Transport Properties of Liquid He^3

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Improved theoretical estimates of transport coefficients in liquid He^3 at various pressures are made. The most recent experimental values of the parameters which enter the Landau phenomenological theory are employed, and the importance of using the correct forward scattering amplitude rather than a self-energy in the estimate of transition probabilities is noted. The results are summarized in a table, in which are also listed the latest empirical values of the parameters of the liquid.

RECENT experiments[1–4] on liquid He^3 have been performed at sufficiently low temperatures for the fluid to exhibit Fermi liquid behavior consistently. Thus, the transport coefficients have been demonstrated to depend on temperature in the manner predicted[5,6] by solutions of the kinetic equation appropriate to the Landau model of a Fermi liquid.[7] With the improved experimental values now available for parameters of liquid He^3 it seems profitable to make more accurate theoretical predictions of the properties of the liquid. This is largely a matter of improving the approximation for the transition probability, $w(\theta,\varphi)$ for the scattering of quasi-particles around the Fermi surface. Here θ is the angle between the initial momenta of the two colliding particles, and φ is the angle between the planes determined by the two initial momenta and by the two final momenta. Then $w(\theta,\varphi)$ can be approximated by $(2\pi/\hbar)[a^2(\theta;\boldsymbol{\sigma},\boldsymbol{\sigma}')]_{\text{av}}$, where $a(\theta;\boldsymbol{\sigma},\boldsymbol{\sigma}')$ is the forward scattering amplitude for quasi-particles of spin $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}'$ on the Fermi surface, and the appropriate average over spin is to be taken. We might expect this neglect of the φ dependence of w to introduce a probable error of about a factor of 2 in the final answer.

The measurements of specific heat C_v by Anderson *et al.*[1] have clearly demonstrated, at least at low pressures, a linear dependence on temperature which, when extrapolated, predicts $C_v=0$ at $T=0$°K. Thus, the value of the effective mass, m^*, for zero pressure obtained from these data must be considered to be reliable. The values for m^* at higher pressures must be taken to be lower limits, since the linear region of the specific heat has not yet been reached. The velocity of sound[2] c, the density[3] ρ, and the magnetic susceptibility[4] χ have been measured as functions of pressure and in sufficiently wide temperature ranges to enable reliable extrapolation to absolute zero. The first two terms of an expansion of $a(\theta;\boldsymbol{\sigma},\boldsymbol{\sigma}')$ in a Legendre polynomial series can be obtained from these parameters. This is accomplished by the establishment of a relationship between $a(\theta;\boldsymbol{\sigma},\boldsymbol{\sigma}')$ and the second functional derivative of the system energy with respect to the quasi-particle distribution functions $n(\mathbf{p},\boldsymbol{\sigma})$ and $n(\mathbf{p}';\boldsymbol{\sigma}')$—that is, the characteristic function of the Fermi liquid, $f(\mathbf{p},\boldsymbol{\sigma};\mathbf{p}',\boldsymbol{\sigma}')$. Here $\mathbf{p}$ and $\mathbf{p}'$ are restricted to the Fermi surface. Following convention we denote the density of states at the Fermi surface by $(d\tau/d\epsilon)_\mu$. Then assuming the spin dependence in f to be wholly of exchange nature, we have, for $\mathbf{p}$ and $\mathbf{p}'$ on the Fermi surface:

$$\left(\frac{d\tau}{d\epsilon}\right)_\mu f(\mathbf{p},\boldsymbol{\sigma};\mathbf{p}',\boldsymbol{\sigma}')=\sum(F_l+\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'Z_l)P_l(\cos\theta),$$

where[7]

$$F_0=\frac{3mm^*}{P_0^2}c^2-1;\quad F_1=3\left(\frac{m^*}{m}-1\right);\quad Z_0=\frac{\beta^2(d\tau/d\epsilon)_\mu}{\chi}-4.$$

Here p_0 is the Fermi momentum $[p_0=\hbar(3\pi^2\rho/m)^{\frac{1}{3}}]$, m is the mass of a bare He^3 atom, and β is twice the magnetic moment of a He^3 nucleus. It is important to note that f and a are not identical,[8] although in the past[5,6] they have been treated as equal as a rough approximation. They are both proportional to the vertex part of a two-particle Green's function in the limit that both momentum transfer q and energy transfer ω vanish. However, in the calculation of f the limits are to be taken so that $\omega/q\rightarrow\infty$, while for the determination of a they must be taken so that $\omega/q\rightarrow 0$. Landau has shown[9] that $a(\theta;\boldsymbol{\sigma},\boldsymbol{\sigma}')$ is given by

$$\left(\frac{d\tau}{d\epsilon}\right)_\mu a(\theta;\boldsymbol{\sigma},\boldsymbol{\sigma}')=\sum(B_n+\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'C_n)P_n(\cos\theta),$$

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[8] The author is grateful to Professor Philippe Nozières for pointing out the importance of using the correct forward scattering amplitude for He^3.

