

Nuclear Structure Studies in the Zirconium Region with Stripping Reactions*

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The single quasi-particle levels in Sr^{89} , Zr^{91} , Zr^{93} , Zr^{95} , and Zr^{97} are located and identified by (d,p) and (d,t) reaction studies. Values of occupation numbers (V_j^2) are determined both by location of levels and by cross section measurements (the two methods agree), and are used to estimate the unperturbed single-particle states in the shell-model potential. The $g_{7/2}$ state is found to be far above the $d_{3/2}$ (and even above the $s_{1/2}$ and $d_{5/2}$) in contrast to the situation in the tin isotopes and the usual theoretical estimates. The ground state of Zr^{97} is found to be $\frac{1}{2}^+$.

INTRODUCTION

IN a previous paper¹ it was pointed out that experiments on stripping reactions in conjunction with pairing theory² constitute a powerful tool for nuclear structure studies. In the first place, angular distributions from stripping reactions may be used to locate the single quasi-particle levels and determine to which shell-model state they belong; pairing theory then gives a simple relationship between their energies, E_j , and the occupation numbers, V_j (we use the notation of reference 2), as

$$E_j = \Delta \left(\frac{1}{2U_j V_j} - \frac{1}{2U_g V_g} \right), \quad (1)$$

where 2Δ is the energy gap, a parameter easily estimated from mass data, the subscript g refers to the ground state, and

$$U_j^2 = 1 - V_j^2. \quad (2)$$

Since V_g is generally easy to estimate with sufficient accuracy, the experimental determination of the E_j may be used in conjunction with (1) to obtain the V_j , the basic quantities from the nuclear structure standpoint.

In the second place, it was shown in reference 1 that the cross sections for stripping reactions may be expressed as

$$\sigma(d,p) = P(l_n, Q, \theta) p(V_j), \quad \sigma(d,t) = T(l_n, Q, \theta) t(V_j), \quad (3)$$

where P and T are quantities derived from reaction theories as functions of l_n , the orbital angular momentum of the stripped [or "picked-up," in the case of (d,t) reactions] neutron in the nucleus; Q , the energy release in the reaction; and θ , the angle at which the proton (or triton) is observed. The functions $p(V_j)$ and $t(V_j)$ are simple functions of V_j given in reference 1.

Thus, if P and T can be determined, (3) gives independent determinations of the V_j . Since energies of single quasi-particle levels may be influenced by mixing with other states of the same spin and parity, this second method based on measured cross sections

should generally be more reliable for determinations of V_j than the first method which is based on observed energies.

In reference 1, these ideas were successfully applied on the isotopes of tin; values of V_j were obtained for each shell-model state in each isotope. It is the purpose of this paper to apply these techniques in the zirconium region.

EXPERIMENTAL

Measurements were made of the energy spectra of protons and tritons emitted from (d,p) and (d,t) reactions induced by 15-Mev deuterons from the University of Pittsburgh cyclotron. The particles were magnetically analyzed and detected by photographic plates; the essential elements of the method have been described previously,³ but two new improvements were made just prior to this experiment. Firstly, a series of careful energy calibrations was carried out using known levels in light nuclei, so that excitation energies of levels given here should be accurate to better than 20 kev. Secondly, a program for improving the energy resolution was carried through; all factors which influence the resolution were analyzed, and all slit sizes were optimized. It was found that multiple scattering in the absorber over the photographic plates worsened the resolution appreciably, so that the plate holder was redesigned to locate the absorber very close to the emulsion. As a result of all these measures, the resolution in these experiments was about 45 kev in (d,p) reactions and 55 kev in (d,t) reactions for the 2.3 mg/cm² natural zirconium target at angles where resolution was optimum. The principal limitation was target thickness; in tests with much thinner targets, resolutions of about 30 kev were obtained.

