vibration. The dispersion relation (2) has a somewhat local character because of its frequency dependence. The effect of adding a classical mode at 400 cm^{-1} was carefully checked. Within experimental uncertainty, even a strong mode at 400 cm^{-1} does not affect the inversion of the over-all reflectivity (with the extra mode included) to obtain the dielectric constant in the region of the low-frequency mode.

Recently, some success has been achieved in explaining the linewidth of optical modes in alkali halides in terms of anharmonic damping.[7] This work suggests that the very broad line in $SrTiO_3$ may result partly from the form of the acoustic phonon dispersion curves. Anharmonic potential terms in the Hamiltonian allow the $k=0$ optic mode to dissipate energy to phonons of other wave vectors. The damping is particularly strong at frequencies $\omega_{la}-\omega_{ta}$ and $\omega_{la}+\omega_{ta}$, where ω_{la} and ω_{ta} are the frequencies of the longitudinal and transverse acoustic phonon branches at the zone boundary.[7,8] For the alkali halides the natural frequency of the optic mode falls in a region of low damping about midway between the above regions of high damping. In $SrTiO_3$, however, a rough calculation using the elastic constants indicates the optic mode frequency to be within 30% of $\omega_{la}-\omega_{ta}$ which is right in the region of large damping.[7,8]

The behavior of $\tan\delta=\epsilon''/\epsilon'$ is of considerable practical interest at frequencies below the dispersion. Here $SrTiO_3$ exhibits nonlinear effects and may be used for harmonic generation, modulation, parametric amplification, etc., with an efficiency usually governed by $\tan\delta$. Rupprecht *et al.*[9] have measured $\tan\delta$ over a wide range of microwave frequencies. Fitting a classical dispersion form to the low-frequency mode observed here gives the frequency and temperature dependence of $\tan\delta$ in the microwave region below 150°K seen by them, namely $\tan\delta\sim\omega/(T-T_c)$. At frequencies so far from the infrared resonance, however, the anharmonic damping factor has decreased considerably,[7] and other damping mechanisms[9] may be important.

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Moving Edge Dislocations in Cubic and Hexagonal Materials*

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Numerical values of the threshold and limiting velocities of edge dislocations in various cubic and hexagonal materials are presented; a number of orientations and directions of motion of the edge dislocation are considered. At the limiting velocity C_∞, the energy of the dislocation becomes infinite; in many instances it is found that the limiting velocity is less than the corresponding velocity of shear sound. At the threshold (Rayleigh) velocity C_R, the shear stress of the moving edge dislocation vanishes on the slip plane. Edge dislocations of like sign moving at velocities between C_R and C_∞ attract rather than repel one another. The lower the value of the ratio C_R/C_∞, the easier it should be to bring edge dislocations into the anomalous velocity range. Theoretically this ratio has zero as a lower limit; however, no value of C_R/C_∞ less than 0.85 was found for those materials and orientations considered.

I. INTRODUCTION

IN a previous paper[1] the dynamical behavior of uniformly moving dislocations in anisotropic media was discussed for those crystal systems for which the edge and screw components can be considered separately. It was found that a screw dislocation behaves normally at all velocities up to its limiting velocity. An edge dislocation, however, displays an anomalous behavior; in general there is a range of velocities for which the shear stress on the slip plane of the moving edge dislocation is negative and edge dislocations of like sign attract rather than repel one another. The existence of this anomalous velocity range in isotropic materials was first pointed out by Weertman.[2] In an isotropic material, the upper limit of this velocity range is the velocity of shear sound, which is also the limiting velocity of the edge dislocation; the lower limit is the Rayleigh wave velocity which can never be less than 0.69 the velocity of shear sound. In the anisotropic case it is possible for both the upper limit (C_∞, the limiting velocity) and the lower limit (C_R, the threshold velocity) to be any velocity from zero up to the shear wave velocity, depending on the elastic constants of the material and the orientation considered.

