Momentum Distributions for Protons in Li^6 in the Cluster Model

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(Received October 16, 1961)

The momentum distributions for protons with two different binding energies as measured in quasi-elastic scattering on Li^6 are calculated in a simplified cluster model. It is assumed that clusters in nuclei are well separated and cluster-model wave functions which are not antisymmetrized are used. The simplification seems to be justifiable in this case. With the cluster-model wave functions of Pearlstein, Wildermuth, and Tang, experimental data from quasi-elastic scattering of protons on Li^6 can be reproduced, whereas this could not be achieved with the simple shell model.

RECENTLY Garron *et al.* measured the angular correlation of protons for quasi-elastic scattering on Li^6 and reported an unexpected behavior of the correlation cross section for protons of the lower binding energy.[1] The suggestion was made that the clustering of nucleons in Li^6 may be held responsible for this effect. Here calculations in a simplified cluster model are presented which seem to confirm the suggestion.

In the quasi-elastic scattering cross section of Li^6 two peaks were observed corresponding to binding energies of 4 ± 1.4 Mev and 20 ± 0.7 Mev.[1] In reproducing $(p,2p)$ scattering data for Li^7 and C^{12}, the concept was proved to be successful that the peak at lower binding energy is due to protons knocked out of the p shell and the other one to protons of the s shell.[2–5] However, this concept cannot be applied to Li^6 because the measured correlation cross section at 4 Mev binding energy has a maximum at a separation angle about 82° rather than a minimum which would be the case for p-shell protons.

Experimental data from quasi-elastic scattering usually are compared with model calculations using the impulse approximation, in which the correlation cross section for symmetrical scattering has the form[4]

$$d\sigma/d\Omega_1 d\Omega_2 dp = \mathrm{const}\,|g(q)|^2, \tag{1}$$

where $q=|\mathbf{q}|$ is the momentum of the knocked out proton in the nucleus before scattering and $|g(q)|^2$ is the corresponding momentum distribution. The magnitude of q is calculated using momentum and energy conservation. The form factor $g(q)$ in the Born-approximation reads for Li^6 ($\hbar=1$)

$$g(q)=\mathrm{const}\int d\tau\,\exp(-i\mathbf{q}\cdot\mathbf{r}_1)\varphi_f^*(\mathbf{r}_2,\cdots,\mathbf{r}_6)\times\varphi_i(\mathbf{r}_1,\cdots,\mathbf{r}_6), \tag{2}$$

with $d\tau=d^3r_1\cdots d^3r_6$; the integration is to be carried out over the whole configuration space.

We consider the problem in the cluster model and take the ground state of the Li^6 nucleus to be an alpha-deuteron configuration.[6] Protons with lower binding energy are knocked out of the deuteron cluster so this process leads to the alpha-neutron configuration which is the ground state of He^5 $(\frac{3}{2}-)$.[7] On the other hand, protons with higher binding energy are knocked out of the alpha-particle cluster so this process leads to the triton-deuteron configuration which is the 16.7-Mev excited state of He^5 $(\frac{3}{2}+)$.[7] Throughout this consideration we have assumed that the clusters in nuclei are well separated. In the following we shall therefore use cluster model wave functions not taking antisymmetrization into account. Of course, this is an idealization, but since the two kinds of protons are experimentally distinguishable one can expect that this procedure may give useful results.

Calculating the form factor of Eq. (2) for protons with the lower binding energy $g_p(q)$, we insert cluster-model wave functions given by Pearlstein, Wildermuth, and Tang[6,7]:

$$\varphi_i=\exp\Big(-\tfrac{1}{2}\alpha_1\sum_{i=3}^{6}r_{i\alpha}^2-\tfrac{1}{2}\bar{\alpha}_1\sum_{i=1}^{2}r_{id}^2-\tfrac{2}{3}\beta_1R_1^2\Big)R_1^2$$

and

$$\varphi_f=\exp\Big(-\tfrac{1}{2}\alpha_2\sum_{i=3}^{6}r_{i\alpha}^2-\tfrac{2}{5}\beta_2R_2^2\Big)R_2Y_1^m(\mathbf{R}_2/R_2), \tag{3}$$

with

$$\mathbf{R}_1=\mathbf{R}_\alpha-\mathbf{R}_d,\quad \mathbf{R}_2=\mathbf{R}_\alpha-\mathbf{r}_2.$$

