

Spin and Parity Assignments to the 5.63-Mev and 5.80-Mev Levels of Ne^{20}

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Angular correlations have been measured, involving spin-zero particles in a special counter arrangement, which have led to the assignment of spins and parities to the 5.63- and 5.80-Mev states of Ne^{20} ; assignments are made of 3- and 1-, respectively. These are the lowest levels above the alpha-particle binding energy whose spin and parity allow formation by alpha-particle capture on O^{16} . A possible role of one or both of these levels in Ne^{20} formation in stars is discussed. The properties of these levels together with those of other levels in Ne^{20} have suggested rotational band structure in Ne^{20} .

INTRODUCTION

RECENT measurements¹⁻⁴ of properties of levels of Ne^{20} have been motivated by two general reasons. Firstly, astrophysical considerations require a knowledge of the properties of levels just above the binding energy of alpha particles⁵ (4.73 Mev) in order to estimate the rate of Ne^{20} production in stars via the $\text{O}^{16} + \text{He}^4$ reaction. The lowest level in Ne^{20} above the alpha-particle binding energy, at 4.97 Mev which has long been assumed to play a major role in Ne^{20} production,⁶⁻⁸ has recently been shown⁹ to be 2- and hence to have the wrong combination of spin and parity for formation via alpha-particle capture on O^{16} . As a consequence of this, the next-higher levels in Ne^{20} , at 5.63 and 5.80 Mev, may play an important role in Ne^{20} formation in stars even though considerably higher temperatures, such as exist in supernova explosions, would be required¹⁰; the 5.80-Mev level is a new level which was recently observed at Florida State University¹¹ in the $\text{Na}^{23}(p, \alpha)\text{Ne}^{20}$ reaction.

Secondly, as a result of the recent intensive study of the properties of levels of Ne^{20} a considerable body of information has accumulated. These properties strongly suggest a collective model description¹² of the low-lying levels of Ne^{20} in which both positive- and negative-parity rotational bands are present. Extension of our knowledge of the levels of Ne^{20} thus becomes of extreme

interest for model comparison since, in this case, detailed shell-model calculations¹³ are also possible.

In this paper measurements are described which have led to the assignment of spins and parities to the 5.63- and 5.80-Mev levels of Ne^{20} . The roles of these levels in the band structure of Ne^{20} are briefly discussed.

THEORETICAL

The reaction $\text{C}^{12}(\text{C}^{12}, \text{He}^4)\text{Ne}^{20}$ was used to produce Ne^{20} excited to the 5.63-Mev (or 5.80-Mev) state and a measurement was made, in a special geometry, of the coincidence angular correlation between the He^4 from the decay of the 5.63-Mev (or 5.80-Mev) state of Ne^{20} and the He^4 leading to this state. Figure 1 contains a level diagram which shows the relevant levels of Ne^{20} and the transitions involved; on the right a diagram indicates the geometrical arrangement employed. In this reaction all particles involved have spin zero except for the Ne^{20} . It can be shown¹⁴ that in these circumstances if the initial He^4 which leads to the formation of Ne^{20} is detected in an axially symmetrical counter at 0° or 180° to the beam the coincidence angular correlation of the He^4 from the Ne^{20} decay (the z axis is taken in the direction of the beam) is

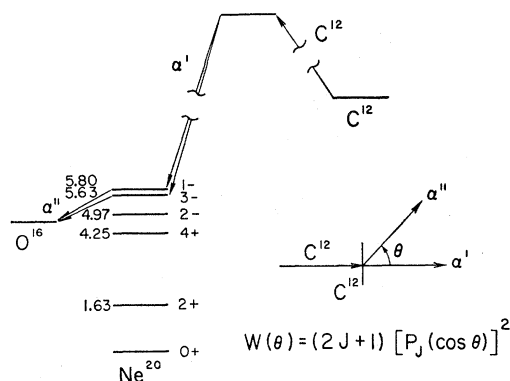


FIG. 1. Levels of Ne^{20} are shown together with the transitions involved in the angular correlation measurements; on the right a diagram indicates the geometrical arrangement employed.