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[1] L. J. Teutonico, Phys. Rev. **124**, 1039 (1961).
[2] J. Weertman, *Response of Metals to High Velocity Deformation*, edited by P. G. Shewmon and V. F. Zackay (Interscience Publishers, Inc., New York, 1961).

TABLE I. Threshold (Rayleigh) and limiting velocities for edge dislocations in cubic materials.

Symbol	Material	Case 1			Case 2			Case 3			Case 4		
		C_R/C_2	C_∞/C_2	C_R/C_∞	C_R/C_1	C_∞/C_1	C_R/C_∞	C_R/C_2	C_∞/C_2	C_R/C_∞	C_R/C_2	C_∞/C_2	C_R/C_∞
Ag	Silver (1)	0.713	0.764	0.933	0.992	1.000	0.992	0.818	0.909	0.900	0.799	0.875	0.913
Ag	Silver (2)	0.725	0.779	0.931	0.991	1.000	0.991	0.823	0.917	0.897	0.806	0.884	0.912
Al	Aluminum (1)	0.909	1.000	0.909	0.954	1.000	0.954	0.917	1.000	0.917	0.915	1.000	0.915
Al	Aluminum (2)	0.892	0.997	0.895	0.964	1.000	0.964	0.907	1.000	0.907	0.903	1.000	0.903
Au	Gold	0.732	0.780	0.938	0.993	1.000	0.993	0.831	0.915	0.908	0.820	0.896	0.915
C	Diamond (1)	0.857	1.000	0.857	0.911	1.000	0.911	0.868	1.000	0.868	0.864	1.000	0.864
C	Diamond (2)	0.849	0.972	0.873	0.959	1.000	0.959	0.875	0.998	0.877	0.867	0.991	0.875
C	Diamond (3)	0.894	1.000	0.894	0.929	1.000	0.929	0.901	1.000	0.901	0.899	1.000	0.899
Cu	Copper	0.694	0.742	0.935	0.992	1.000	0.992	0.807	0.899	0.898	0.783	0.856	0.915
Ge	Germanium	0.825	0.948	0.870	0.960	1.000	0.960	0.860	0.993	0.866	0.849	0.976	0.870
K	Potassium	0.525	0.544	0.965	0.998	1.000	0.998	0.738	0.807	0.914	0.696	0.741	0.939
Li	Lithium	0.439	0.451	0.973	0.999	1.000	0.999	0.704	0.767	0.918	0.649	0.685	0.947
Mo	Molybdenum	0.953	1.000	0.953	0.892	1.000	0.892	0.948	1.000	0.948	0.949	1.000	0.949
Na	Sodium (1)	0.465	0.481	0.967	0.998	1.000	0.998	0.704	0.779	0.904	0.637	0.675	0.944
Na	Sodium (2)	0.489	0.504	0.970	0.998	1.000	0.998	0.722	0.789	0.915	0.674	0.715	0.943
Ni	Nickel	0.748	0.816	0.917	0.985	1.000	0.985	0.826	0.938	0.881	0.806	0.897	0.899
Pb	Lead	0.652	0.684	0.953	0.996	1.000	0.996	0.794	0.870	0.913	0.775	0.837	0.926
Si	Silicon	0.841	0.966	0.871	0.958	1.000	0.958	0.869	0.997	0.872	0.861	0.987	0.872
Th	Thorium	0.654	0.699	0.936	0.992	1.000	0.992	0.784	0.881	0.890	0.748	0.816	0.917
W	Tungsten	0.925	1.000	0.925	0.925	1.000	0.925	0.925	1.000	0.925	0.925	1.000	0.925
AgBr	Silver bromide	0.977	1.000	0.977	0.867	0.966	0.898	0.972	1.000	0.972	0.972	1.000	0.972
AgCl	Silver chloride	0.984	1.000	0.984	0.834	0.918	0.908	0.979	1.000	0.979	0.980	1.000	0.980
$Ba(NO_3)_2$	Barium nitrate	0.970	1.000	0.970	0.843	0.952	0.886	0.963	1.000	0.963	0.964	1.000	0.964
CaF_2	Fluorspar	0.968	1.000	0.968	0.844	0.956	0.883	0.961	1.000	0.961	0.963	1.000	0.963
Cr_2FeO_4	Chromite	0.885	0.999	0.886	0.951	1.000	0.951	0.899	1.000	0.899	0.895	1.000	0.895
