The counting data were analyzed statistically by the "F" method[11] and found to be within a 95% confidence level. From the ratio of the extrapolated zero-bias integral counting rates in the single-channel and coincidence systems a value of 1.5 kev was obtained for the figure of merit, the energy required to be dissipated in the scintillator solution to produce on the average one electron at the photocathode.[9] For this value the observed extrapolated zero-bias integral counting rates were calculated to be 93 and 81% of the disintegration rate for single-channel and coincidence systems, respectively. A 95% confidence level error of 2% for the counting method was due to the uncertainty in the standards used to prove this method.[9]

The errors involved in the calculation of the half-life are listed below, with all errors quoted at the 95% confidence level.

(a)	Mass spectrometer	2.4%
(b)	Counting method	2.0%
(c)	Chemical analysis	1.0%
(d)	Counting statistics	<0.1%
(e)	Figure of merit	1.0%

These errors were propagated in the standard manner to give the final error in the calculation. The best value for the half-life of Ni^{63} is 91.6±3.1 yr with a 95% confidence level.

The beta spectrum appears to be that of pure Ni^{63}. A Kurie plot[12] was linear (above 7.5 kev) and gave an end-point energy of 67±2 kev for the maximum energy, which is in agreement with other published values.[13]

From the mass spectrometric data and the integrated flux a value of 14±1 barns was calculated for the $Ni^{62}(n,\gamma)Ni^{63}$ cross section. This agrees with other published values.[3,14] In this calculation we have neglected any appreciable capture cross section for Ni^{63}.

ACKNOWLEDGMENTS

The authors wish to acknowledge the helpful discussions with M. H. Studier, L. E. Glendenin, and K. F. Flynn.

[11] C. A. Bennett and N. L. Franklin, *Statistical Analysis in Chemistry and the Chemical Industry* (John Wiley & Sons, Inc., New York, 1954), p. 108.

[12] K. F. Flynn and L. E. Glendenin, Phys. Rev. **116**, 744 (1959).

[13] D. Strominger, J. M. Hollander, and G. T. Seaborg, Revs. Modern Phys. **30**, 585 (1958).

[14] H. Pomerance, Phys. Rev. **76**, 195 (1949).

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Interpretation of Experiments on the Beta Decay of Eu^{152}†

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The 1483-kev beta decay of Eu^{152} has previously been interpreted in terms of the "modified B_{ij}" approximation. This decay has been reconsidered using more complete theoretical expressions and including the recent determination of the beta-circularly polarized gamma correlation along with the beta-gamma directional correlation and the beta spectral shape. The data indicate that the nuclear matrix element parameter ζ_1 lies between 0.5 and 1. For some suitable sets of matrix element parameters the quantities u and x are small in qualitative agreement with the "modified B_{ij}" approximation. However, other suitable sets of matrix element parameters may be found which are in disagreement with the approximation. The method of analysis described here is generally applicable to first forbidden beta decays with spin change 1 and the results from various types of experiments may be incorporated.

INTRODUCTION

THE theoretical expressions for the various observables, such as the spectral shape, beta-gamma directional correlation, and the beta-polarized gamma directional correlation in first-forbidden beta decay with spin change 1 are given by Kotani[1] in terms of the following nuclear matrix element parameters:

$$\zeta_1 = -(\xi - W_0/3)u - iC_V\int \boldsymbol{\alpha} \Big/ C_A\int B_{ij} - (\xi + W_0/3)x,$$

$$u = i\int \boldsymbol{\sigma}\times\mathbf{r} \Big/ \int B_{ij},$$

$$x = -C_V\int \mathbf{r} \Big/ C_A\int B_{ij},$$

where W_0 is the end-point energy of the betas in mc^2 units and ξ is a dimensionless parameter dependent

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[1] T. Kotani, Phys. Rev. **114**, 795 (1959).

upon the atomic and mass numbers of the nucleus. With the elucidation of beta-decay theory it is now important to establish, through various beta-ray experiments, what values of the parameters are consistent with the data. For a single experiment, such as a beta-gamma directional correlation, it is generally possible to obtain a suitable set of parameters by a trial and error procedure. However, it is unlikely that such a set of parameters represents a unique solution. When two or more experiments are simultaneously analyzed, the trial and error procedure becomes increasingly cumbersome and it is still likely that a satisfactory set of parameters, if found, represents only one of a family of suitable solutions. Furthermore, if there is no set of parameters that fits the experimental data, it is difficult to establish this fact using the trial and error procedure.