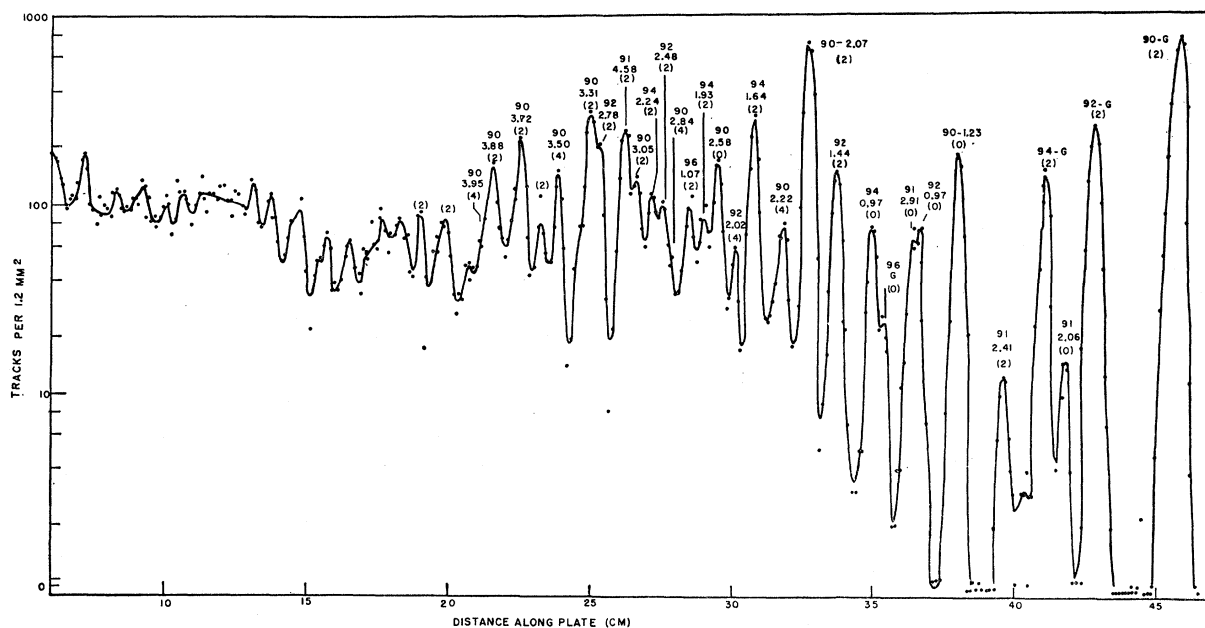
Typical data from (d,p) and (d,t) reactions in the natural zirconium target are shown in Figs. 1 and 2. Data were taken at enough angles to determine values of l_n for each observed level from known characteristic angular distributions.¹ The principal complication arises from the fact that natural zirconium has several isotopes, so that the assignment of observed levels to the proper isotope is difficult. To facilitate this, isotopically enriched targets were prepared from oxide

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¹ B. L. Cohen and R. E. Price, Phys. Rev. **121**, 1441 (1961).

² L. S. Kisslinger and R. A. Sorenson, Kgl. Danske Videnskabs Selskab, Mat.-fys. Medd. **32**, 9 (1960).

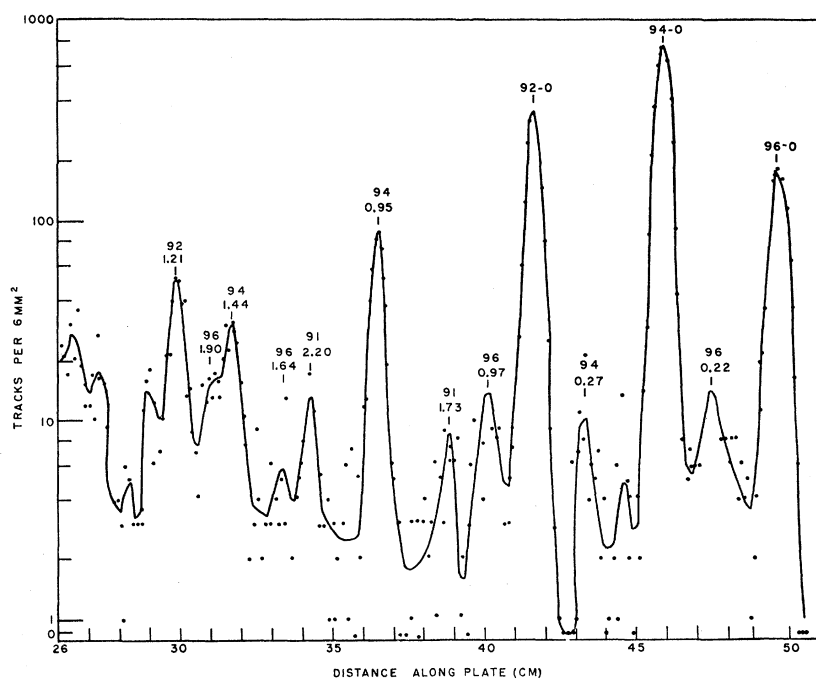
³ B. L. Cohen, J. B. Mead, R. E. Price, K. S. Quisenberry, and C. Martz, Phys. Rev. **118**, 499 (1960).



