Conversion-Electron Angular Correlations for Stripping Reactions*

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The $(d,p;e)$ and $(d,n;e)$ angular correlation between the outgoing proton or neutron of a stripping reaction and a possible internal conversion electron is theoretically investigated. Expressions for the correlation function similar to those for $(d,p;\gamma)$ and $(d,n;\gamma)$ angular correlations are obtained, differing only in the presence of the particle parameters $b_\nu(LL';e)$. It is suggested that conversion-electron correlations for stripping reactions may prove useful as a nuclear spectroscopic tool for the heavier nuclei.

I. INTRODUCTION

THE (d,p) and (d,n) stripping reactions, as is well known, can leave the residual nucleus in an excited state which can then decay by the emission of a gamma ray. The $(d,p;\gamma)$ and $(d,n;\gamma)$ angular correlations between the outgoing proton or neutron and the radiated gamma ray have been studied theoretically and experimentally by a number of investigators,[1] and yield information not obtainable from the differential stripping cross sections. In certain cases, however, internal conversion electrons may be emitted with high probability instead of gamma rays by excited nuclei. The purpose of this investigation is to consider theoretically the angular correlation between the outgoing particle of a stripping reaction and a possible internal conversion electron. It may be anticipated that $(d,p;e)$ and $(d,n;e)$ angular correlations, when they can be measured, will prove to be useful as a nuclear spectroscopic tool in the determination of nuclear spins and parities.

We shall first briefly sketch in Sec. II the theory of conversion-electron angular correlations for stripping processes, and then in Sec. III consider the feasibility and utility of the experimental measurements. Since, to our knowledge, no experiments of the proposed type have as yet been performed, it is not possible at the present time to present a detailed comparison between the theoretical and experimental results.

II. THEORY

We consider the stripping reaction $d+X\rightarrow Y^*+f$, where X is the target nucleus, Y^* is the excited residual nucleus, and f is the outgoing particle (either a neutron or a proton), followed by the conversion-electron process $Y^*\rightarrow Y+e$, where Y is the final nucleus of the two-step process. We wish to obtain the angular correlation function $W(\mathbf{k}_d,\mathbf{k}_f,\mathbf{k}_e)$ for this sequence, where W is the relative probability for the emission of f and e in directions specified by the center-of-mass wave vectors $\mathbf{k}_f$ and $\mathbf{k}_d$, respectively, for a reaction initiated by an incoming deuteron having a wave vector $\mathbf{k}_d$.

Let J_0, M_0; J, M; J_f, M_f be the total angular momentum and magnetic quantum numbers corresponding to the nuclear states of X, Y^*, and Y, respectively.[2] Let l_c and m_c denote the orbital and magnetic quantum numbers of the captured particle of the deuteron, and L the angular momentum transfer quantum number of the conversion electron.

The correlation function may then be written in the general form[3]

$$W(\mathbf{k}_d,\mathbf{k}_f,\mathbf{k}_e)=\sum_{M,M'}E_{MM'}{}^{(1)}E_{M'M}{}^{(2)},\qquad(1)$$

where

$$E_{MM'}{}^{(1)}=S_1\sum_{M_0}(JM|H_1|J_0M_0)(JM'|H_1|J_0M_0)^*,\qquad(2a)$$

$$E_{M'M}{}^{(2)}=S_2\sum_{M_f}(J_fM_f|H_2|JM)(J_fM_f|H_2|JM')^*.\qquad(2b)$$

H_1 and H_2 are the interaction Hamiltonians responsible for the stripping process and for the emission of the conversion electron, respectively. S_1 and S_2 represent summations over the unobserved variables for the two types of processes, respectively. As usual in angular correlations, the initial and final transitions may be considered independently of each other.

For the case in which the polarization of the conversion electron is not observed one obtains in the usual way the expression[4,5]

$$\begin{aligned}E_{M'M}{}^{(2)}=\sum_{L,L',\nu}&(-1)^M[(2L+1)(2L'+1)(2\nu+1)^{-1}]^{\frac{1}{2}}\\&\times(J_F\|T(L\pi)\|J)^*(J_F\|T(L'\pi)\|J)\\&\times C(LL'\nu;1-1)C(JJ\nu;M-M')\\&\times W(JJLL';\nu J)Y_\nu{}^{M'-M}(\Omega)b_\nu(LL';e),\end{aligned}\qquad(3)$$

where nonessential constant factors have been omitted.

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[1] See the discussion by S. T. Butler and O. H. Hittmair in *Nuclear Stripping Reactions* (Horwitz Publications, Inc., Sydney, 1957), pp. 84 ff. References to other theoretical and experimental studies will here be found.

[2] The notation used here is that of reference 1.

[3] S. Devons and L. J. B. Goldfarb, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. **42**, p. 382.

[4] L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953). See especially Eqs. (13), (42), and (62).

[5] J. L. Richter, Ph.D. dissertation, University of Texas, Austin, Texas, 1958 (unpublished). The conversion-electron angular correlation for stripping reactions is here considered using a somewhat different approach.