CuZn	β-brass (1)	0.460	0.473	0.973	0.998	1.000	0.998	0.713	0.777	0.918	0.665	0.704	0.945
CuZn	β-brass (2)	0.455	0.467	0.974	0.999	1.000	0.999	0.712	0.773	0.921	0.664	0.702	0.946
Cu_3Au	Copper-gold	0.758	0.822	0.922	0.989	1.000	0.989	0.838	0.937	0.894	0.823	0.909	0.905
Fe_3O_4	Magnetite	0.901	1.000	0.901	0.938	1.000	0.938	0.908	1.000	0.908	0.906	1.000	0.906
FeS_2	Pyrites	0.951	1.000	0.951	0.760	0.888	0.856	0.929	1.000	0.929	0.936	1.000	0.936
GaAs	Gallium arsenide	0.821	0.926	0.887	0.972	1.000	0.972	0.862	0.984	0.876	0.850	0.965	0.881
GaSb	Gallium antimonide	0.817	0.926	0.882	0.969	1.000	0.969	0.858	0.985	0.871	0.846	0.964	0.878
InSb	Indium antimonide	0.804	0.898	0.895	0.977	1.000	0.977	0.854	0.973	0.878	0.841	0.943	0.892
$KAl(SO_4)_2 \cdot 12H_2O$	Potassium alum	0.907	1.000	0.907	0.940	1.000	0.940	0.912	1.000	0.912	0.911	1.000	0.911
KBr	Potassium bromide	0.987	1.000	0.987	0.713	0.776	0.919	0.981	1.000	0.981	0.982	1.000	0.982
KCl	Sylvine	0.985	1.000	0.985	0.727	0.794	0.916	0.979	1.000	0.979	0.980	1.000	0.980
KI	Potassium iodide	0.985	1.000	0.985	0.729	0.798	0.914	0.979	1.000	0.979	0.980	1.000	0.980
LiF	Lithium fluoride (1)	0.805	0.921	0.874	0.964	1.000	0.964	0.848	0.984	0.862	0.833	0.957	0.870
LiF	Lithium fluoride (2)	0.822	0.943	0.872	0.960	1.000	0.960	0.857	0.991	0.865	0.845	0.972	0.869
MgO	Magnesium oxide	0.842	0.977	0.862	0.948	1.000	0.948	0.867	0.999	0.868	0.858	0.992	0.865
NaBr	Sodium bromide	0.958	1.000	0.958	0.856	0.979	0.874	0.950	1.000	0.950	0.952	1.000	0.952
$NaBrO_3$	Sodium bromate (1)	0.950	1.000	0.950	0.883	1.000	0.883	0.943	1.000	0.943	0.945	1.000	0.945
$NaBrO_3$	Sodium bromate (2)	0.942	1.000	0.942	0.902	1.000	0.902	0.938	1.000	0.938	0.939	1.000	0.939
NaCl	Rock salt	0.956	1.000	0.956	0.863	0.987	0.874	0.948	1.000	0.948	0.950	1.000	0.950
$NaClO_3$	Sodium chlorate	0.962	1.000	0.962	0.856	0.974	0.879	0.954	1.000	0.954	0.956	1.000	0.956
$(NH_4)Al(SO_4) \cdot 12H_2O$	Ammonium alum	0.912	1.000	0.912	0.937	1.000	0.937	0.916	1.000	0.916	0.915	1.000	0.915
NH_4Br	Ammonium bromide	0.980	1.000	0.980	0.772	0.857	0.901	0.973	1.000	0.973	0.974	1.000	0.974
NH_4Cl	Ammonium chloride	0.981	1.000	0.981	0.761	0.841	0.905	0.974	1.000	0.974	0.976	1.000	0.976
PbS	Galena (1)	0.975	1.000	0.975	0.804	0.901	0.892	0.967	1.000	0.967	0.970	1.000	0.970
PbS	Galena (2)	0.952	1.000	0.952	0.892	1.000	0.892	0.947	1.000	0.947	0.948	1.000	0.948
TlBr	Thallium bromide	0.967	1.000	0.967	0.866	0.978	0.885	0.960	1.000	0.960	0.962	1.000	0.962
TlCl	Thallium chloride	0.971	1.000	0.971	0.852	0.960	0.888	0.964	1.000	0.964	0.966	1.000	0.966
ZnS	Zinc sulfide (1)	0.768	0.845	0.909	0.984	1.000	0.984	0.838	0.949	0.883	0.820	0.915	0.896
ZnS	Zinc sulfide (2)	0.777	0.851	0.913	0.986	1.000	0.986	0.845	0.951	0.889	0.831	0.924	0.899