powders⁴ on Mylar backing. However, these give much poorer energy resolution (typically 200 keV) and are heavily contaminated with oxygen and carbon. Principally due to the latter fact, very little angular distribution data could be obtained from the isotope targets, and they were used principally to assign levels in the

spectra from natural zirconium. This process is limited only to the first few Mev of excitation energy since the opportunities for confusion increase rapidly as level densities become higher.

The strontium target was prepared by evaporation on a gold backing. Here again there was considerable



⁴ G. Fodor and B. L. Cohen, Rev. Sci. Instr. **31**, 73 (1960).

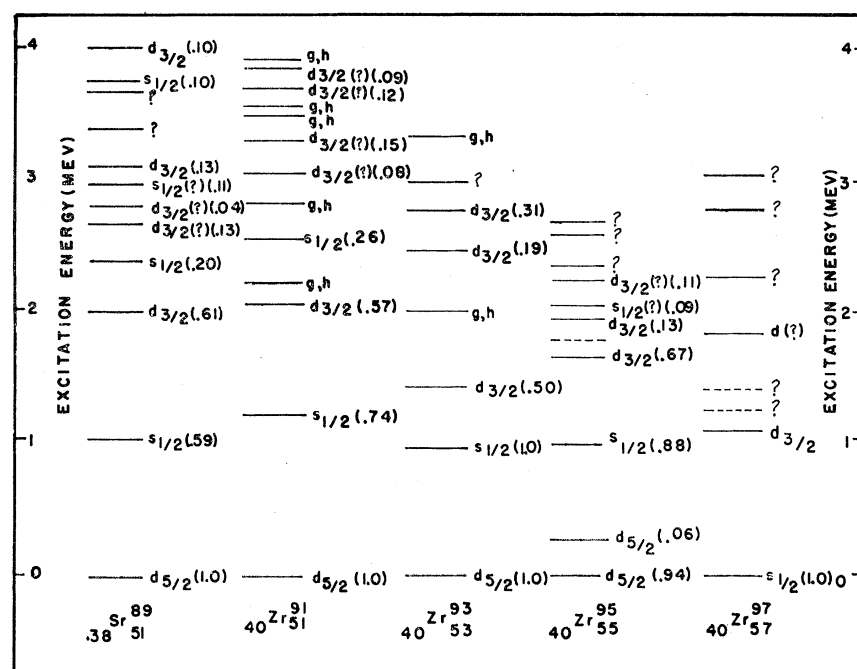


FIG. 3. Energy levels of odd isotopes found in this work, and their shell model assignments. Numbers in parenthesis are relative cross sections for excitation by (d,p) reactions for various levels belonging to the same shell model state.

carbon and oxygen contamination which obliterated parts of the angular distributions, but the situation was not quite so serious because the resolution was reasonably good (about 70 kev). Since natural strontium is 88% Sr^{88} , the most intense particle groups are assumed to be from that isotope.

RESULTS AND DISCUSSION

The energy levels observed and their shell model assignment from angular distribution data are listed in Table I, and shown graphically in Fig. 3. The highly excited $l_n=2$ states are assigned as $d_{3/2}$, partly on theoretical grounds, but partly from the fact that the ratio of cross sections for exciting any two $d_{3/2}$ levels, or any two $d_{5/2}$ levels should be roughly the same in

(d,p) and (d,t) reactions (see below). The numbers in parenthesis in Fig. 3 give relative intensities for exciting these levels in (d,p) reactions corrected for the Q -value dependence of the P function (see below); they are normalized to sum to unity for each shell model state.

