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Extrapolation to Cuts and the Scattering of Electrons and Positrons*

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The low momentum transfer region of strong-interaction processes may be discussed in terms of one-pion-exchange graphs. This success of "extrapolation to poles" suggests, as an extension, "extrapolation to cuts" corresponding to low-mass intermediate states containing two or more particles. We find that this extrapolation may be performed in the case of electromagnetic interactions, owing to the vanishing photon mass. For small-angle scattering of relativistic electrons in a pure Coulomb field, $\sin^4(\theta/2)(d\sigma/d\Omega)=a_0+a_1\sin(\theta/2)+\cdots$, where a_0 is given precisely by the one-photon pole and a_1 (obtained exactly to all orders in $Z\alpha$) comes from the many-photon terms. For scattering by finite nuclei we further find that, for fixed momentum transfer, the ratio of second to first Born approximation decreases as $(\text{energy})^{-1}$ at high energies. This leads to a simple approximate formula for the ratio $R\equiv(\sigma_- - \sigma_+)/(\sigma_- + \sigma_+)$ of electron and positron scattering, and to a simple method of determining the charge form factors of intermediate-Z nuclei.

I. INTRODUCTION

RECENT studies have emphasized the importance of the pole term arising from the exchange of a single pion in low momentum transfer interactions at high energies.[1] The elastic proton-neutron scattering amplitude, for instance, has a pole corresponding to exchange of a single "real" pion between the nucleons. The contribution of the one-pion exchange term is proportional to the pion propagator which, in the center-of-mass system, is

$$-\{2|P|^2[1+(\tfrac{1}{2}\mu^2/|P|^2)-\cos\theta]\}^{-1}, \tag{1}$$

where $|P|$ is the nucleon momentum, θ is the scattering angle, and $\Delta^2=-2|P|^2(1-\cos\theta)$ is the invariant square of the four-momentum transfer. The pole appears in the nonphysical region at $\Delta^2=\mu^2$, or $\cos\theta=1+\mu^2/2|P|^2>1$; however, this term is important when $|P|^2>\mu^2$ and $\cos\theta=1$, and the propagator (1) becomes large. In fact, it has proved possible to isolate this contribution and measure the pion-nucleon coupling strength by extrapolating to the pole which is the singularity in the scattering amplitude lying nearest to the physical region.[1] There are additional singularities due to the exchange of two or more "real" pions or heavier particles between the nucleons. These give rise to a branch line, or continuous sequence of poles, starting further from the physical region, as shown in Fig. 1, at momentum transfers $\Delta^2=(2\mu)^2$, the square of the mass of the lightest such intermediate states, and continuing to $\Delta^2\to\infty$.

For certain processes, selection rules prohibit the exchange of a single pion. Two examples are elastic pion-pion and pion-nucleon scattering as illustrated in Fig. 2. A vertex with only three (or any odd number of) pion lines is ruled out by the odd intrinsic parity of pions and therefore the momentum transfer t must be carried by at least two pions as indicated. The success of the procedure of extrapolating to one-pion poles near to the physical region encourages inquiry into the feasibility and fruitfulness of similar extrapolations to branch cuts in processes such as illustrated which do not admit the pole term. This is the subject of the present paper.

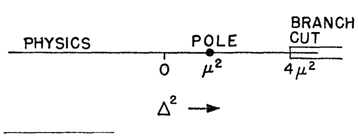

FIG. 1. Singularities of the elastic proton-neutron scattering amplitude.

We show in Sec. II that it in general is not possible to isolate the contributions from amplitudes for the graphs in Fig. 2. However, for massless intermediate quanta, i.e., photons, the pole and the onset of the branch cuts for $n>1$ photon exchange coincide at $\cos\theta=1$; such an extrapolation can actually be made in this case. This has led us to re-examine Coulomb scattering of an electron by a point charge at small angles θ from this point of view. This is discussed in Sec. III and a simplified derivation of the cross section is given which is exact for the $\csc^4(\theta/2)$ and $\csc^3(\theta/2)$ contributions. In Sec. IV we extend our considerations to examine Coulomb scattering from finite nuclei and, working in first and second Born approximation, derive several new and useful results. Starting from the formulation of Lewis for the scattering amplitude in second Born approximation we show that at high energies for any fixed momentum transfer the ratio of the second to the first Born amplitude vanishes as 1/energy of the incident electron. We also construct a simplified integral which accurately represents the second Born contribution at high energy and small angles. Finally, in Sec. V these results are applied to recent electron and positron scattering experiments of Pine and Yount.