In some cases, the data analysis has been simplified by making approximations that restrict the number of parameters. There has been considerable use of the "modified B_{ij}" approximation as suggested by Morita and Morita.[2] In this approximation it is assumed that all first-forbidden matrix elements are small compared to the B_{ij} matrix element. As a result, the theoretical formulas for various observables can be expressed in terms of the single nuclear matrix element parameter $Y=\zeta_1+W_0(x-u)/3$. With the theoretical expressions reduced to a single parameter, the value of Y needed to fit the data may be readily determined provided a fit is possible. There has been considerable success in fitting the data of a single experiment with the "modified B_{ij}" approximation. However, attempts to fit the results of two or more experiments simultaneously with this approximation have been only partially successful.

In the course of our recent investigations on the interpretation of beta-decay experiments it has been found that even for decays that can be successfully analyzed using the "modified B_{ij}" approximation, the "modified B_{ij}" solution is usually only one of a number of satisfactory solutions. For instance, the beta-gamma directional correlation and the spectral shape for the 1483-kev beta group of Eu^{152} can be reasonably fitted with the "modified B_{ij}" approximation.[3,4] However, when no approximation to the formulas of Kotani[1] is made, fits of the two experiments are obtained with several different sets of parameters including sets for which u and x are not small, contrary to the assumption of the "modified B_{ij}" approximation. Thus the use of the "modified B_{ij}" approximation results in the neglect of many equally valid solutions.

Since the experimental determination of beta-decay matrix elements is capable of furnishing information of value in the construction and validation of nuclear models it is important not to be guided by misleading or incomplete interpretations of the data. Hence, it seems imperative that the analysis of beta-decay experiments be carried through in such a way that a complete picture emerges of the acceptable range of values for the matrix element parameters.

In the present paper a systematic method of determining all suitable sets of parameters consistent with a series of beta-decay experiments is presented for the case of first forbidden beta decays with spin change 1. We might note that extension to the case of spin change 0 will prove difficult in practice because of the increased number of independent matrix-element parameters. Our tentative conclusion is that study of decays with spin change 1 will prove more useful, in regard to the determination of nuclear beta-decay matrix elements, than study of decays with spin change 0. Our work further convinces us that approximations to reduce the number of matrix element parameters can be expected to yield misleading results.

The features of this analytical method may be briefly outlined. No approximations beyond those made by Kotani[1] are used. The method may be applied to the simultaneous analysis of the data from any number of experiments for a given beta decay. It illustrates the effect that each experiment plays in restricting the parameters and thus illustrates which experiments are most useful in this respect. It illustrates how the error limits on the experimental data affect the range of suitable values for the parameters. If no suitable set of parameters exists, the method establishes this fact.

APPLICATION TO Eu^{152}

Work on the 1850-kev beta transition in the decay of Eu^{154} indicated that the beta-gamma directional correlation and the beta spectral shape correction factor[5] for that decay could not be simultaneously fitted with the "modified B_{ij}" approximation.[6,7] In an attempt to fit the Eu^{154} data, all terms in the theoretical expressions were retained and the present method of analysis using the three parameters u, x, and ζ_1 was developed. It was found[7] that all the available data for Eu^{154} could be simultaneously fitted. However, no unique solution for the parameters was found. On the contrary, there are many suitable sets of parameters with values of ζ_1 ranging from about 1 to 3. For many of these sets u and x are not small, contrary to the assumptions of the "modified B_{ij}" approximation. This suggested the possibility that there are solutions for the parameters in the case of Eu^{152} other than the "modified B_{ij}" approximation. Recently the beta-circularly polarized gamma directional correlation as a function of angle has been reported for Eu^{152}.[8] Therefore,

[2] M. Morita and R. S. Morita, Phys. Rev. **109**, 2048 (1958).
[3] H. Dulaney, C. H. Braden, and L. D. Wyly, Phys. Rev. **117**, 1092 (1960).
[4] H. J. Fischbeck and R. G. Wilkinson, Phys. Rev. **120**, 1762 (1960).
[5] L. M. Langer and D. R. Smith, Phys. Rev. **119**, 1308 (1960).
[6] K. S. R. Sastry, R. F. Petry, and R. G. Wilkinson, Phys. Rev. **123**, 615 (1961).
[7] L. D. Wyly, E. T. Patronis, H. Dulaney, and C. H. Braden, Phys. Rev. **124**, 841 (1961).
[8] J. Berthier, R. Lombard, and J. W. Sunier, Compt. rend. **252**, 257 (1961). The scale on the vertical axis of the curves of ω vs $\cos\theta$ is apparently displaced here.