The relative cross sections for exciting the lowest lying $d_{3/2}$, $s_{1/2}$, and $d_{5/2}$ states in each isotope by (d,p) and (d,t) reactions are listed in Table II, column 3. Measurements were made at several angles and averaged; the standard deviations of the mean in these averages were typically 5–12%. These deviations are largely due to slight differences in angular distributions for different Q values. Column 4 of Table II gives the theoretical expression for these cross sections from (3). It may be noted that we assume that all $V_j=0$ for Zr^{90} since that isotope contains 50 neutrons. Thus cross sections involving Zr^{90} as target or residual nucleus give direct determinations of the P and T functions in (3). This is the basic "trick" that allows us to derive absolute values of the V_j here; in many ways it is more satisfactory than the methods used in reference 1.

Before comparing the observed cross sections of column 3 with the theoretical expressions of column 4, two corrections must be applied:

(a) The dependence of P and T on Q must be corrected for. This was done as in reference 1 by assuming

$$\begin{aligned} P(l, Q, \theta) &= P_l(\theta) K^{-Q(\text{Mev})}, \\ T(l, Q, \theta) &= T_l(\theta) K^Q(\text{Mev}). \end{aligned} \quad (4)$$

A direct determination of K is available from comparison of the $d_{3/2}$ and $d_{5/2}$ states in $\text{Zr}^{90}(d,p)$; this gives $K=1.15 \pm 0.10$. A few distorted wave Born approxi-

TABLE I. Excitation energies and l_n for observed levels.

Sr^{89}		Zr^{91}		Zr^{93}		Zr^{95}		Zr^{97}	
E	l	E	l	E	l	E	l	E	l
0	2	0	2	0	2	0	2	0	0
1.05	0	1.23	0	0.97	0	0.97	0	1.10	2
2.02	2	2.07	2	1.44	2	1.64	2	1.24	
						1.77	a		
2.40	0	2.22	4, 5	2.02	4, 5	1.93	2	1.39	
2.68	2(?)	2.58	0	2.48	2	2.01	0(?)	1.81	2(?)
2.81	2(?)	2.84	4, 5	2.78	2	2.24	2	2.24	
2.99	0(?)	3.05	2	3.01	...	2.33	...	2.78	
3.12	2	3.31	2	3.35	4, 5	2.57	...	3.03	
3.40	...	3.50	4, 5			2.68	...		
3.69	...	3.58	4, 5						
3.76	0	3.72	2						
4.04	2	3.88	2						
		3.95	4, 5						

a Seen only in (d,t) reaction.

TABLE II. Cross-section data on (d,p) and (d,t) reactions.

