Beck[1] found in their low-temperature specific heat measurements with Cr-Fe alloys a strong variation of the Debye temperature with the composition. Near the peak of the density-of-states curve, θ_D was very large, but alloys with 10 at. % and 15 at. % Fe had an unusually low θ_D. It seems, therefore, likely that the low-temperature specific heat term proportional to T^3 is not due to lattice vibrations only, but that it comprises other contributions to the specific heat as well.

CONCLUSIONS

The specific heat of chromium-rich Cr-Fe alloys measured in the temperature range from $-140°$ to 350°C can be separated into electronic and lattice specific heats. The lattice specific heat is described by the Debye function, with θ_D between 440° to 465°K. Because these values agree reasonably well with accepted θ_D values for iron and chromium, it is suggested that the different θ_D found at low temperatures are not real, but due to additional T^3 terms in the specific heat. The electronic specific heat in our experiments is within the experimental accuracy described by $c_E=\gamma T$. γ shows a maximum at the composition of 19 at. % Fe, the same composition at which a maximum was observed at low temperatures. This indicates that the term linear in T in the specific heat is essentially an electronic contribution and it reflects a high density of states.

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Interband Transitions in Superconductors

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When the photon energy is such that an interband transition occurs between the valence and conduction bands in the normal state, small differences in the optical properties of the normal and superconducting metals can arise. Using a simple energy-gap model for the superconductor, the change in the reflectivity and transmission is calculated and is shown to be measurable over a narrow frequency range, assuming realistic values for the pertinent parameters. This type of experiment may be useful for studies of both interband transitions in normal metals and anisotropy of the energy gap in superconductors.

I. INTRODUCTION

THE optical properties of superconductors have been studied extensively in the far infrared region[1] ($h\nu \approx 3.5kT_c$) and in the infrared region[2,3] ($h\nu \gg 3.5kT_c$). Ramanathan[3] found the change in the absorptivity of normal and superconducting Sn at 14 μ to be less than the 0.3% limit of his sensitivity. (At 14 μ, Sn has $h\nu/kT_c \approx 280$.) The far infrared measurements of Richards and Tinkham[1] show the absorptivity to be essentially the same in the normal and superconducting states for $h\nu/kT_c > 6$. It has also been noted by Tinkham and Ferrell[4] that the sum rule on the real part of the conductivity also implies that for $h\nu \gg 3.5kT_c$, the conductivity is the same in the normal and superconducting states.

This note points out that when the photon energy is such that an interband transition occurs between the valence and conduction bands in the normal state, small differences in the optical properties of the normal and superconducting metals can arise. Using a simple energy-gap model for the superconductor, the change in the reflectivity and transmission is calculated and is shown to be measurable, assuming realistic values for the pertinent parameters. This type of experiment may be useful for studies of both the interband transitions in normal metals and the anisotropy of the energy gap in superconductors.

II. CALCULATION

For simplicity, a parabolic two-band model is chosen with the energy extrema of the valence and conduction bands taken at $\mathbf{k}=0$. In the superconducting state, an energy gap of 2Δ is introduced symmetrically about the Fermi energy in the conduction band, as is shown in Fig. 1. The valence bands for the normal metal and

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the superconductor are assumed to be identical. According to this picture, the density of states in the conduction band is considered to be the same for $\mathcal{E}_c < \epsilon_F - \Delta$ and $\mathcal{E}_c > \epsilon_F + \Delta$. However, the states which occur in the normal metal between $\epsilon_F - \Delta < \mathcal{E}_c < \epsilon_F$ and $\epsilon_F < \mathcal{E}_c < \epsilon_F + \Delta$ are replaced in the superconductor by δ-function contributions at $\mathcal{E}_c = \epsilon_F - \Delta$ and $\mathcal{E}_c = \epsilon_F + \Delta$, respectively.

The complex conductivity of a metal can be separated into terms involving a single band (intraband conductivity) and terms involving two bands (interband conductivity).[5] Since interband processes occur at high photon frequencies ($h\nu \gg 3.5kT_c$), the intraband conductivity can be taken the same for the normal metal and superconductor. At optical frequencies nonlocal terms in the current-field relation can be neglected. For allowed transitions, the matrix elements of **p** are expanded about $\mathbf{k}=0$ and the lowest order terms are retained. Then, the contribution to the interband conductivity from transitions between the valence and conduction bands can be written as[5]

$$\sigma_{c,v}{}^{\text{inter}} = \frac{\hbar^2 e^2(\omega - i\tau_{cv}{}^{-1})}{2\pi^3 i m_0{}^2} |(v,0|p_x|c,0)|^2 \times \int d\mathbf{k} \frac{[f_0(\mathcal{E}_v) - f_0(\mathcal{E}_c)]}{[\mathcal{E}_c - \mathcal{E}_v][\hbar^2(\omega - i\tau_{cv}{}^{-1})^2 - (\mathcal{E}_c - \mathcal{E}_v)^2]}, \quad (1)$$