The practical motivation behind this study of Coulomb scattering near forward angles comes from the desire to develop convenient and simple approximations for analyzing scattering of high-energy electrons and positrons (or muons) by heavy as well as light nuclei. Present analyses are based on elaborate machine calculations and fit nuclear charge distributions to observed

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[1] *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960). Extensive references are given in the report of S. D. Drell, in *Proceedings of the 1960 Conference on Strong Interactions, Berkeley, December, 1960* [Revs. Modern Phys. **33**, 458 (1961)].

structure in the scattering angular distributions at large angles.[2] At small scattering angles the cross sections are enormous due to the $\csc^4(\theta/2)$ Coulomb factor and, therefore, very accurate data are in principle available. We ask then, how much can be learned about charge distributions of nuclei by simple hand calculations, together with accurate small-angle measurements on electron and positron scattering.

II. EXTRAPOLATION TO A BRANCH CUT

To show the difficulties with attempts to extrapolate to branch cuts for processes shown in Fig. 2, we write a dispersion relation for the pion-nucleon scattering amplitude $A_{\pi N\to\pi' N'}\equiv A$. From the Mandelstam representation[3] a one-dimensional dispersion relation can be derived for $A(u,t,s)$ considered, for fixed energy s, as a function of momentum transfer t and of crossed momentum transfer $u=2(\mu^2+m^2)-t-s$ between incident pion and outgoing nucleon. It takes the form

$$A(u,t,s)=\frac{1}{\pi}\int_{m^2}^{\infty}du'\frac{A_1(u';M_t^2-u'-s;s)}{u'-u+i\epsilon}+\frac{1}{\pi}\int_{4\mu^2}^{\infty}dt'\frac{A_2(M_t^2-t'-s;t';s)}{t'-t+i\epsilon},\quad(2)$$

where $M_t^2\equiv 2(\mu^2+m^2)=$ sum of squares of external particle masses, and A_1 and A_2 denote the imaginary parts of the amplitudes for the reactions $\pi+\bar{N}'\to\pi'+\bar{N}$ and $\pi+\pi'\to N'+\bar{N}$, respectively. No pole term of the form $1/(t-\mu^2)$, as in Eq. (1), occurs here because of the selection rule prohibiting one-pion exchange. The dispersion integral in the momentum transfer, t', in (2) starts at $4\mu^2$, the square of the lowest mass state of two pions connecting the two parts of the graph in Fig. 2

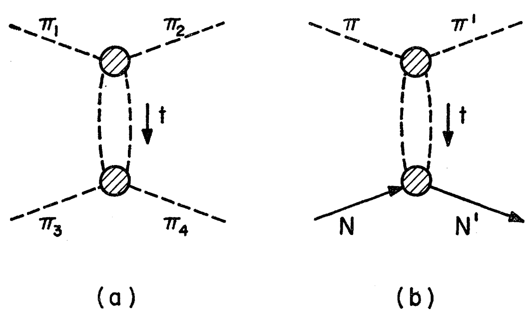

FIG. 2. Scattering processes in which the exchange of a single particle is forbidden. (a) Pion-pion scattering. (b) Pion-nucleon scattering.

[2] B. Hahn, D. G. Ravenhall, and R. Hofstadter, Phys. Rev. **101**, 1131 (1956), Dr. R. Yennie, D. G. Ravenhall, and R. N. Wilson, *ibid.* **95**, 500 (1954). Extensive references to earlier work are also given by R. Herman and R. Hofstadter, *High-Energy Electron Scattering Tables* (Stanford University Press, Stanford, California, 1960). Differences between electron and positron scattering have recently been discussed by G. H. Rawitscher and C. R. Fischer, Phys. Rev. **122**, 1330 (1961).

[3] S. Mandelstam, Phys. Rev. **112**, 1344 (1958).

in the reaction $\pi+\pi'\to N+\bar{N}'$. The threshold in the crossed amplitude $\pi+\bar{N}'\to\pi+\bar{N}$ occurs at the pole $u'=m^2$ for a single nucleon line.

