, and q is the center-of-mass meson momentum. The total energy and the total nucleon energy in the c.m. system are denoted by W and E, respectively. The dispersion relations are now used to evaluate the right-hand side of Eq. (3), where we keep only s and $p_{\frac{3}{2}}$ terms under the dispersion integrals, and only keep terms to first order in the recoil (i.e., $1/M$).

For our purposes, it is only necessary to record the correction terms due to the coupling of s wave into p wave to first order in recoil. These correction terms are denoted by a bar over the appropriate quantities. We then have that

$$-\frac{6}{q^2}\operatorname{Re}\bar{f}_{P\frac{3}{2}}\approx\frac{1}{2\pi^2M}\int_1^\infty d\nu_L'\frac{\sigma_s q'}{(\nu_L'+\nu_L)^2}\equiv\epsilon(\nu_L),\quad (4)$$

where σ_s is the total s-wave π-Λ cross section. We might note that the correction approaches zero as M becomes large, contrary to the situation in pion-nucleon scattering where the coupling of p wave into s wave goes linearly with the mass. If one estimates the $p_{\frac{3}{2}}$ contribution to the dispersion integrals by the global symmetry solution to the p-wave equations, then one sees that the shift $\Delta\omega$ in the resonance energy (ω_0) is given approximately by

$$\Delta\omega/\omega_0=-(f^2/3)(q_0^6/\omega_0)\epsilon(\nu_L^0).\quad (5)$$

This gives a decrease in the resonant energy of only 10 Mev or less, even if the maximum value for σ_s ($4\pi/q^2$) is taken.[8] This is then a maximum estimate of the effect within the framework given here.

This simple calculation (which has ignored any possible nonglobal effects in the p wave) argues that if global symmetry is a valid pion-baryon coupling scheme, then the Y_1^* should, at least in part, be a $p_{\frac{3}{2}}$ resonance. Should this later turn out not to be the case, we would then have an argument against global symmetry.

[7] The invariant scattering amplitude T is given by $T=-A+i\gamma\cdot QB$ with $Q=\frac{1}{2}(q_1+q_2)$ where q_1 and q_2 are the initial and final pion four-momenta.

[8] Note that no low- or high-energy cutoff is required.

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Wave Functions*

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A convenient definition of wave functions, in terms of S-matrix quantities, may be obtained from the dispersion relations for hypothetical reactions in which a probe interacts impulsively with a target. The wave functions defined in this way are useful for constructing an impulse approximation for complicated reactions. These wave functions also satisfy a wave equation.

I. INTRODUCTION

DISPERSION theory techniques have been applied with great success to such problems as the analysis of the form factors of "simple" particles (those which do not lead to anomalous thresholds) and the analysis of a transition amplitude $T(a+b\rightarrow c+d)$ involving four simple particles.[1] There is no doubt that dispersion theory is also applicable to more complicated problems, in which additional particles are present, either in the final state of a production process, or through the virtual decay of a nonsimple particle. In these more complicated problems, techniques for practical calculation are relatively undeveloped although, partial results have already been obtained by straightforwardly extending the systematic methods used in simpler problems.[2,3] However, in production processes it is possible that the systematic method of writing down a dispersion relation in one or more variables will not turn out to be the most useful one, because there are so many independent variables, and because the singularities, which lie on complicated curves in complex regions, are very difficult to treat exactly. It is certainly clear, that more intuitive or "physical" approaches which would help single out the most

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† Now at NORDITA, Copenhagen, Denmark.
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