(1) Shell- state	(2) Target mass	(3) σ of lowest state (rel.)	(4) Theory	(5) σ corrected (rel.)	(6) Conclusion
(a) (d,p) reactions					
$d_{\frac{1}{2}}$	90	1.00 (std.)	$6P_2$	1.00 (std.)	$P_2=0.167$
	91	0.034 ± 0.003	$P_2V_{\frac{1}{2}}^2(92)$	0.043 ± 0.04	$V_{\frac{1}{2}}^2(92)=0.26 \pm 0.03$
	92	0.68 ± 0.03	$6P_2U_{\frac{1}{2}}^2(92)$	0.63 ± 0.03	$U_{\frac{1}{2}}^2(92)=0.63 \pm 0.03$
	94	3.82 ± 0.30	$6P_2U_{\frac{1}{2}}^2(94)$	0.33 ± 0.03	$U_{\frac{1}{2}}^2(94)=0.33 \pm 0.04$
$d_{\frac{3}{2}}$	90	0.50 ± 0.06	$4P_2$	0.63 ± 0.10	$P_2=0.158 \pm 0.20$
	92	0.47 ± 0.09	$4P_2U_{\frac{1}{2}}^2(92)$	0.70 ± 0.14	$U_{\frac{1}{2}}^2(92)=1.05 \pm 0.20$
	94	0.70 ± 0.11	$4P_2U_{\frac{1}{2}}^2(94)$	7.63 ± 0.12	$U_{\frac{1}{2}}^2(94)=0.95 \pm 0.20$
	96 ^a	1.35 ± 0.20	$4P_2U_{\frac{1}{2}}^2(96)$	0.89 ± 0.14	$U_{\frac{1}{2}}^2(96)=1.33 \pm 0.21^a$
$s_{\frac{1}{2}}$	90	1.00 (std.)	$2P_0$	1.00 (std.)	$P_0=0.50$
	92	0.96 ± 0.10	$2P_0U_{\frac{1}{2}}^2(92)$	0.69 ± 0.11	$U_{\frac{1}{2}}^2(92)=0.69 \pm 0.11$
	94	1.17 ± 0.12	$2P_0U_{\frac{1}{2}}^2(94)$	0.90 ± 0.14	$U_{\frac{1}{2}}^2(94)=0.90 \pm 0.14$
	96	1.93 ± 0.58	$2P_0U_{\frac{1}{2}}^2(96)$	1.31 ± 0.30	$U_{\frac{1}{2}}^2(96)=1.31 \pm 0.30$
(b) (d,t) reactions					
$d_{\frac{1}{2}}$	91	1.00 (std.)	T_2	1.00 (std.)	$T_2=1.00$
	92	1.37 ± 0.15	$6T_2V_{\frac{1}{2}}^2(92)$	1.70 ± 0.19	$V_{\frac{1}{2}}^2(92)=0.28 \pm 0.03$
	94	2.90 ± 0.32	$6T_2V_{\frac{1}{2}}^2(94)$	3.41 ± 0.37	$V_{\frac{1}{2}}^2(94)=0.57 \pm 0.07$
	96	2.28 ± 0.24	$6T_2V_{\frac{1}{2}}^2(96)$	5.75 ± 0.60	$V_{\frac{1}{2}}^2(96)=0.96 \pm 0.10$
$d_{\frac{3}{2}}$	94	0.108 ± 0.013	$4T_2V_{\frac{1}{2}}^2(94)$	~ 0.18	$V_{\frac{1}{2}}^2(94) \approx 0.05$
	96	0.146 ± 0.018	$4T_2V_{\frac{1}{2}}^2(96)$	~ 0.22	$V_{\frac{1}{2}}^2(96) \approx 0.06$
$s_{\frac{1}{2}}$	92	1.00 (std.)	$2T_0V_{\frac{1}{2}}^2(92)$	1.00 (std.)	$V_{\frac{1}{2}}^2(92)=0.057^b$
	94	1.12	$2T_0V_{\frac{1}{2}}^2(94)$	0.73	$V_{\frac{1}{2}}^2(94)=0.042^b$
	96	1.34	$2T_0V_{\frac{1}{2}}^2(96)$	0.92	$V_{\frac{1}{2}}^2(96)=0.053^b$

^a In the highest resolution data, the group corresponding to this state appeared to be multiple, so that there is probably another level included in the cross section.

^b Based on $T_0=4T_2$ which is only very approximate. Relative values among the three isotopes should be good to $\sim 10\%$.

mation calculations on (d,p) reactions have recently become available⁵; these suggest $K \approx 1.20$. In reference 1, we used $K=1.18$, so we again adopt this value here. Actually, the significant corrections for Q values in this work are always less than 40%, so that the effects of uncertainties in K are negligible.

(b) The effect of the higher excited states must be taken into account by dividing the cross section for the level measured by the fraction of the single-particle level contained in that state (i.e., the numbers in parenthesis in Fig. 3). This introduces appreciable uncertainties—perhaps about 15%—into the corrected cross sections for (d,p) reactions leading to $d_{\frac{1}{2}}$ states. For (d,t) reactions leading to $d_{\frac{1}{2}}$ states, the uncertainties are very large. In general, the higher excited states are not observed in (d,t) reactions so that the fractions were assumed to be the same as in (d,p) reactions. In the two cases where this assumption could be tested, it was found to be very bad:

(i) The ratio of the cross sections for exciting the 1.90- and 1.64-Mev levels in Zr^{96} is 0.20 in (d,p) but 1.6 in (d,t) reactions. This discrepancy is so large that one might suspect the 1.90-Mev level of being $d_{\frac{3}{2}}$, but its ratio to the ground ($d_{\frac{1}{2}}$) state is 0.3 in (d,p) but 0.06 in (d,t) reactions. Thus there is a large discrepancy whether it is assigned as $d_{\frac{1}{2}}$ or $d_{\frac{3}{2}}$. Actually one expects discrepancies when levels are weakly excited [as in the $d_{\frac{1}{2}}$ case for (d,t) reactions] since the portion of the wave function that is not pure quasi-particle may be effective. Thus the $d_{\frac{1}{2}}$ assignment seems more likely.