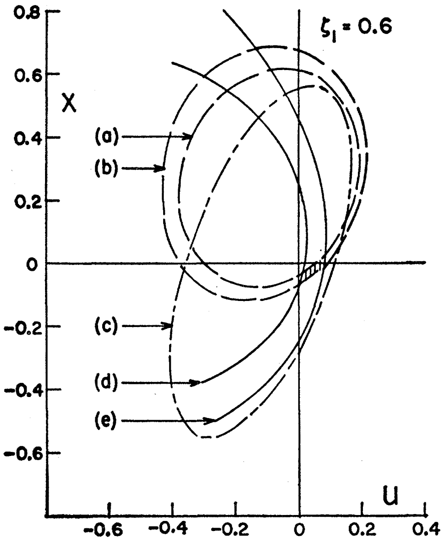

FIG. 1. Plots of constraining conditions for $\zeta_1=0.6$ for the following experimental values: (a) $A_2^*=-0.147$ at $W=3.35$; (b) $A_2^*=-0.139$ at $W=3.35$; (c) $S=1.36$; (d) $\omega=-0.75$ at $\cos\theta=0.6$; and (e) $\omega=-0.65$ at $\cos\theta=0.6$.

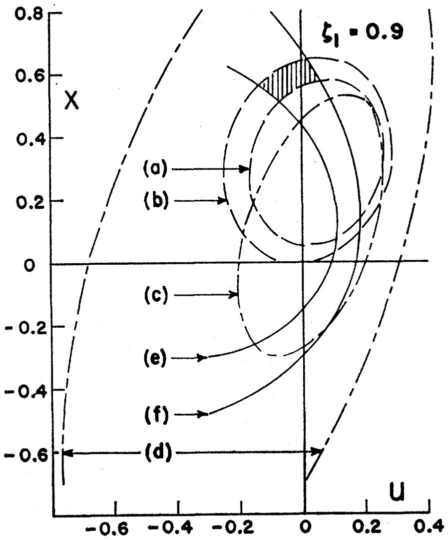

FIG. 3. Plots of constraining conditions for $\zeta_1=0.9$ for the following experimental values: (a) $A_2^*=-0.147$ at $W=3.35$; (b) $A_2^*=-0.139$ at $W=3.35$; (c) $S=1.26$; (d) $S=1.36$; (e) $\omega=-0.75$ at $\cos\theta=0.6$; and (f) $\omega=-0.65$ at $\cos\theta=0.6$.

now the data for three observables must be simultaneously fitted.

In the present analysis it has been found convenient to partially characterize the experimental data by the following three constraining conditions on the beta-decay matrix element ratios:

I. The modified beta-gamma directional correlation coefficient,[9] $A_2^*=WA_2/p^2\lambda_2$, is set equal to -0.143

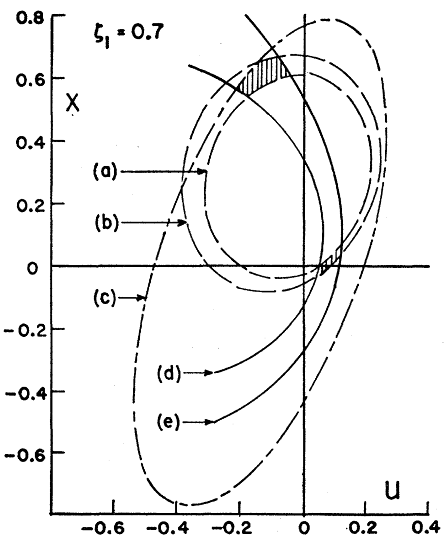

FIG. 2. Plots of constraining conditions for $\zeta_1=0.7$ for the following experimental values: (a) $A_2^*=-0.147$ at $W=3.35$; (b) $A_2^*=-0.139$ at $W=3.35$; (c) $S=1.36$; (d) $\omega=-0.75$ at $\cos\theta=0.6$; and (e) $\omega=-0.65$ at $\cos\theta=0.6$.