The notation is as defined in reference 4. Only the particle parameters $b_\nu(LL';e)$ differentiate this expression from that for gamma emission. Numerical tables and expressions for these quantities are available.[4,6]

The quantities $E_{MM'}^{(1)}$, on the other hand, are characteristic of the stripping portion of the angular correlation process and are independent of whether the particle emitted in the conversion process is a gamma ray or an electron. They depend on the amplitudes for the absorption of the captured particle,[7] and can be evaluated for a particular nuclear model and coupling scheme.[8]

For example, if one first couples the spin of the captured particle $\mathbf{s}_c$ to $\mathbf{J}_0$ to form the channel spin $\mathbf{s}$, and then couples $\mathbf{s}$ with $\mathbf{l}_c$ to obtain $\mathbf{J}$, then

$$(JM|H_1|J_0M_0)=\sum_{l_c,s} C(\tfrac{1}{2}J_0s;\mu_c'M_0)C(sl_cJ;m_sm_c) \times C(\tfrac{1}{2}\tfrac{1}{2}1;\mu_c'\mu_f')\int d\xi d\mathbf{r}_c d\mathbf{r}_f \psi_f(\mathbf{r}_f)^* v_{l_c}(\xi;J_0M_0)^* \times Y_{l_c}^{m_c}(\Omega_c)^* v(\mathbf{r}_c;l_c)^* V_{cf}\psi_d(\mathbf{r}_c,\mathbf{r}_f)u_0(\xi;J_0M_0), \quad (4)$$

where the notation used is essentially that of reference 1. The integral in this expression cannot of course be evaluated unless some assumption is made concerning the nature of the wave functions and that of the interaction potential V_{cf} between the neutron and proton of the deuteron.

These considerations show that the angular correlation function $W(\mathbf{k}_d,\mathbf{k}_f,\mathbf{k}_e)$ for the $(d,f;e)$ process may be written in the general form

$$W(\mathbf{k}_d,\mathbf{k}_f,\mathbf{k}_e)=\sum_{\nu,\mu,L,L'} A(\nu,\mu;L,L')b_\nu(LL';e)Y_\nu^\mu(\theta,\varphi), \quad (5)$$

where θ and φ are spherical polar angles defined with respect to an arbitrary axis of quantization and $A(\nu,\mu:L,L')$ are the parameters appropriate for gamma-ray correlation.[8a] Letting the particle parameters $b_\nu(LL';e)$ equal to 1, the angular correlation function $W(\mathbf{k}_d,\mathbf{k}_f,\mathbf{k}_\gamma)$ for the $(d,f;\gamma)$ process is obtained.

If the multipole transition for electron emission is of a pure 2^L-pole type, then $L=L'$, and

$$W(\mathbf{k}_d,\mathbf{k}_f,\mathbf{k}_e)=\sum_{\nu,\mu} A(\nu,\mu;L,L)b_\nu(LL;e)Y_\nu^\mu(\theta,\varphi). \quad (6)$$

If, in addition, the assumptions of the Butler theory are introduced, then the above expression further simplifies to yield

$$W(\mathbf{k}_d,\mathbf{k}_f,\mathbf{k}_e)=\textstyle\sum_\nu A_\nu b_\nu(LL;e)P_\nu(\cos\theta), \quad (7)$$

where θ is referred to the direction of recoil of the residual nucleus.[9] Explicit forms for the parameters A_ν may be found in references 1 and 9.

As a numerical example, consider the stripping reaction–conversion electron process $d+\mathrm{Cd}^{110}\rightarrow\mathrm{Cd}^{111*}+p+e$, in which the first excited, 0.247-Mev state of Cd^{111} is involved. Taking $J_0=0$, $J=\frac{5}{2}$, $J_f=\frac{1}{2}$, $l_c=2$, and $L=2$, one obtains for the gamma-ray correlation function the expression $1+0.571P_2(\cos\theta)-0.571P_4(\cos\theta)$ for Butler stripping.[1] Using the values $b_2=1.806$, $b_4=-1.015$ for the particle parameters[4] for K conversion, the K conversion electron correlation function becomes $1+1.032P_2(\cos\theta)+0.580P_4(\cos\theta)$.

III. DISCUSSION

Experimentally, a major source of difficulty in investigating the $(d,p;\gamma)$ or $(d,n;\gamma)$ gamma-ray correlations lies in the inability to distinguish between radiations which are too close in energy. For light nuclei with widely spaced levels it is still possible to obtain useful results, but for the heavier nuclei the closely spaced levels make gamma ray correlation measurements very difficult or impossible at the present time.