Theoretically the ratio C_R/C_∞ has a lower limit of zero; the lower the value of this ratio the easier it should be to bring edge dislocations into the anomalous velocity range. The optimum material and orientation for an experimental test of the attraction hypothesis will be that combination (material and orientation) for which the ratio C_R/C_∞ has its smallest value.

This paper presents calculations of the threshold and limiting velocities of an edge dislocation in various cubic and hexagonal materials. A number of orientations and directions of motion for the edge dislocation are considered.

II. REVIEW OF THEORY

In order that the plane strain analysis of reference 1 be applicable, it is necessary that the edge dislocation lie along a diad axis or be normal to a plane of reflection symmetry. This demands

$$F_{14}=F_{15}=F_{24}=F_{25}=F_{46}=F_{56}=F_{34}=F_{35}=0, \quad (1)$$

where the F_{ij}'s are the generalized elastic constants. However, application of the analysis is tedious unless one also assumes

$$F_{16}=F_{26}=0. \quad (2)$$

TABLE II. Threshold (Rayleigh) and limiting velocities for edge dislocations in hexagonal materials.

Symbol	Material	Case 1			Case 2			Case 3		
		C_R/C_1	C_∞/C_1	C_R/C_∞	C_R/C_2	C_∞/C_2	C_R/C_∞	C_R/C_2	C_∞/C_2	C_R/C_∞
$BaTiO_3$	Barium titanate ceramic *A* (1)	0.930	1.000	0.930	0.924	1.000	0.924	0.920	1.000	0.920
$BaTiO_3$	Barium titanate ceramic *A* (2)	0.930	1.000	0.930	0.931	1.000	0.931	0.932	1.000	0.932
$BaTiO_3$+5% $CaTiO_3$ by weight	5% calcium barium titanate ceramic	0.930	1.000	0.930	0.925	1.000	0.925	0.926	1.000	0.926
Cd	Cadmium (1)	0.925	1.000	0.925	0.908	1.000	0.908	0.938	1.000	0.938
Cd	Cadmium (2)	0.922	1.000	0.922	0.929	1.000	0.929	0.950	1.000	0.950
Co	Cobalt	0.935	1.000	0.935	0.964	1.000	0.964	0.962	1.000	0.962
H_2O	Ice	0.933	1.000	0.933	0.959	1.000	0.959	0.958	1.000	0.958
Mg	Magnesium	0.928	1.000	0.928	0.944	1.000	0.944	0.943	1.000	0.943
SiO_2	β-quartz	0.898	1.000	0.898	0.930	1.000	0.930	0.932	1.000	0.932
Zn	Zinc	0.907	1.000	0.907	0.811	0.885	0.916	0.877	0.999	0.878
$Ca_5FP_3O_{12}$	Apatite	0.888	1.000	0.888	0.847	0.963	0.880	0.858	0.983	0.873
$Be_3Al_2Si_6O_{18}$	Beryl (1)	0.921	1.000	0.921	0.958	1.000	0.958	0.959	1.000	0.959
$Be_3Al_2Si_6O_{18}$	Beryl (2)	0.920	1.000	0.920	0.950	1.000	0.950	0.953	1.000	0.953
Be	Beryllium	0.890	1.000	0.890	0.869	1.000	0.869	0.863	1.000	0.863
Y	Yttrium	0.922	1.000	0.922	0.934	1.000	0.934	0.934	1.000	0.934
Tl	Thallium	0.932	1.000	0.932	0.961	1.000	0.961	0.957	1.000	0.957
$CaMg_2$	Calcium-magnesium Laves compound	0.914	1.000	0.914	0.939	1.000	0.939	0.938	1.000	0.938