in which τ_{cv} and $(v,0|p_x|c,0)$ are, respectively, the relaxation time and matrix element of momentum for the transitions between the valence and conduction bands. For simplicity, these two parameters are assumed to be unchanged in going from the normal to the superconducting state. The Fermi function for energies $\mathcal{E}_v(\mathbf{k})$ and $\mathcal{E}_c(\mathbf{k})$ of the valence and conduction bands denoted by $f_0(\mathcal{E}_v)$ and $f_0(\mathcal{E}_c)$, respectively. The change in the interband conductivity on this model at $T=0°$K is

$$(\sigma_S{}^{\text{inter}} - \sigma_N{}^{\text{inter}}) = -\frac{\epsilon_g C b \beta^{\frac{1}{2}}}{16\pi\hbar(\alpha_{cv} + i\mu)} \left(\frac{\Delta}{\epsilon_g}\right)^2 \times [2(1+\eta\beta)^{-2} - (1+\mu - i\alpha_{cv} + \eta\beta)^{-2} - (1-\mu + i\alpha_{cv} + \eta\beta)^{-2}]. \quad (2)$$

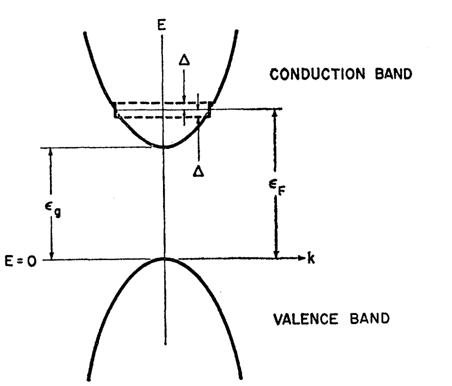

FIG. 1. The parabolic curves give the $\mathcal{E}(\mathbf{k})$ vs $\mathbf{k}$ for the valence and conduction bands of the normal metal with extrema at $\mathbf{k}=0$. In the superconducting state an energy gap of 2Δ appears, modifying the $\mathcal{E}(\mathbf{k})$ vs $\mathbf{k}$ for the conduction band.

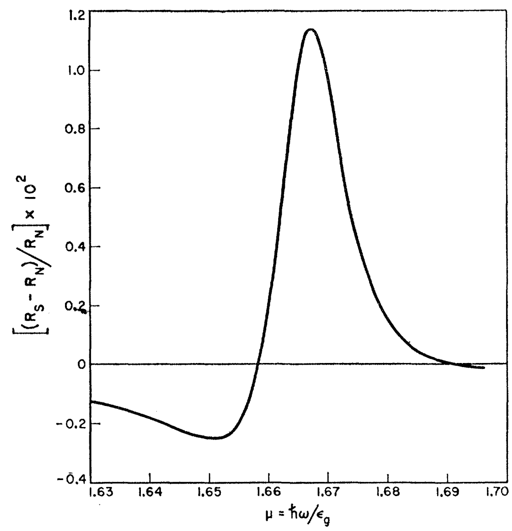

FIG. 2. Frequency variation of the change in reflectivity for the superconducting and normal metal relative to the normal state reflectivity. The frequency is expressed in dimensionless form as the photon energy relative to the band gap. See text for the definition of the pertinent parameters and for their values.

This effect is largest in materials with a high transition temperature, with small band gaps, and with strongly coupled bands. The strength of coupling between the valence and conduction bands is expressed by the parameter $C = 3m_c{}^*|(v,0|p_x|c,0)|^2/m_0{}^2\epsilon_g$, in which $m_c{}^*$ is the effective mass in the conduction band. The dimensionless parameters of Eq. (2) are defined as the Fermi energy parameter $\beta = (\epsilon_F - \epsilon_g)/\epsilon_g$, the interband relaxation parameter $\alpha_{cv} = \hbar/(\tau_{cv}\epsilon_g)$, the frequency parameter $\mu = \hbar\omega/\epsilon_g$, the effective-mass parameter $\eta = 1 + m_c{}^*/m_v{}^*$, and the parameter $b = 4\pi N e^2\hbar^2/\epsilon_g{}^2\beta^{\frac{3}{2}}$. The change $(\sigma_S{}^{\text{inter}} - \sigma_N{}^{\text{inter}})$ is appreciably different from zero only for a narrow range of photon energies $\hbar\omega \approx \epsilon_g(1+\eta\beta)$, the width of the range being characterized by the superconducting gap and by the interband relaxation parameter α_{cv}. The real part of $(\sigma_S{}^{\text{inter}} - \sigma_N{}^{\text{inter}})$ follows a dispersion-type curve, with a negative swing for $\mu < 1+\eta\beta$ and a positive swing for $\mu > 1+\eta\beta$. The imaginary part follows an absorption type curve, with a large positive swing about $\mu = 1+\eta\beta$, surrounded by broad shallow negative portions on either side.