For these heavier nuclei certain of the internal conversion coefficients may be appreciable, however, and the possibility then exists of measuring angular correlations involving conversion electrons. Experimentally, this offers certain advantages; in particular, the energy resolution could be greatly increased by the use of a magnetic spectrometer. The desirability of large internal conversion coefficients limits the utility of the method for the most part to the heavier nuclei. This requirement, in turn, necessitates the use of fairly high bombarding energies (of the order of the Coulomb barrier, or higher) in order to attain sufficiently high stripping probabilities. The two angular correlation methods—gamma rays for light nuclei, and conversion electrons for heavier nuclei—thus experimentally complement each other.

The experimental problems involved in making measurements of internal electron conversion angular distributions for stripping reactions are, in fact, similar to those encountered in determining the conversion electron-gamma ray angular correlations following radioactive decay. The experimental techniques used in studying the latter type of processes are well known, and have led to results for the nuclear properties of certain levels of Pb^{207}, Cd^{111}, $\mathrm{Te}^{121,123}$, and Br^{80}, among others.[10] Such stripping reactions as $\mathrm{Pb}^{206}(d,p)\mathrm{Pb}^{207*}$ or $\mathrm{Cd}^{110}(d,p)\mathrm{Cd}^{111*}$, for example, yielding nuclei previously studied by conversion electron techniques, would, therefore, possibly be appropriate processes for initial conversion-electron stripping-reaction investigations.

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[7] R. Huby, M. Y. Refai, and G. R. Satchler, Nuclear Phys. **9**, 94 (1958/59).

[8] W. Tobocman, Case Institute of Technology Report No. 29, 1956 (unpublished), Chap. 6.

[8a] *Note added in proof.*—It has been called to the author's attention that the result here obtained, Eq. (5), is anticipated in the paper by G. R. Satchler in Proc. Phys. Soc. (London) **A66**, 1081 (1953).

[9] L. C. Biedenharn, K. Boyer, and R. A. Charpie, Phys. Rev. **88**, 517 (1952).

[10] For specific references see, for example, the article by H. Frauenfelder in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1958).

The disadvantage of using thin targets to reduce multiple electron scattering in conversion-electron studies should be more than offset by the higher efficiencies possible for electron counters as compared with those for gamma-ray detectors, and by the reduction in spurious background counts. Conversion-electron angular correlation studies for stripping reactions appear especially promising for tandem accelerators with sufficiently high energies. The possibility of conversion electron polarization measurements, yielding additional nuclear information, might also be mentioned in conclusion.

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Alpha Particles from Be^9 and C^{12} by 25-Mev Alpha-Particle Bombardment

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Alpha-particle spectra from thin beryllium and carbon targets have been taken at 47.5° laboratory angle and an alpha energy of 25.41 Mev. The center-of-mass spectra of both nuclei indicate that the four-body reactions $Be^9(\alpha;3\alpha,n)$ and $C^{12}(\alpha;4\alpha)$ are greatly preferred over the three-body reactions $Be^9(\alpha;\alpha,n)Be^8$ and $C^{12}(\alpha;2\alpha)Be^8$. Several inelastic levels of both nuclei appear, and the recoil-decay alpha particles from the 9.61-Mev level of C^{12*} are seen. The Fermi statistical model is invoked in an attempt to establish that the multibody reactions are reasonably representative of the proportions of the two- and three-cluster configurations in the ground states of the two nuclei, and that the three-cluster configurations are preferred.

INTRODUCTION

WITHIN the past two years there has been a renewed interest in the alpha-particle model of the nucleus, which was first proposed over twenty years ago.[1–3] The more recent models do not precisely resemble the earlier models, but tend toward the form of the "cluster model," which takes into account the possible nucleon subgroups within the nuclear structure. In many of the light nuclei the natural subgroups are alpha particles, which have been used, for example, to calculate ground-state magnetic moments,[4,5] and thus the older and newer theories have a superficial similarity. However, the cluster model is designed to be compatible with the shell model, but with this difference; instead of having random azimuthal phase relations, the particles are no longer independent, but have a sufficiently strong pairing interaction to cause some phase grouping. The j-j coupling of the shell model is thus modified to a partially L-S coupled model. A particularly clear exposition of this idea has been given by Phillips and Tombrello,[6] in the case of the highest known level of the mass-5 nuclei. In this paper a more extreme assumption is made in the case of several levels in the light nuclei; specifically that a two-body (two-cluster) model will describe many of the ground- and low-excited levels, although these levels may be mixed states of two or more possible sets of cluster states. The Be^9 nucleus is believed to consist primarily of Be^8 [ground state (g.s.)] and a neutron, while the more highly bound internal structure of the Be^8 is attributed to two alpha particles in several rotational-vibrational states. The Be^8 structure has been examined by an ingenious calculation by Kallenopoulus and Wildermuth,[7,8] which tends to confirm the assumption.

Another nucleus which has received considerable theoretical attention is C^{12}. As in the case of all nuclei which can be said to consist of a whole number of alpha particles, there has been speculation that it might be fitted by a model of three alpha particles. In contrast to this, one might extend the two-cluster model of Phillips and Tombrello to attribute the ground state of C^{12} to an alpha and a Be^8 (ground) nucleus, although

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