What we are interested in here is the strength of the singularity in $A(u,t,s)$ as the momentum transfer t is analytically continued from the physical region where $t<0$ to the onset of the cut at $t=+4\mu^2$. Equation (2) shows that at most it will be logarithmic if the absorptive amplitude for the process $\pi+\pi'\to N'+\bar{N}$ is finite at $t=4\mu^2$; i.e., for

$$A_2(M_t^2-4\mu^2-s;4\mu^2;s)\neq 0.\quad(3)$$

The variable t represents the total energy in this channel of the scattering amplitude and A_2 must be analytically continued as a function of t to the value $t=4\mu^2$ which lies below the physical threshold of $t=4m^2$ for the physical interaction $\pi+\pi'\to N'+\bar{N}$. The possibility of this continuation is assumed in the Mandelstam representation.

Extrapolation to a logarithmic singularity which is introduced at $t=4\mu^2$ if (3) applies is, perhaps, not beyond possibility since it would mean looking for a first order pole in

$$\frac{\partial}{\partial t}[\sigma(t,s)]^{\frac{1}{2}}\bigg|_{t=4\mu^2}.$$

It turns out, however, that (3) is not true and in fact $A_2(M_t^2-4\mu^2-s;4\mu^2;s)=0$ for two-pion exchange. This is easily seen by using the unitarity condition in the region of energies $16\mu^2>t>4\mu^2$ to express A_2 as the product of the invariant amplitude $a_{\pi\pi;\pi\pi}$ for π-π scattering times A itself:

$$A_2(u,t,s)=\int\frac{d^3q_1}{2\omega_1}\int\frac{d^3q_2}{2\omega_2}\delta^4(p_{\pi_1}+p_{\pi_2}-q_1-q_2)\times a_{\pi\pi;\pi\pi}(p_{\pi_1}p_{\pi_2};q_1q_2)A_{\pi\pi';N'\bar{N}}(q_1q_2;p'\bar{p}).\quad(4)$$

Here $(p_{\pi_1}+p_{\pi_2})^2=t=(p'+\bar{p})^2$ and the 2π state is the only physical state which connects the initial 2π state with the $N\bar{N}$ state in this region of t. The unitarity condition (4) contains an integral over all energy-momentum conserving states as constrained by the δ function. Since A_2 is expressed by Lorentz-invariant factors, we can simply evaluate the integrals in (4) by transforming to the center-of-mass frame $p_{\pi_1}+p_{\pi_2}=(t^{\frac{1}{2}},\mathbf{0})$. This gives the factor

$$\int\frac{d^3q_1}{2\omega_1}\int\frac{d^3q_2}{2\omega_2}\delta^4(p_{\pi_1}+p_{\pi_2}-q_1-q_2)=\int d^4q_1\,\theta(q_1^0)\theta(t^{\frac{1}{2}}-q_1^0)\times\delta(t-2\sqrt{2}q_1^0)=\tfrac{1}{8}[(t-4\mu^2)/t]^{\frac{1}{2}}\int d\Omega_1,\quad(5)$$

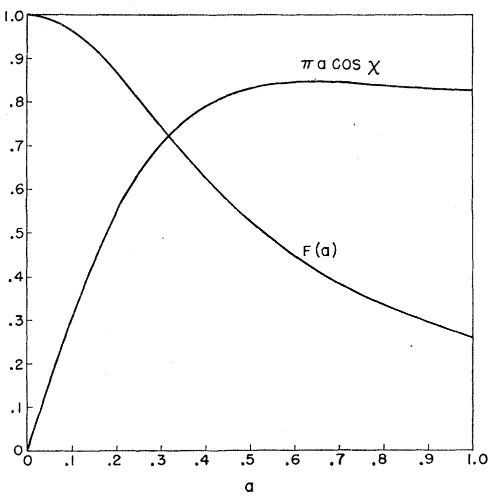

FIG. 3. Strength of the $\csc^3(\theta/2)$ singularity in point Coulomb scattering. The functions $\cos\chi$ and $\pi a \cos\chi$ of Eq. (7) are plotted as a function of a.

and shows that A_2 vanishes with $[(t-4\mu^2)/t]^{\frac{1}{2}}$ as $t\to 4\mu^2$.