[9] We arbitrarily adopt the experimental results of Dulaney *et al.*[3] which lie intermediate between those of Fischbeck and Wilkinson[4] and S. K. Bhattacherjee and S. K. Mitra, Nuovo cimento **16**, 175 (1960).

±0.04 at a beta energy $W=3.35$. Here A_2 is the usual coefficient of the second-order Legendre polynomial in an expansion of the directional correlation, W is the total beta-ray energy in mc^2 units, p is the beta-ray momentum in mc units, and λ_2 is a Coulomb correction.[10]

II. The ratio of the beta spectrum shape correction factor[5] at $W=3.90$ to that at $W=3.15$, $S=C(3.90)/C(3.15)$, is set equal to 1.31 ± 0.05.

III. The beta-circularly polarized gamma directional correlation[11] ω is set equal to -0.70 ± 0.05 at $W=3.3$ and $\cos\theta=0.6$.

Each of these constraining conditions is represented by a set of two equations. These are obtained by separately equating the appropriate theoretical expression[12] to the two limits of the experimental value in question. The two equations representing a given constraining condition are then plotted in the u-x plane for various values of the third matrix element parameter ζ_1. Generally, for a particular ζ_1, the equations will be quadratic in u and x, so that conic sections are produced in the u-x plane. Sets of parameters which satisfy a particular constraining condition within the prescribed error limits lie within the area bounded by the two curves resulting from the constraining conditions.[13] Solution of the equations has been programmed

[10] T. Kotani and M. H. Ross, Phys. Rev. **113**, 622 (1959).

[11] No experimental values are directly quoted by Berthier *et al.*[8]; hence, this represents our estimate of their results.

[12] We redefine ζ_1 of Kotani[1] to be $\zeta_1=Y+W_0(u-x)/3$ which brings the results into agreement with Morita and Morita.[2]

[13] Analysis of beta-decay data using procedures that may be regarded as approximations to the present method have been employed previously, e.g., F. T. Porter, M. S. Freedman, T. B. Novey, and F. Wagner, Jr., Phys. Rev. **103**, 921 (1956); G. Hartwig and H. Schopper, Phys. Rev. Letters **4**, 293 (1960); and G. E. Bradley, F. M. Pipkin, and R. E. Simpson, Phys. Rev. **123**, 1824 (1961).

for an IBM 650 or a Burroughs 220 digital computor. In order to satisfy all three constraining conditions, the parameters must be represented by a point that is simultaneously within the error limit curves of all three conditions. In Figs. 1–3 the equations resulting from the three constraining conditions are plotted for values of $\zeta_1=0.6$, 0.7, and 0.9.

For $\zeta_1=0.6$ and 0.7, the quadratic equation resulting from the lower limit of the spectral shape constraining condition has no real roots. In this case the constraining condition is represented by a single ellipse and any point within the ellipse represents a set of parameters consistent with the shape constraining condition. For $\zeta_1=0.7$ there are two regions in which the three sets of constraining curves overlap while for $\zeta_1=0.6$ or 0.9 there is only one such region. From the regions of overlap the following sets of matrix element parameters are selected as typifying the satisfactory fits to the constraining conditions:

$$\begin{array}{llll} A. & \zeta_1=0.6, & u=\ \ 0.05, & x=-0.02; \\ B. & \zeta_1=0.7, & u=\ \ 0.1, & x=\ \ 0.03; \\ C. & \zeta_1=0.7, & u=-0.1, & x=\ \ 0.63; \\ D. & \zeta_1=0.9, & u=-0.03, & x=\ \ 0.60. \end{array}$$

We note that it may prove useful to impose additional constraining conditions from the available experimental data by specifying the value of A_2^* at additional values of the beta-ray energy or by specifying the value of ω at additional values of the beta-ray energy and/or angle. For the present investigation we did not feel the need for the imposition of additional conditions. The use of constraining conditions at more than one energy (or angle) might be especially indicated in instances where information is available only for a