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TABLE I. Parameters of liquid He^3.

Pressure (atm)	0	2.75	7.75	16.5
Density (g/cm^3)	0.082	0.089	0.097	0.106
$(p_0/\hbar)\times 10^{-8}$ (cm^{-1})	0.785	0.808	0.831	0.856
m^*/m	2.82	3.06	3.40	4.04
$(d\tau/d\epsilon)_\mu \times 10^{-38}$ (erg^{-1} cm^{-3})	1.01	1.13	1.29	1.72
F_0	9.25	14.8	24.3	40.6
F_1	5.46	6.18	7.20	9.12
Z_0	−2.80	−2.87	−2.97	−2.95
Z_1	−1.75	−1.46	−0.76	−1.52
T^* [a]	0.347	0.321	0.281	0.234
C (m/sec)[b]	182	223	276	335
ηT^2 (poise °K^2) (theory)	1.5×10^{-6}	1.2×10^{-6}	0.87×10^{-6}	0.73×10^{-6}
(expt)[c]	2.8×10^{-6}			
κT (erg cm^{-1} sec^{-1}) (theory)	57	45	30	24
(expt)[d]	48 ± 3			
DT^2 (cm^2 sec^{-1} °K^2) (theory)	4.2×10^{-6}	2.9×10^{-6}	1.5×10^{-6}	0.85×10^{-6}
(expt)[d]	1.54×10^{-6}	1.0×10^{-6}	0.62×10^{-6}	0.35×10^{-6}

[a] Defined as the ratio of the product of the magnetic susceptibility and the temperature at 1.18°K to the magnetic susceptibility at $T=0$°K.
[b] Velocity of sound at $T<0.1$°K.
[c] See reference 2.
[d] See reference 4.

where B_n and C_n are related to F_n and Z_n, respectively, by[5]

$$B_n=\frac{F_n}{1+F_n/(2n+1)},\quad C_n=\frac{Z_n}{1+Z_n/4(2n+1)}.$$

We note that the reduction of the coefficients for small n can be considerable, so that the use of f instead of a can modify the results appreciably. As we want to retain the P_1 as well as the P_0 terms we must choose C_1. A consistent way to do this is to require that the scattering amplitude for parallel spins and $\theta=0$ vanish, so that the Pauli principle is satisfied.

The Landau theory is formulated so that the spin of a quasi-particle is approximately a good quantum number. Thus, a description of a collision as involving parallel or antiparallel spins is consistent with the approximations of the theory. For the viscosity η and the thermal conductivity κ the two types of collisions must be weighted equally in the average of a^2 over spin, as transport of momentum and energy rather than explicitly spin-dependent quantities is involved. However, as was pointed out in a previous paper,[6] for the calculation of the diffusion coefficient D we must consider anti-parallel spin collisions only.

Solutions of the kinetic equation give the following expressions for the transport coefficients[5,6]:

$$\eta=\frac{64}{45}\frac{\hbar^3 p_0^5}{m^{*4}T^2}\left\{\left[\frac{w_\eta(\theta,\varphi)}{\cos(\theta/2)}(1-\cos\theta)^2\sin^2\varphi\right]_{\rm av}\right\}^{-1},$$

$$\kappa=\frac{8}{3}\frac{\pi^2\hbar^3 p_0^3}{m^{*4}T}\left\{\left[\frac{w_\kappa(\theta,\varphi)}{\cos(\theta/2)}(1-\cos\theta)\right]_{\rm av}\right\}^{-1},$$

$$D=[1+\tfrac{1}{4}Z_0]\frac{32\pi^2\hbar^6 p_0^2}{3m^{*5}T^2}\times\left\{\left[\frac{w_D(\theta,\varphi)}{\cos(\theta/2)}(1-\cos\theta)(1-\cos\varphi)\right\}_{\rm av}\right]^{-1}.$$

The subscripts on w are introduced in recognition of the differences in transition probabilities due to the different spin averages. The calculated as well as the empirical values for η, κ, and D, plus the latest experimental values for the parameters characterizing the liquid are given in Table I. We estimate the probable error in the theoretical coefficients at zero pressure to be a factor of 2 or 3. At higher pressures the errors are expected to be greater, due to uncertainties in the experimental values of m^*. However, the general behavior of the transport coefficients with pressure should be as indicated. This is substantiated by the experimental results for D.