For the orientations considered in this paper both conditions (1) and (2) will hold.

An edge dislocation moving on its slip plane can be treated as a wave (actually a sum of waves) propagating on an internal surface. In order that boundary conditions along the slip plane be satisfied, the direction of propagation of the wave will, in general, be inclined to the slip plane; the angle of inclination is a function of dislocation velocity. There are four possible classes of behavior:

(i) The wave propagates parallel to the slip plane at all velocities; the limiting velocity is then that of shear sound.

(ii) The angle of inclination decreases with increasing velocity and becomes zero at some velocity less than the shear sound velocity; again the limiting velocity is that of shear sound.

(iii) The direction of propagation is always inclined to the slip plane; the limiting velocity is then less than that of shear sound.

(iv) The wave propagates parallel to the slip plane initially, but at some velocity less than the shear sound velocity the direction of propagation becomes inclined to the slip plane; the limiting velocity is again less than the shear sound velocity.

In general, at some velocity between zero and the limiting velocity, the shear stress on the slip plane will vanish, thus making the slip plane a stress-free (internal) surface; depending on whether or not the direction of propagation is inclined to the slip plane at that velocity, the moving edge dislocation will correspond to a generalized or ordinary Rayleigh wave.[3]

We can summarize the above discussion in mathematical form. The threshold velocity C_R (at which the shear stress on the slip plane vanishes) is given as a solution of an equation analogous to the Rayleigh wave equation:

$$\begin{aligned} &F_{22}F_{66}^2(F_{22}-F_{66})f^6 \\ &\quad -F_{22}F_{66}[F_{66}(F_{22}-F_{11})+2(F_{11}F_{22}-F_{12}^2)]f^4 \\ &\quad +(F_{11}F_{22}-F_{12}^2)[2F_{22}F_{66}+F_{11}F_{22}-F_{12}^2]f^2 \\ &\quad -(F_{11}F_{22}-F_{12}^2)^2=0. \end{aligned} \tag{3}$$

f is the dislocation velocity C normalized to the shear wave velocity i.e., $f=C/(F_{66}/\rho)^{\frac{1}{2}}$, where ρ is the density of the material.

The limiting velocity of the edge dislocation is the velocity of shear sound, i.e., $C_\infty=(F_{66}/\rho)^{\frac{1}{2}}$ if $Q>0$, where

$$Q=(F_{11}F_{22}-F_{12}^2-2F_{12}F_{66})-F_{66}(F_{22}+F_{66}). \tag{4}$$

If, however, $Q<0$, the limiting velocity is less than the velocity of shear sound and is given as a solution of

$$\begin{aligned} &F_{66}^2(F_{22}-F_{66})^2f^2 \\ &\quad +2F_{66}\{2F_{66}[F_{22}(F_{11}+F_{12})+F_{66}(F_{22}+F_{12})] \\ &\quad -(F_{22}+F_{66})(F_{11}F_{22}-F_{12}^2)\}f \\ &\quad +(F_{11}F_{22}-F_{12}^2)[F_{11}F_{22}-(F_{12}+2F_{66})^2]=0. \end{aligned} \tag{5}$$

III. RESULTS FOR CUBIC MATERIALS

Four cases of moving edge dislocations in cubic materials have been studied.