(ii) The ratio of the cross sections for exciting the 2.45- and 1.45-Mev levels in Zr^{98} is 0.4 in (d,p) but 1.5 in (d,t) reactions. If the 2.45-Mev level is assigned as $d_{\frac{3}{2}}$, its ratio to the ground ($d_{\frac{1}{2}}$) state is 0.22 in (d,p) but 0.07 in (d,t) reactions. Since there is a large discrepancy in either case, the $d_{\frac{1}{2}}$ assignment is better for reasons discussed above.

The only conclusion one can draw from this is that the information obtained on $d_{\frac{1}{2}}$ states from (d,t) reactions cannot be considered quantitative.

The cross sections obtained after applying these corrections are listed in column 5 of Table II; Column 6 gives the conclusions from comparing columns 4 and 5. Since there is no direct determination of T_0 , it was estimated by assuming that the ratio T_0/T_2 averaged over several angles (35° , 43° , and 60° were used) is four⁵; this is a rather crude assumption, but it is not likely to be in error by more than about 50%. The relative values of $V_{\frac{1}{2}}^2$ for the various isotopes is, of course, much more accurate.

The results listed in column 6 of Table II, in conjunction with (2), give two or three independent determinations for each of the V_j . In general, these are quite self-consistent. The only important discrepancy is in $V_{\frac{1}{2}}^2(92)$ which the (d,p) and (d,t) reactions give as 0.31 ± 0.11 and 0.05 ± 0.03 , respectively. This can be explained if an $s_{\frac{1}{2}}$ level at high excitation in Zr^{93} was missed; this would also explain the low value of $V_{\frac{1}{2}}^2(94)$

⁵ B. L. Cohen, S. Mayo, and R. E. Price, Nuclear Phys. **20**, 360, 370 (1960).

TABLE III. Results for V_j^2 in even isotopes. $V_j^2(\sigma)$ are from Table II, $n_j(\sigma)$ are obtained from use of (5) on the $V_j^2(\sigma)$, and $V_j^2(E_j)$ are obtained by interpolating from Table IV. Values in parenthesis are obtained indirectly from use of (5).

State	$V_j^2(\sigma)$	Zr ⁹² $n_j(\sigma)$	$V_j^2(E_j)$	$V_j^2(\sigma)$	Zr ⁹⁴ $n_j(\sigma)$	$V_j^2(E_j)$	$V_j^2(\sigma)$	Zr ⁹⁶ $n_j(\sigma)$	$V_j^2(E_j)$
$d_{5/2}$	0.30 ± 0.03	1.8 ± 0.2	(0.30)	0.62 ± 0.05	3.7 ± 0.3	(0.60)	0.96 ± 0.10	5.8 ± 0.6	...
$s_{1/2}$	~ 0.05	0.1	0.055	~ 0.05	0.1	0.06	~ 0.05	0.1	...
$d_{3/2}$	0.025	~ 0.05	~ 0.2	0.025	$\lesssim 0.05$	$\lesssim 0.2$	0.04
$g_{7/2}$	($\gtrsim 0.03$)	($\gtrsim 0.02$)	0.025	($\gtrsim 0.04$)	($\gtrsim 0.3$)	0.025	($\gtrsim 0.06$)	$\gtrsim 0.5$	< 0.04

from (d,t) reactions relative to the corresponding quantity in the other isotopes. In the analysis, we will assume the existence of such a level.

Weighted averages of the V_j^2 from Table II are listed in Table III. Also listed in that table are the number of particles in the level, n_j , given by

$$n_j = (2j+1)V_j^2. \quad (5)$$

Since the sum of all n_j for each isotope must be equal to the number of particles outside the closed shell, an upper limit to the number of particles in the $g_{7/2}$ state can be deduced, and this with (5) gives an upper limit to $V_{7/2}^2$. These are also listed in Table III in parenthesis.