Here

$$\mathbf{r}_{i\alpha}=\mathbf{r}_i-\mathbf{R}_\alpha\quad(i=3,4,5,6),\quad \mathbf{r}_{id}=\mathbf{r}_i-\mathbf{R}_d\quad(i=1,2)$$

are the relative vectors in the alpha particle and in the deuteron cluster, respectively, and

$$\mathbf{R}_\alpha=\tfrac{1}{4}\sum_{i=3}^{6}\mathbf{r}_i,\quad \mathbf{R}_d=\tfrac{1}{2}\sum_{i=1}^{2}\mathbf{r}_i$$

are the corresponding center-of-mass vectors.

The form factor for protons with the higher binding

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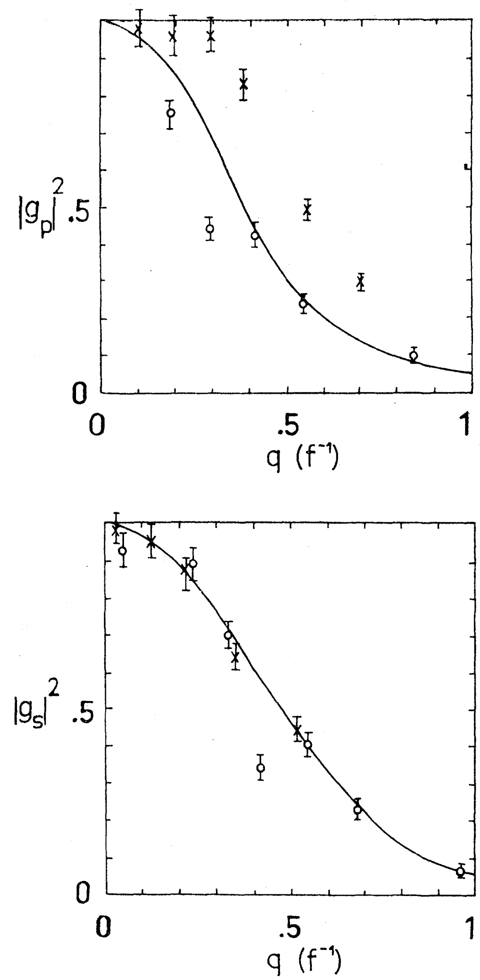

FIG. 1. Momentum distributions for protons in Li[6]. The distributions are normalized to unity at $q=0$. The points are those of Garron *et al.*[1] Since the correlation cross sections[1] are not completely symmetrical, the points corresponding to separation angles smaller than 90° are drawn with crosses.

energy $g_s(q)$ is calculated by means of Eq. (2) and[6,7]

$$\varphi_i=\exp(-\tfrac{1}{2}\alpha_1\sum_{i=1}^{4}r_{i\alpha}^2-\tfrac{1}{2}\bar{\alpha}_1\sum_{i=5}^{6}r_{id}^2-\tfrac{2}{3}\beta_1R_1^2)R_1^2$$

and (4)

$$\varphi_f=\exp(-\tfrac{1}{2}\alpha_3\sum_{i=2}^{4}r_{it}^2-\tfrac{1}{2}\bar{\alpha}_3\sum_{i=5}^{6}r_{id}^2-\tfrac{3}{5}\beta_3R_3^2)R_3^2$$

with

$$\mathbf{R}_1=\mathbf{R}_\alpha-\mathbf{R}_d,\quad \mathbf{R}_3=\mathbf{R}_t-\mathbf{R}_d.$$

Here again

$$\mathbf{r}_{i\alpha}=\mathbf{r}_i-\mathbf{R}_\alpha\quad(i=1,2,3,4),\quad \mathbf{r}_{it}=\mathbf{r}_i-\mathbf{R}_t\quad(i=1,2,3),$$
$$\mathbf{r}_{id}=\mathbf{r}_i-\mathbf{R}_d\quad(i=5,6)$$

are the relative vectors in the alpha particle, triton, and deuteron cluster, respectively, and

$$\mathbf{R}_\alpha=\tfrac{1}{4}\sum_{i=1}^{4}\mathbf{r}_i,\quad \mathbf{R}_t=\tfrac{1}{3}\sum_{i=2}^{3}\mathbf{r}_i,\quad \mathbf{R}_d=\tfrac{1}{2}\sum_{i=5}^{6}\mathbf{r}_i$$

are the corresponding center-of-mass vectors.