This result rules out any possibility of extrapolation to the onset of the two-pion exchange cut at $4\mu^2$. On the other hand, for a two-photon exchange, the factor in (5) is finite since $\mu^2=0$ for electromagnetic quanta. This suggests an attempt to extrapolate to the cut for electromagnetic interactions. For $\mu^2=0$, the cut starts at the edge of the physical region at $\cos\theta=1$ as does also the one-photon exchange pole, indicated by Eq. (1) for $\mu^2=0$.

III. COULOMB SCATTERING BY A POINT CHARGE

The pole term in relativistic Coulomb scattering by a point charge produces a $\csc^2(\theta/2)$ singularity in the matrix element, leading to the familiar $\csc^4(\theta/2)$ singularity of the Rutherford scattering cross section in the forward direction. Many-photon exchanges do not alter the strength of this singularity in the cross section, but simply build up a phase factor in the matrix element. Extrapolating to cuts hence requires picking out weaker singularities from the dominant Rutherford term. A convenient experimental method to do this is to measure the *difference* of electron and positron scattering at small angles, since this difference is nonzero only due to the existence of many-photon exchange terms.

For any heavy target particle in nature, the Coulomb scattering law will be modified due to finite size of the charge distribution. Our interest in this section, before including such effects, is to identify the leading terms in the Born series at small angles.

The two-photon exchange diagram produces a $\csc(\theta/2)$ singularity in the matrix element and so a $\csc^3(\theta/2)$ singularity in the cross section, the McKinley-Feshbach term.[4] One asks whether exchanges of more photons also contribute to this singularity. To answer this question, and to specify more completely the singularities of the matrix element, we have used Mott's phase shift formulation of the Coulomb scattering problem.[5] For simplicity we treated the extreme-relativistic case, though the same techniques are easily applied to any energy. Then the cross section is given by $GG^*/p^2\cos^2(\theta/2)$ where, by summing over partial waves, we obtain

$$G\approx\tfrac{1}{2}a[\sin(\theta/2)]^{2ia-2}\frac{\Gamma(1-ia)}{\Gamma(1+ia)}\times\{1+\tfrac{1}{2}\pi a e^{i\chi}\sin(\theta/2)+[(-\tfrac{1}{2}+ia-\tfrac{1}{8}\pi^2a^2)-\tfrac{1}{2}(\sin(\theta/2))^{-2ia}(N+2iaM)]\sin^2(\theta/2)\},\quad(6)$$

$$e^{i\chi}=\Gamma(\tfrac{1}{2}-ia)\Gamma(1+ia)/\Gamma(\tfrac{1}{2}+ia)\Gamma(1-ia),$$

$$N\approx 1+\pi a-(4-\tfrac{1}{12}\pi^2)a^2,$$

$$M\approx 1+\pi a,$$

where $a=ze^2$.

This gives the complete analytic form of all terms in G which do not vanish in the forward direction; the only approximation is in the a-dependence of M and N.[6] The terms which have been neglected are of orders $a^3\theta^2$ and $a^2\theta^3$.

On squaring and dropping terms of higher order, the cross section is given as

$$(2p)^{-2}a^2\csc^4(\theta/2)\{1+[\pi a\cos\chi]\sin(\theta/2)-[2aM\sin(2a\ln\sin(\theta/2))+N\cos(2a\ln\sin(\theta/2))]\sin^2(\theta/2)\}.\quad(7)$$

There are no terms which simply have a $\csc^2(\theta/2)$ singularity, without the oscillations of the logarithmic factor. In answer to our original question, we see that higher photon exchanges do contribute to the $\csc^3(\theta/2)$ singularity of the cross section.[7] The coefficient πa of the McKinley-Feshbach term is replaced by $\pi a\cos\chi$, which is plotted in Fig. 3. However, since $\cos\chi$ is even in a, it is still true that the difference between electron and positron scattering directly measures this singularity. In point Coulomb scattering one can thus extrapolate to the cuts corresponding to many-photon exchange.

Finally, to test how well Eq. (7) approximates exact point Coulomb cross sections we compare with the

[4] W. A. McKinley and H. Feshbach, Phys. Rev. **74**, 1759 (1948), also R. H. Dalitz, Proc. Roy. Soc. (London) **A206**, 509 (1951).

[5] N. F. Mott, Proc. Roy. Soc. (London) **A135**, 429 (1932).