As discussed in the introduction, the V_j^2 can also be determined from the observed energies by use of (1). There is a slight dependence on V_j for the ground state, but this can easily be estimated or taken from Table III; the results are quite insensitive to this. The values of V_j^2 for the odd isotopes determined in this way are listed in Table IV. Interpolating these gives independent determinations of the V_j^2 for the even isotopes, and these are listed in Table III. It is readily seen that V_j^2 determined by the two methods are in reasonably good agreement. Where they differ the determinations from the cross sections are more reliable for the $d_{5/2}$, but the determinations from observed energies are probably more reliable for the other states since, as discussed previously, the cross section method tends to break down when cross sections are small.

Among other applications, the V_j^2 may be used to calculate the energies of shell model states, e_j ($\epsilon_j - \lambda$ in Kisslinger-Sorenson² notation). From reference 1,

$$e_j = (U_j^2 - V_j^2)/2U_jV_j. \quad (6)$$

The values of e_j relative to e_j for the ground state, as calculated from (6), are listed in Table V. There is some inconsistency among the isotopes. This is probably due to the fact that the small V_j^2 are rather inaccurately

determined (percentage-wise), whereas (6) is very sensitive to V_j^2 when it is small. However, it is very clear that there is an energy gap of about 2 Mev between the $d_{5/2}$ and $s_{1/2}$, another gap of about 1 Mev between the $s_{1/2}$ and $d_{3/2}$, and that the $g_{7/2}$ lies even higher, more than 3 Mev above the $d_{5/2}$.

In the usual theory,^{2,6} the values of the e_j are derived from the shell model potential well so that they should be independent of mass. It is therefore of interest to compare the values determined here with those in the tin region. The values in the Sn isotopes, as determined experimentally in reference 1 and as theoretically estimated^{2,6} are listed in the last two columns of Table V.

In comparing the e_j in the Zr and Sn regions, by far the most glaring discrepancy is in the relative positions of the $d_{5/2}$ and $g_{7/2}$ states. In the Sn region, they are very close, with the $g_{7/2}$ perhaps slightly lower, whereas in the Zr region, the $g_{7/2}$ is at least 3-Mev higher. This very rapid shift, 3 Mev in a mass interval of 20, is without precedent in other experiences with shell model in heavy nuclei. Talmi⁷ has suggested that this shift may be due to an extra strong coupling between the $g_{7/2}$ neutrons and the $g_{9/2}$ protons (which fill between Zr and Sn) arising from the fact that both are g states.

Another interesting application of the V_j^2 is in the calculation of the energies of collective oscillation levels, $\hbar\omega_l$; the appropriate relationship is⁸

$$\frac{1}{X_l} = \sum_{j,j'} \frac{M_{jj'}(U_jV_{j'} + U_{j'}V_j)}{(E_j + E_{j'})^2 - (\hbar\omega_l)^2}, \quad (7)$$

where X_l is the strength of the P_l force, j, j' are any two shell-model states which can couple to give the spin (l) and parity of the collective state, $M_{jj'}$ is a matrix element between these states, and $E_j, E_{j'}$ are the single quasi-particle energies of these states. The summation in (7) includes many terms between states of completely open and completely closed shells, but the contribution of these is very slowly varying with mass number, A . Thus, the detailed energy variation of collective levels with A depends on contributions from the shell that is filling. In the Zr region, the

TABLE IV. Determinations of V_j^2 from energies of observed levels by use of (1). Quantities in parenthesis were obtained indirectly by use of (5).

State	Zr ⁸⁹	Zr ⁹¹	Zr ⁹³	Zr ⁹⁵	Zr ⁹⁷
$d_{5/2}$	(0.11)	(0.11)	(0.43)	(0.78)	≥ 0.96
$s_{1/2}$	0.04	0.04	0.07	0.05	...
$d_{3/2}$	0.02	0.02	0.03	0.02	0.06
$g_{7/2}$	< 0.03	0.02	0.03	< 0.02	< 0.06

⁶ S. G. Nilsson, Kgl. Danske Videnskab, Selskab. Mat.-fys. Medd. **29**, 16 (1955).