¹³ J. P. Elliott, Proc. Roy. Soc. (London) A245, 128 (1958).

¹⁴ A. E. Litherland and A. J. Ferguson, Can. J. Phys. 39, 788 (1961).

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² H. E. Gove, A. E. Litherland, and M. A. Clark, Can. J. Phys. 39, 1243 (1961).

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⁹ H. E. Gove, A. E. Litherland, and M. A. Clark, Nature 191, 1381 (1961).

¹⁰ A. G. W. Cameron (private communication).

¹¹ H. S. Adams, J. D. Fox, N. P. Heydenburg, and G. M. Temmer, Bull. Am. Phys. Soc. 6, 250 (1961) (and private communication).

¹² A. E. Litherland, J. A. Kuehner, H. E. Gove, M. A. Clark, and E. Almqvist, Phys. Rev. Letters 7, 98 (1961).

given by

$$W(\theta) = \frac{1}{4\pi} \sum_{k,m} (-)^{m+k/2} P(m)(JJm-m|k0) \times Z(JJJJ,0k) Q_k P_k(\cos\theta), \quad (1)$$

where J is the spin of the excited state in Ne^{20} and $P(m)$ is the population of its m th substate. $(JJm-m|k0)$ is a vector addition coefficient and the coefficient $Z(JJJJ,0k)$ has been tabulated by Sharp *et al.*¹⁵ Q_k is an attenuation coefficient¹⁶ which corrects for the finite size of the movable counter. The factor $1/4\pi$ is included so that the correlation function integrates to unity over the sphere. An alternative way of writing the angular correlation function, utilizing the associated Legendre function, is

$$W(\theta) = \sum_m P(m) \frac{(2J+1)(J-|m|)!}{4\pi(J+|m|)!} [P_J^m(\cos\theta)]^2. \quad (2)$$

In the reaction $\text{C}^{12}(\text{C}^{12}, \text{He}^4)\text{Ne}^{20}$ in which the He^4 is detected at 0° , since both the C^{12} and He^4 have spin zero, and since the orbital angular momentum must be perpendicular to the beam direction, there can be no component of angular momentum in the beam direction; thus the Ne^{20} states must all have $m=0$. Thus, providing a sufficiently small solid angle detector is used at zero degrees, the angular correlation given by Eqs. (1) or (2) has only $P(m=0)$ terms. Under these circumstances the angular correlation function reduces to

$$W(\theta) = \frac{(2J+1)}{4\pi} [P_J(\cos\theta)]^2. \quad (3)$$

In practice, since the 0° counter must have a finite solid angle, the angular correlation may contain a small term due to $P(m=\pm 1)$ and this possibility must be taken into account.

Inspection of Eq. (3) suggests a variation of the experiment described in this paper in which only one coincidence measurement would be required to yield J , specifically a measurement in which $\theta=0^\circ$. In practice such a measurement could be accomplished using a broad-range magnetic spectrograph set at 0° , which would separate the two alpha-particle groups in space, in conjunction with two detectors connected in a coincidence circuit, one of which is movable along the focal plane of the spectrograph. With such an arrangement control of the magnetic field and of the position of one of the detectors would allow coincidences to be measured. The ratio of coincidence counts to direct counts due to the first alpha particle, when corrected for the second detector solid angle, would equal

$(2J+1)/4\pi$ and thus would determine J in a single measurement.

EXPERIMENTAL

The fixed counter, at 0° , was a silicon pn diffused junction detector of half-angle $\sim 5^\circ$. It was covered with three thicknesses of 0.0006-in. Al foil to stop the direct beam; it was found that a single foil of the same total thickness was so nonuniform that a considerable amount of the direct beam reached the detector. The movable counter consisted of an uncovered Au-Si surface barrier detector of half-angle $\sim 2^\circ$, movable in the angular range 28° – 150° . A 16-Mev carbon beam was used to bombard a self-supporting carbon foil of thickness $\sim 30 \mu\text{g}/\text{cm}^2$.