Now, let us calculate the momentum distributions in an approximation taking the reference system in which the residual nucleus is at rest. In this case the integrations are not too complicated and can be performed in a closed form.[8]

After some lengthy manipulations one gets for $g_p(q)$ finally

$$g_p(q)=\text{const}[F(2,\tfrac{3}{2};-\tfrac{1}{2}q^2/q_p^2)+K_pF(3,\tfrac{3}{2};-\tfrac{1}{2}q^2/q_p^2)],\quad(5)$$

where $F(a,c;z)$ is the confluent hypergeometric function,

$$q_p^2=\tfrac{1}{6}[25\bar{\alpha}_1\beta_1+(5\beta_2-\alpha_1-\alpha_2)(3\bar{\alpha}_1+2\beta_1)]/(2\bar{\alpha}_1+3\beta_1+5\beta_2-\alpha_1-\alpha_2),$$

and

$$K_p=\tfrac{1}{3}(\bar{\alpha}_1-\beta_1)(16+24u+9u^2)/[(8+15u)q_p^2],$$

with the abbreviation

$$u=4(\bar{\alpha}_1-\beta_1)/(2\bar{\alpha}_1+3\beta_1+5\beta_2-\alpha_1-\alpha_2).$$

Similarly for $g_s(q)$ one finds

$$g_s(q)=\text{const}[\exp(-\tfrac{1}{2}q^2/q_s^2)+K_sF(\tfrac{5}{2},\tfrac{3}{2};-\tfrac{1}{2}q^2/q_s^2)+K_s'F(\tfrac{7}{2},\tfrac{3}{2};-\tfrac{1}{2}q^2/q_s^2)],$$
$$q_s^2=\tfrac{1}{6}[48\alpha_1\beta_1-18\alpha_1^2-(9\alpha_1+\beta_1)(2\alpha_3-5\beta_3)]/(9\beta_1+10\beta_3-3\alpha_1-4\alpha_3),\quad(6)$$
$$K_s=(1/180)(9\beta_1+10\beta_3-3\alpha_1-4\alpha_3)\times(1+10v+30v^2)/q_s^2,$$
$$K_s'=(1/144)(\alpha_1-\beta_1)^2(1+6v+9v^2)/q_s^4$$

with the abbreviation

$$v=3(\alpha_1-\beta_1)/(9\beta_1+10\beta_3-3\alpha_1-4\alpha_3).$$

In Tang *et al.*[6] and Pearlstein *et al.*,[7] the values of the parameters α_j, $\bar{\alpha}_j$ and $\beta_j(j=1,2,3)$ are given in terms of $x_j=\beta_j/\alpha_j$, $y_j=k/\alpha_j$, and $z_j=\bar{\alpha}_j/\alpha_j$, where $k=0.416$ f^{-2}. Using these values, tabulated as follows:

Nucleus	j	x_j	y_j	z_j
Li^6	1	0.76	0.96	1.52
He^5 ($\frac{3}{2}-$)	2	0.75	0.96	
He^5 ($\frac{3}{2}+$)	3	0.15	1.48	1.7

one can calculate all introduced quantities and there-

[8] A. Erdelyi *et al.*, *Higher Transcendental Functions* (McGraw-Hill Book Company, Inc., New York, 1953), Vol. I., p. 254.

with the momentum distributions[9]:

$$q_p = 0.64\ \mathrm{f}^{-1},\quad u = 0.43,\quad K_p = 0.53,$$
$$q_s = 0.59\ \mathrm{f}^{-1},\quad v = 0.33,\quad K_s = 0.12,\quad K_s' = 0.0027.$$

In Fig. 1 the calculated momentum distributions are compared with the momentum distributions deduced from experiment.[1] Taking into account that this is not a fit since fixed values are used for α_j, $\bar{\alpha}_j$, and β_j, the agreement seems to be satisfactory.