⁷ I. Talmi (private communication).

⁸ M. Baranger, Phys. Rev. **122**, 992 (1961).

TABLE V. Shell-model energies e_j ($=\epsilon_j-\lambda$), relative to $d_{5/2}$ state. Energies are in Mev.

State	Zr ⁹²	Zr ⁹⁴	Sn ^a	Sn ^b
$d_{5/2}$	0	0	0	0
$s_{1/2}$	1.7	2.1	0.95	1.90
$d_{3/2}$	2.7	3.3	1.23	2.20
$g_{7/2}$	2.6	3.3	-0.16	0.22

^a See reference 1.^b See reference 2.

important terms for the collective quadrupole ($l=2$) and octopole ($l=3$) oscillations are $U_{\frac{5}{2}}V_{\frac{5}{2}}$ and $U_{11/2}V_{\frac{5}{2}}$, respectively (where the subscripts $\frac{5}{2}$ and $11/2$ refer to the $d_{\frac{5}{2}}$ and $h_{11/2}$ states).

These terms, as well as the experimental positions of the collective states,¹⁰ are plotted in Fig. 4; the close relationship is readily observed. If the $g_{7/2}$ and $d_{\frac{5}{2}}$ states were very close in energy and filling simultaneously in this region as expected from the usual shell model wells,⁶ the $d_{\frac{5}{2}}$ state would be only about half full at Zr⁹⁶ so that the A dependence of $U_{\frac{5}{2}}V_{\frac{5}{2}}$ and $U_{11/2}V_{\frac{5}{2}}$ would be as shown by the squares in Fig. 4. (It turns out¹⁰ that the $g_{7/2}$ state does not contribute appreciably to the collective states because of small values of M_{jj} .) It is readily seen that the agreement with observed energies is destroyed. Also shown in Fig. 4 (triangles) are calculations by Yoshida¹⁰ of the energies of the 3⁻ collective levels; these are based on the usual shell-model wells,⁶ so that they agree with the squares in Fig. 4, but not with the experimental data on energies.

It is interesting to note that the ground state of Zr⁹⁷ is found to be $\frac{1}{2}^+$ rather than $\frac{5}{2}^+$ as has usually been assumed heretofore. This state is known to β decay with a 17-hr half life to the $\frac{1}{2}^-$ state of Nb⁹⁷. The $\log ft$ values is 7.1, which is about typical for a first forbidden transition. If the spin of Zr⁹⁷ had actually been $\frac{5}{2}^+$, the transition would have been first-forbidden unique;

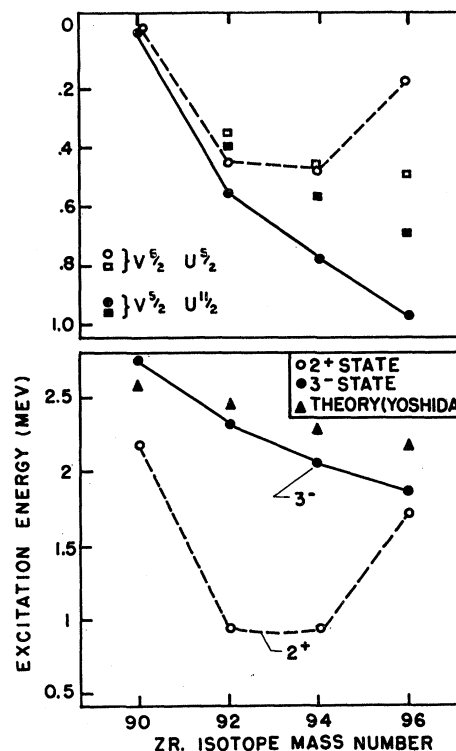
⁹ B. L. Cohen (to be published).¹⁰ S. Yoshida (private communication).

FIG. 4. Energies of 2⁺ and 3⁻ collective levels in the even isotopes of zirconium, and functions responsible for their variation in the theory. In upper figure, circles are from the results of this paper, and squares are from reference 2 and 6. See text.

no known transition of this type has a $\log ft$ value smaller than about 8.0.

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