The change in the reflectivity and transmission is found in a straightforward manner from the change in the interband conductivity (e.g., see reference 5). The results are shown as a function of photon energy in

[5] M. S. Dresselhaus and G. Dresselhaus (to be published).

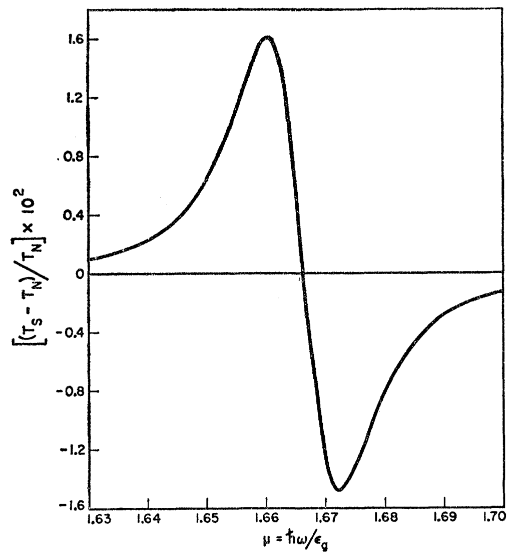

FIG. 3. Frequency variation of the change in transmission for the superconducting and normal metal relative to the normal state transmission. The frequency is expressed in dimensionless form as the photon energy relative to the band gap. See text for the definition of the pertinent parameters and for their values.

Figs. 2 and 3. The superconducting gap relative to the band gap is taken as $(\Delta/2\epsilon_g)=10^{-3}$, and equal effective masses for the valence and conduction bands are assumed. The other parameters are chosen as $\beta=\frac{1}{3}$, $\alpha_{cv}=0.01$, $b=2.5$, $C=1.5$, and a core dielectric constant of unity. With this choice of parameters, there is as much as a 1% change in the relative reflectivity $(R_S-R_N)/R_N$ and a 2% change in the relative transmission $(T_S-T_N)/T_N$. The range of photon energy which is of interest is $-0.03<(\mu-1-\eta\beta)<0.03$, where $\mu=1+\eta\beta$ denotes the onset of the interband transitions in the normal state and infinite τ_{cv}. The magnitude of the effect does not appear sensitive to the position of the Fermi energy, but is reduced as the core dielectric constant increases and as the interband relaxation time decreases. Only when the condition $\alpha_{cv}<(\Delta/2\epsilon_g)$ is satisfied, is the energy difference between the minimum and maximum in the reflectivity and transmission curves a reliable measure of the superconducting gap. This condition is difficult to realize experimentally.

III. DISCUSSION

The change in the reflectivity and transmission shown in Figs. 2 and 3 are large enough to be observable with presently available high-resolution equipment. If observable, this interband effect can be useful in the study of both normal and superconducting metals.

In the case of the normal metal, the interband effects are generally difficult to locate in ordinary reflection and transmission experiments. On reflection, interband effects correspond to a very small absolute change in reflectivity, which is difficult to measure as a function of frequency. The preparation of sufficiently thin samples for the more sensitive transmission studies is usually prohibitively difficult for good conductors. The application of a static magnetic field is extremely useful for the study of interband transitions in those metals which have low mass carriers to enhance the magnetic effects. In those materials for which the magnetic field is not a practicable tool, the comparison method provided by the normal-superconducting transition might prove useful in locating interband transitions.

The interband method is also of interest in the study of the superconducting state. The energy difference between the minimum and maximum in the reflectivity and transmission is a measure of the superconducting gap, but not a convenient or accurate measure of 2Δ. Broadening associated with interband relaxation processes complicate the interpretation of this energy difference. The changes in reflectivity and transmission can also be smeared out by anisotropy of the Fermi energy in the normal metal and by anisotropy of the superconducting gap on a particular piece of Fermi surface.

Although the superconducting energy gap can be measured more easily and accurately by other means such as tunneling, this method is particularly well suited to the study of the anisotropy of the superconducting gap associated with different bits of Fermi surface. Superconductors are, for the most part, metals which have a rather complex band structure with little pieces of isotropic Fermi surface at various spots in the Brillouin zone.[6] Since $|\mathbf{k}|$ is conserved in direct interband transitions, the superconducting gap associated with each band gap could be separated.

ACKNOWLEDGMENT

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[6] W. A. Harrison in *The Fermi Surface*, edited by W. A. Harrison and M. B. Webb (John Wiley & Sons, Inc., New York, 1960), p. 28; D. Shoenberg, *ibid*, p. 74.