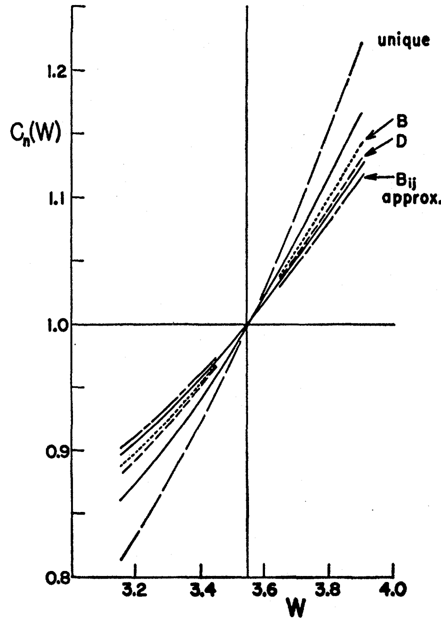

FIG. 5. Theoretical curves for the normalized beta spectrum shape correction factor, $C_n(W)$, as a function of the total beta energy W for the following sets of parameters: B. $\zeta_1=0.7$, $u=0.1$, $x=0.03$ and D. $\zeta_1=0.9$, $u=-0.03$, $x=0.60$. The theoretical curves for the unique shape and the "modified B_{ij}" approximation with $\zeta_1=0.8$ are also shown. The solid lines represent the limit curves for the expression given by Langer and Smith[5] as producing the best fit of the experimental shape correction factor. The theoretical curves for parameter sets A and C also are within the error limits.

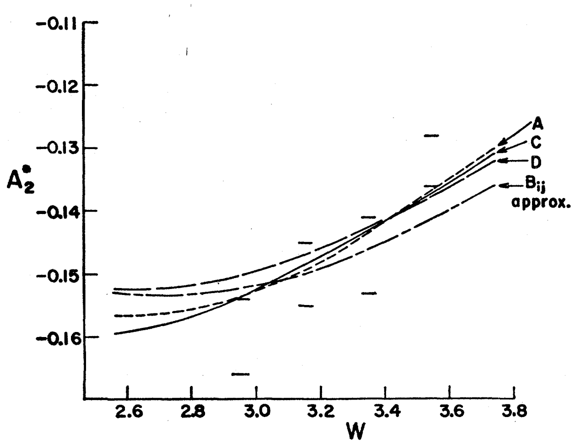

FIG. 4. Theoretical curves for the modified beta-gamma directional coefficient, $A_2^*=WA_2/p^2\lambda_2$, as a function of the total beta energy, W, for the following sets of parameters: A. $\zeta_1=0.6$, $u=0.05$, $x=-0.02$; C. $\zeta_1=0.7$, $u=-0.1$, $x=0.63$; and D. $\zeta_1=0.9$, $u=-0.03$, $x=0.60$. The "modified B_{ij}" approximation curve is shown for $\zeta_1=0.8$. The theoretical curve for parameter set B is omitted since it is very close to the curve for set A. The error limits of the experimental values[9] are also shown in the figure.

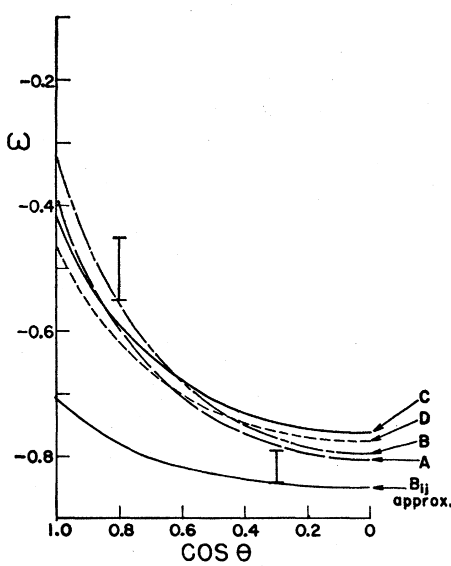

FIG. 6. Theoretical curves for the β-circularly polarized gamma correlation coefficient,[8,11] ω, as a function of $\cos\theta$ for the following sets of parameters: A. $\zeta_1=0.6$, $u=0.05$, $x=-0.02$; B. $\zeta_1=0.7$, $u=0.1$, $x=0.03$; C. $\zeta_1=0.7$, $u=-0.1$, $x=0.63$; and D. $\zeta_1=0.9$, $u=-0.03$, $x=0.60$. The "modified B_{ij}" approximation curve is shown for $\zeta_1=0.8$. The error limits of the experimental values are also shown in the figure.

single type of experiment, e.g., only for the beta-gamma directional correlation.