[6] It is interesting to note that, although both the $\csc^2(\theta/2)$ and $\csc(\theta/2)$ singularities are multiplied by the same characteristic Coulomb phase factor, there do exist terms which do not contain this factor.

[7] We have since learned that J. H. Bartlett and R. E. Watson, Proc. Am. Acad. Arts Sci. **74**, 53 (1940), computed $\cos\chi$; however their treatment of the $\csc^2(\theta/2)$ singularity is apparently incorrect. One can extract the small-angle portions of the cross sections recently calculated by B. Nagel, Trans. Roy. Inst. Technol. Stockholm No. 157 (1960), and by W. R. Johnson, T. A. Weber, and C. J. Mullin, Phys. Rev. **121**, 933 (1961), by making a consistent expansion of Eq. (7) in a through relative order a^2, for fixed θ.

TABLE I. Small-angle approximation to the differential cross section for point Coulomb scattering. Ratios of the results predicted by Eq. (7) to the numerical calculations of Doggett and Spencer[a] are given as a function of charge Z and angle θ, for electron velocity $\beta=1$.

θ \ Z	6	13	29	50	82
15°	1.00	1.00	1.00	0.99	1.04
30°	1.00	1.00	0.99	0.97	1.07
45°	1.00	1.00	0.99	0.94	1.01
60°	1.00	1.00	0.99	0.90	0.92

[a] J. A. Doggett and L. V. Spencer, Phys. Rev. **103**, 1597 (1956).

tabulation of Doggett and Spencer. The results shown in Table I encourage us to pursue this small-angle approach in studying scattering from finite nuclei, to which we now turn.

IV. HIGH-ENERGY APPROXIMATION FOR SCATTERING FROM FINITE NUCLEI

Replacing a point charge by a scatterer with a finite charge distribution $\rho(r)$ simply multiplies the one-photon-exchange matrix element by a form factor $F(\Delta) = \int d^3r\,\rho(r)e^{i\Delta\cdot r}$, where $\Delta=2K\sin(\theta/2)$ is the momentum transfer. $F(\Delta)$ reduces the cross section for large Δ. For distributions with fairly sharp edges, approximating the physical situation, $F(\Delta)$ is oscillatory and gives rise to zeros in the first-Born-approximation cross section. We are interested in forward angles θ such that $\Delta\cong K\theta$ is small enough to prevent $F(\Delta)$ from going through its first diffraction minimum. Our expansion parameter is $\Delta/K\ll 1$; however ΔR, where R is the "size" of the source distribution, may be large or small. Then we anticipate a rapid convergence of the Born series (as in the point Coulomb case) and we study the second Born amplitude. In this section we show that at high energies this second Born contribution is characterized completely by a second function of momentum transfer, $G(\Delta)$, and that its ratio to first Born decreases as 1/energy.

Our discussion is based on the formulation by Lewis[8] of the Born series for scattering of a relativistic electron by a charge distribution. The validity of the Born series iteration is estimated by comparing the amplitudes of the first and second Born approximations.[9] The accuracy of our high-energy limit is determined by comparing with exact numerical results for the second Born amplitude where available, and by estimating correction terms.

Since the one-photon matrix element is real, only the real part of the second Born matrix element contributes to the cross section in relative order Z.[10] Following Lewis, the cross section neglecting order Z^2 may be written

$$d\sigma=d\sigma_{\mathrm{Mott}}|F(\Delta)|^2(1+R), \tag{8}$$

where R is odd in Z and to this order represents the difference between electron and positron scattering divided by their sum. R is given by a double integral:

$$\begin{gathered} R=[2Ze^2K\Delta^2/\pi^2P^2F(\Delta)]I,\\ I=P.V.\int d^3K'\frac{\mathbf{P}\cdot\mathbf{K}'+\frac{1}{2}P^2}{K'^2-K^2}\frac{F(\mathbf{K}'-\mathbf{K}_f)}{|\mathbf{K}'-\mathbf{K}_f|^2}\frac{F(\mathbf{K}'-\mathbf{K}_i)}{|\mathbf{K}'-\mathbf{K}_i|^2},\\ \mathbf{P}=\mathbf{K}_i+\mathbf{K}_f,\quad \mathbf{\Delta}=\mathbf{K}_i-\mathbf{K}_f, \end{gathered} \tag{9}$$

where $\mathbf{K}_i$ is the incident electron momentum and $\mathbf{K}_f$ is the final electron momentum.