[9] The final expressions for the two form factors are $g_p(q) = 0.54 - 0{\cdot}043q^2/q_p^2 + (0.46 - 0.9q^2/q_p^2 + 0.043q^4/q_p^4)F(1, \frac{3}{2}; -\frac{1}{2}q^2/q_p^2)$, and $g_s(q) = (1 - 0.017q^2/q_s^2)\exp(-\frac{1}{2}q^2/q_s^2)$.

ACKNOWLEDGMENTS

This work was done partially during a stay at the Institute for Theoretical Physics in Heidelberg. The author wishes to express sincere thanks to Professor J. H. D. Jensen and to Professor B. Stech for hospitality at the Institute and to some members of the Institute for discussions. A fellowship of the International Atomic Energy Agency is gratefully acknowledged.

PHYSICAL REVIEW VOLUME 125, NUMBER 5 MARCH 1, 1962

Decay of Pm^{148}, Pm^{148m}, and Eu^{148}†

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(Received August 28, 1961)

The beta decay of Pm^{148} and Pm^{148m} (5.0 days and 45.5 days) and the electron capture of Eu^{148} (58 days) proceed to levels in Sm^{148}. These activities have been investigated with scintillation and magnetic spectrometers, and coincidence and directional correlation techniques. Pm^{148} (5 day) decays via three beta branches to levels at 1460 kev, 551 kev, and the ground state, with the available energy being 2460 ± 20 kev. The decay of Pm^{148} (45.5 day) and Eu^{148} each populate levels of 551, 1181, 1596, 1908, and 2098 kev in Sm^{148}. The long-lived Pm populates additional levels at 2027 and 2197 kev, and Eu^{148} populates levels at 1886, 2148, 2201, 2697(?), and 2780 kev. The experimental data are consistent with the 45-day state being approximately 140 kev above the 5-day state. Decay schemes consistent with the experimental data are presented.

I. INTRODUCTION

EARLY investigators[1] established a 5.3-day activity associated with Pm^{148}. Long and Pool[2] also established a 48-day activity associated with Pm^{148}. They found beta particles of 1.7 ± 0.1 and 0.6 ± 0.1 Mev endpoints and a 0.54-Mev gamma ray. Folger, Stevenson, and Seaborg[3] found an activity in the products of the high-energy fission of uranium which had beta spectra with endpoints of approximately 2.3 and 0.5 Mev and a gamma ray of ~ 1 Mev associated with it.

Recently Bhattacherjee *et al.*[4] and Eldridge and Lyon[5] have investigated Pm^{148}. Bhattacherjee[4] found that the 4.2-day activity had γ rays of 560, 900, and 1460 kev associated with it, while the 46-day activity had γ rays of 105, 195, 295, 400, 560, 630, 720, 930, 1015, and 1200 kev. Eldridge and Lyon[5] further found that the 1200-kev photopeak could be almost fully accounted for by the "sum" of the 550- and 630-kev transitions. Both groups proposed decay schemes for Pm^{148} based on beta-gamma and gamma-gamma coincidences.

No decay scheme has been available for the electron capture of Eu^{148} to Sm^{148}. Mack and Pool[6] have bombarded enriched samarium isotopes with protons and found that Eu^{148} decayed with a 54-day half-life and had a 0.58-Mev gamma ray associated with it. Hoff, Rasmussen, and Thompson[7] reported that there was no evidence for positrons.

We have investigated the activities of Pm^{148} and Eu^{148} as part of a program to obtain as much information as possible on the nuclei in the so-called "transition" region ($A = 143$ to 150) between spherical and strongly deformed nuclei.

II. EXPERIMENTAL PROCEDURE

The sources were obtained by bombarding enriched isotopes of Nd^{148} (81.7%) and Sm^{148} (83.1%) with protons in the 86-in. cyclotron at the Oak Ridge National

† Work accomplished in part under contract with the U. S. Atomic Energy Commission.

* Based on a thesis submitted to the University of Notre Dame, Notre Dame, Indiana, in partial fulfillment of the requirements for the Ph.D. degree.

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