In order to be acceptable, a parameter set must produce agreement between the theoretical expressions and the experimental data not just at the points of constraint but over the entire experimental range. Therefore, the final step in the analysis is to plot, for each set of parameters, theoretical curves of the shape correction factor $C(W)$ vs energy, A_2^* versus energy, and ω versus $\cos\theta$ and compare with the experimental data. This calculation has also been programmed for a digital computor. Such curves are presented in Figs. 4–6. Theoretical curves for the "modified B_{ij}" approximation with $\zeta_1 = Y = 0.8$ and $u = x = 0$ are given also. This value of Y is taken as producing the best simultaneous fit of all the experimental data, within this approximation.

DISCUSSION

Each of the four parameter sets, A, B, C, and D, yield theoretical expressions that satisfactorily fit A_2^* and $C(W)$ over the observed energy range. The theoretical expressions for ω have a somewhat different angular dependence than our estimate of the experimental data.[8,11] However, in view of the uncertainty in the nature of the experimental results all four sets of parameters must be considered as yielding satisfactory fits to the data.

It may be noted that suitable parameter sets other than the four considered may be obtained. For example, additional sets may be found if ζ_1 is chosen as 0.8. However, there are no sets of parameters that satisfy the three constraining conditions for ζ_1 less than about 0.5 or greater than about 1. A point of particular significance is that the satisfactory parameter sets fall into two distinct groups. For one group u and x are small (less than about 0.1) in qualitative agreement with the assumption of the "modified B_{ij}" approximation. For the other group $x = 0.6$ which is in disagreement with the assumption of the "modified B_{ij}" approximation. This illustrates the fact that the use of parameter eliminating approximations may result in erroneous conclusions about the parameters due to the neglect of certain suitable solutions.

From an examination of the conic sections produced by the constraining conditions, it is apparent in this instance that a measurement of the spectral shape correction factor is somewhat less effective than the other measurements in restricting the matrix element parameters. For $|\zeta_1| < 0.5$, the shape factor constraining equations have no real roots. In this case the theoretical shape factor is too close to the "unique" shape regardless of the values of u and x. For $|\zeta_1| > 0.5$, the shape factor constraining condition is satisfied by a wide range of u and x values.

The directional correlation constraining condition by itself is satisfied by parameter sets with values of ζ_1 ranging from about -10 to 1.2. However, the sets with negative ζ_1 that simultaneously satisfy the directional correlation and shape constraining conditions yield plots of A_2^* versus energy that have an incorrect slope. Furthermore, these sets are in poor agreement with the polarization measurements. For example, the set $\zeta_1 = -1$, $u = -0.37$, $x = -0.1$ gives agreement with the shape measurement ($S = 1.33$) and with the directional correlation at high energy ($A_2^* = -0.14$ for $W = 3.45$), but the theoretical directional correlation is no longer in agreement at lower energy ($A_2^* = -0.11$ for $W = 3$). This parameter set also gives $\omega = -0.11$ for $\cos\theta = 0.6$ which disagrees markedly with the polarization constraining condition.

For values of ζ_1 in the range 0.5 to 1 there is a wide range of values for u and x that simultaneously satisfy the directional correlation and shape factor constraining conditions. In fact, for $\zeta_1 = 0.7$, almost all sets that satisfy the directional correlation also satisfy the shape factor condition. Generally, for ζ_1 in the range 0.5 to 1, sets that satisfy both constraining conditions produce theoretical expressions for A_2^* and $C(W)$ with a satisfactory energy dependence.

The addition of the polarization correlation constraining condition produces little effect on the suitable range of values for ζ_1. However, for a particular value of ζ_1 between 0.5 and 1 the polarization measurement sharply reduces the acceptable values for u and x.

For the decays that we have investigated using this method of analysis, experimental data of other types, e.g., longitudinal polarization of the betas, have not been available. The incorporation of such data in the analysis would be straightforward for those experiments where the appropriate theoretical formulas are available. Clearly, data on several types of experiments are very desirable in delimiting the acceptable values for the matrix element parameters.

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