We want to show that for high energies and small angles I is a function of Δ only, so that $R\approx(Ze^2/K)G(\Delta)$. To do this we change variables to $l=K-\frac{1}{2}P$ and insert for the form factors

$$\begin{aligned} F(|l+\tfrac{1}{2}\Delta|)&=\int_0^\infty dx\,F(x^{\frac{1}{2}}\Delta)\delta(x-(l+\tfrac{1}{2}\Delta)^2/\Delta^2),\\ F(|l-\tfrac{1}{2}\Delta|)&=\int_0^\infty dy\,F(y^{\frac{1}{2}}\Delta)\delta(y-(l-\tfrac{1}{2}\Delta)^2/\Delta^2). \end{aligned} \tag{10}$$

We can carry out the momentum integrals to obtain

$$\begin{aligned} I=\frac{\pi}{2\Delta}\int_0^\infty\frac{dx}{x}F(x^{\frac{1}{2}}\Delta)\int_0^\infty\frac{dy}{y}F(y^{\frac{1}{2}}\Delta)\theta(4xy-(1-x-y)^2)&\\ \times\left\{1+\frac{\Delta}{P}\frac{(2P^2/\Delta^2+1-x-y)\epsilon(x+y-1)\theta[(1-x-y)^2(1+\Delta^2/P^2)-4xy]}{[(1-x-y)^2(1+\Delta^2/P^2)-4xy]^{\frac{1}{2}}}\right\}&=\{J_1+J_2\}. \end{aligned} \tag{11}$$

The first term in the brackets has the desired property, depending on Δ alone. In the second term the two step functions limit the integration region to the shaded area in Fig. 4, between the parabola $(1-x-y)^2=4xy$ and the ellipse $(1-x-y)^2=4xy/(1+\Delta^2/P^2)$. For $\Delta^2/P^2\ll 1$, this approximates a single line integral along the narrow strip between the two curves. At high energies the dominant contributions to J_2 come from values of x, $y\ll P^2/\Delta^2$ where the strip has width Δ^2/P^2. To carry out the integral we introduce the variables

$$\begin{aligned} w&=(1-x-y)^2/4xy,\\ z&=[\tfrac{1}{2}(x+y)-\tfrac{1}{4}]^{\frac{1}{2}}, \end{aligned} \tag{12}$$

[8] R. R. Lewis, Phys. Rev. **102**, 537 (1956).

[9] *Amplitudes* should be compared, not magnitudes: The Born series iteration is not necessarily poor at a zero of the form factor.

[10] We do not discuss the imaginary part of second Born approximation, which contributes to the cross section in order Z^2 and "washes out" the zeros of $F(\Delta)$. This is consistent with the restriction of our discussion to the region before the first diffraction zero.

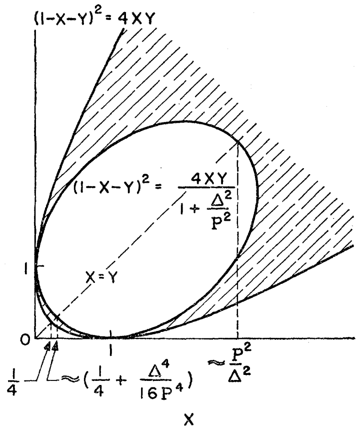

FIG. 4. Region of integration of the integral J_2, Eq. (11), in the xy plane, is the shaded area between parabola and ellipse.

and integrate dw.[11] Keeping the leading terms in P/Δ, we obtain

$$J_2 = 4\pi P.V.\int_0^\infty dl \frac{F(|l+\Delta/2|)F(|l-\Delta/2|)}{l^2-\Delta^2/4}, \tag{13}$$

where neglected terms are $(\Delta/P)^2 \ll 1$ or $(\Delta/P)(1/PR)^2 \ll 1$, R the size or radius of the charge distribution.[12] For a nucleus such as Cu, $PR\approx 6$ at 300 Mev. J_2 is a function of Δ only and in this approximation is now of the desired form.