Case	Direction along which edge dislocation lies	Direction along which edge dislocation moves
1	[001]	[100]
2	[001]	[110]
3	[101]	[10$\bar{1}$]
4	[101]	[010]

(6)

Calculations were made for those cubic elements and compounds for which elastic constants are tabulated in

[3] J. L. Synge, J. Math. and Phys. 35, 323 (1956).

Huntington's review article.[4] In Table I are listed the limiting and threshold velocities (each normalized to the appropriate shear sound velocity) as well as the ratio C_R/C_∞. For cases 1, 3, and 4 the shear sound velocity is

$$C_2=(C_{44}/\rho)^{\frac{1}{2}}, \tag{7}$$

whereas for case 2 it is

$$C_1=[(C_{11}-C_{12})/2\rho]^{\frac{1}{2}}. \tag{8}$$

(The C_{ij} are the elastic constants referred to the crystal coordinates.) All tabulated results apply at room temperature except those for potassium (83°K), lithium (78°K), and sodium (−183°C). A number of important points to be obtained from Table I are the following:

(i) In all cases the threshold velocity is less than the limiting velocity, i.e., $C_R<C_\infty$; there is always a range of velocities for which two like edge dislocations on the same slip plane will attract.

(ii) There are many instances (93 of the 216 listed) in which the limiting velocity is less than the corresponding shear sound velocity. The lowest normalized limiting velocity is 0.451 (lithium, case 1).

(iii) The normalized threshold (Rayleigh) velocity varies from 0.439 (lithium, case 1) to 0.999 [lithium and β-brass (2), case 2].

(iv) The ratio C_R/C_∞ always has values between 0.85 and unity despite the large variation in the values of C_R and C_∞. The lowest value of C_R/C_∞ is 0.857 [diamond (1), case 1].

IV. RESULTS FOR HEXAGONAL MATERIALS

Three cases of moving edge dislocations in hexagonal materials were considered:

Case 1: edge dislocation along [0001] moving in any direction in the base plane;

Case 2: edge dislocation in the base plane moving along [0001];

Case 3: edge dislocation in the base plane and moving in a direction (normal to itself) in the base plane.

Cases 1 and 2 correspond to slip in the prism planes, case 3 to slip in the base plane. The latter appears to be the only primary slip system to which the plane strain analysis is applicable. In Table II are listed the normalized threshold and limiting velocities and the ratio C_R/C_∞. For case 1 the shear sound velocity is C_1; for cases 2 and 3 it is C_2. [C_1 and C_2 are defined in (7) and (8), respectively.]

Elastic constant data for the first ten materials listed in Table II ($BaTiO_3$ through Zn) were taken from reference 4. (The three titanate ceramics are piezoelectric; for the first, elastic-constant measurements were made at constant D, whereas for the other two they were made at constant E.) The constants for apatite and beryl were obtained from Hearmon's review[5]; those for beryllium,[6] yttrium,[7] thallium[8] and the calcium-magnesium Laves compound[8] have been determined recently and did not appear in either review article. The tabulated results apply at room temperature except those for ice (−16°C), β-quartz (600°C), thallium (78°K), and $CaMg_2$ (0°C). From Table II we obtain the following points:

(i) In all instances $C_R<C_\infty$, i.e., there is a velocity range of anomalous dynamical behavior for edge dislocations.

(ii) Only for zinc and apatite (cases 2 and 3) is the limiting velocity less than that of shear sound. It has been shown (reference 1) that for case 1 the limiting velocity must be the shear sound velocity.

(iii) The normalized threshold (Rayleigh) velocity varies from 0.811 (zinc, case 2) to 0.964 (cobalt, case 2).

(iv) The smallest value found for the ratio C_R/C_∞ is 0.863 (beryllium, case 3).

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[4] H. B. Huntington, *Solid-State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1958), Vol. 7, p. 213.

[5] R. F. S. Hearmon, *Advances in Physics*, edited by N. F. Mott (Taylor and Francis Ltd., London, 1956), Vol. 5, p. 323.

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