This establishes the high-energy limit and leads to an experimental consequence. The cross section in our approximation takes the form

$$d\sigma = d\sigma_{\mathrm{Mott}}|F(\Delta)|^2\left[1+\frac{Ze^2}{K}G(\Delta)\right], \tag{14}$$

and therefore predicts the dependence

$$R = \frac{d\sigma_{e^-}-d\sigma_{e^+}}{d\sigma_{e^-}+d\sigma_{e^+}} = \frac{Ze^2}{K}G(\Delta), \tag{15}$$

TABLE II. Form factors $F(\Delta a)$ for various charge distributions, taken mainly from Herman and Hofstadter.[a]

Model \ Δa	2.0	2.5	3.0
uniform	0.474	0.279	0.114
trapezoidal, $t=1.5$	0.480	0.287	0.132
trapezoidal, $t=2.5$	0.487	0.298	0.146
trapezoidal, $t=3.5$	0.486	0.306	0.152
hollow Gaussian	0.492	0.312	0.163
Gaussian	0.513	0.353	0.432
exponential	0.563	0.432	0.327
Yukawa I	0.600	0.490	0.400

[a] See reference 2.

[11] The form factors depend on w through Eq. (12) but the dependence is very weak since $1/(1+\Delta^2/P^2)\leq w \leq 1$. We carried through a numerical integration for a shell distribution $F(x^{\frac{1}{2}}\Delta) = (\sin x^{\frac{1}{2}}\Delta R)/x^{\frac{1}{2}}\Delta R$ and for w at both extremes; i.e., integrating along both the ellipse and hyperbola. The results differed by less than 1% for Co at 300 Mev.

[12] In addition to the error estimate of footnote 11, we have compared Eq. (13) with the results which Lewis obtained for Yukawa and Gaussian distributions. The differences were only a few percent, and generally less than $\Delta^2/4P^2$, etc.

which can be checked by programming at fixed Δ and different energies K. Conversely, the sum $(d\sigma_{e^-}+d\sigma_{e^+})$ can be interpreted *directly* in terms of the nuclear form factors. Since our results are limited to Δ within the first minimum of $F(\Delta)$, very high accuracy is required to distinguish different nuclear models. It must be remembered, however, that Coulomb scattering cross sections are enormous at small angles because of the $\csc^4(\theta/2)$ factor so that high precision is possible.[13] To illustrate the required accuracy we give in Table II the computed form factors for different nuclear models corresponding to the same root-mean-square radius value and for various momentum transfers.

V. APPLICATION TO ELECTRON AND POSITRON SCATTERING

Equation (14) shows that the sum of electron and positron scattering can provide a direct measure of the form factor, and that the ratio R of difference to sum has the simple form $(Ze^2/K)G(\Delta)$ at high energies. These predictions are independent of the charge distribution.

From Eqs. (11) and (13) we can also calculate the $G(\Delta)$ predicted by any charge distribution. For a point charge, $G(\Delta)=\frac{1}{2}\pi\Delta$. For a Yukawa distribution, $\rho(r) = e^{-\lambda r}$,

$$G(\Delta) = \frac{\Delta^2}{\lambda}\left[-\frac{6\lambda^2}{\Delta^2+4\lambda^2}+\frac{\Delta^2+\lambda^2}{\Delta\lambda}\tan^{-1}\left(\frac{2\lambda^3}{\Delta(\Delta^2+3\lambda^2)}\right)\right]. \tag{16}$$

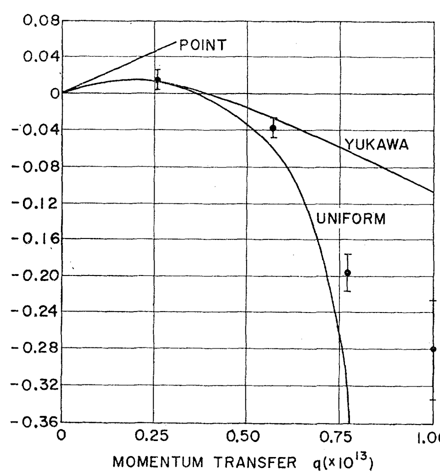

FIG. 5. Comparison of theory and experiment for R, the ratio of the difference to the sum of electron and positron scattering. Predictions of point, Yukawa, and uniform distributions in the high-energy approximations are compared with the preliminary experimental values of Pine and Yount for Co at 300 Mev.

[13] When we are concerned with high precision we must correct for other contributions not considered here, due to nuclear polarizability and to exchange of transverse photons in addition to the "Coulomb" photons between the electron and nucleus. These effects have been considered previously and shown to be small. In the present context they can be neglected entirely since the matrix elements for these effects have no singularities at $\theta=0$.

For a more realistic uniform distribution, writing $R=(Ze^2/y)G(x)$, where $y=ka$, $x=\Delta a$, with a the rms radius, and defining $\beta=1/[(5/3)^{\frac{1}{2}}x]$, then

$$G(x)=\frac{x}{F(x)}[\tfrac{1}{2}\pi-\mathrm{Si}(1/\beta)+\beta^2(-150\beta^4+48\beta^2+5)$$
$$\times\sin(1/\beta)-\beta(-150\beta^4-2\beta^2+1)\cos(1/\beta)], \quad (17)$$

where $\mathrm{Si}(z)$ is the sine integral. $G(\Delta)$ has a zero for these finite charge distributions, and in the case of a distribution with an edge it also oscillates, like $F(\Delta)$, and generally out of phase. $G(\Delta)$ passes through its first zero before $F(\Delta)$.

Pine and Yount[14] have recently determined R in the scattering of 300-Mev positrons and electrons from Co through angles $\theta=10°$, 20°, 30°, 40°. Predictions for R from various nuclear models are compared with experiment in Fig. 5. Within the first zero of the form factor the uniform model gives a good prediction. To distinguish details between different realistic models, however, a better estimate of the importance of higher Born contributions must be made.

[14] J. Pine and D. Yount (private communication).

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Vector Mesons and Nucleon-Nucleon Potentials*

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The role of the π-π resonances in the nucleon-nucleon force is considered on a model which treats these resonances as particle exchanges. The nucleon form factors are used to obtain the resonance parameters; thus the coupling constant of the "particles" to the nucleon are the only free parameters. It is found that the qualitative features of the central, spin-orbit, and tensor potentials in all states can be reproduced, except that the central repulsive core has too long a range. These results suggest that π-π effects dominate rather than supplement the usual uncorrelated 2π, 3π contribution to the nucleon-nucleon force.

THERE is considerable current interest in the role of π-π resonances in other strong interactions. In this article we discuss the effect of the $J=1$, $T=1$ and $J=1$, $T=0$ resonances, considered as ρ and ω mesons, respectively, on nucleon-nucleon scattering. In particular, it is conjectured that π, ρ, and ω exchange are the dominant contributions to the long-range part of the nucleon-nucleon force.

Instead of using dispersion-theoretic techniques to compute the contribution of what would there appear as poles, we propose obtaining a useful orientation in terms of standard one-particle exchange potential theory. This procedure allows one to compare the results directly with phenomenological potentials.[1,2] Thus, as has been pointed out by Breit[3] and Sakurai,[4] the repulsive core and strong L-S forces needed to understand the scattering data can be qualitatively understood by a neutral vector meson. The difference in our approach lies in the use of the nucleon form factors. Including both "charge" and "magnetic" contributions to the meson-nucleon currents, we can identify the mesons in the nucleon structure as shown below.

The nucleon's electromagnetic structure has been shown to require a $J=1$ strong π-π interaction for its understanding. Bergia *et al.*[5] have given a simple discussion of the electromagnetic (e.m.) form factors, F, in the context of dispersion theory and found that $T=0$, $J=1$ resonances lead to ($\hbar=c=1$)

$$F_{1,2}{}^{S,V}=1-a_{1,2}{}^{S,V}+\frac{a_{1,2}{}^{S,V}}{1-q^2/m^2}, \quad (1)$$

where S-V refer to the isoscalar-vector, 1-2 to the charge-anomalous moment parts of the photon-nucleon current [e.g., see (2) below]; and m is the resonance energy, 1-a being the fraction of charge remaining in the core or short-range part of the cloud.

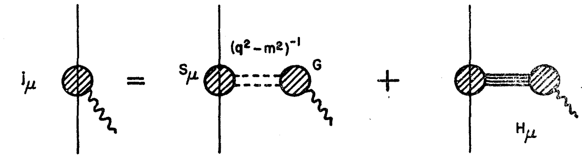

FIG. 1. The composition of the photon-nucleon vertex.

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