Figure 2 contains on the left a direct spectrum obtained with the 0° counter. This spectrum shows groups of alpha particles corresponding to the energy levels of Ne^{20} up to an excitation of approximately 7.5 Mev, at which point the spectrum is obscured due to a proton peak corresponding to protons passing completely through the sensitive volume of the junction. Note the low 0° yield of α_3 , corresponding to an excited state at 4.97 Mev. Since this level has been shown⁹ to have spin and parity 2^- , its yield at 0° must be identically zero.¹⁷ The observed yield is due to the finite solid angle and corresponds to $m \geq 1$ substates.

The group of interest in this experiment, $\alpha_{4,5}$, is not resolved into two components by the 0° counter due to range straggling in the Al absorbing foil. The right-hand side of Fig. 2 contains a spectrum obtained in a subsequent measurement using a magnetic spectrometer¹⁸ and no Al absorbing foil. The solid angle used, half-angle $= 1.5^\circ$, was somewhat less than that used in the spectrum on the left. This spectrum shows the 5.63- and 5.80-Mev groups resolved and allows a determination of the relative feeding of these

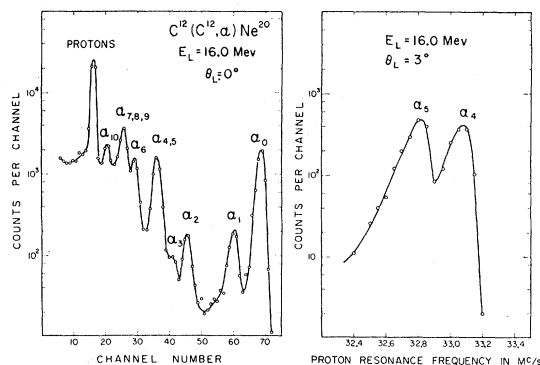


FIG. 2. On the left is the charged-particle spectrum obtained in the Si- pn detector at 0° . The group of interest in this experiment, $\alpha_{4,5}$, is not resolved into two components due to range straggling in the Al foil. On the right is a portion of a spectrum obtained using a magnetic spectrometer and no Al absorbing foil which shows α_4 and α_5 resolved.

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¹⁷ A. E. Litherland, Can. J. Phys. **39**, 1245 (1961).

¹⁸ H. A. Eng, Rev. Sci. Instr. **29**, 885 (1958).

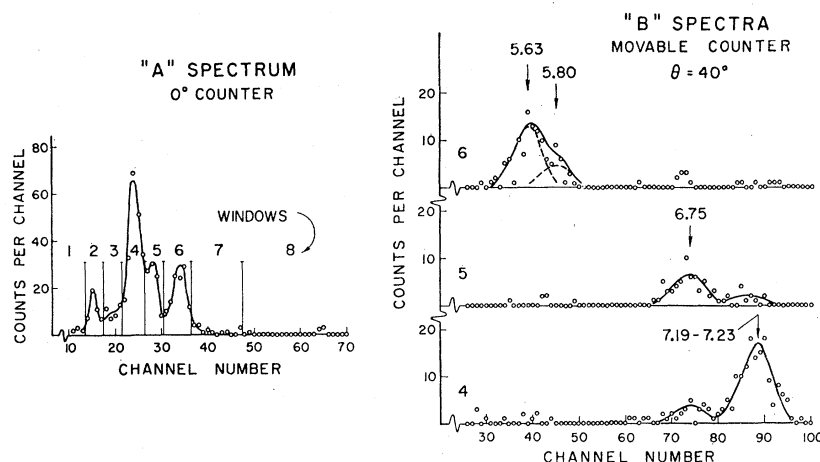


FIG. 3. Part of a two-dimensional spectrum for a coincidence run is shown. The spectrum on the left shows the "A" spectrum (0° counter) on which are set 8 windows. The "B" spectra (movable counter) in coincidence with each of these windows are divided into eight separate, 100-channel analyzers. Those spectra in coincidence with windows 4, 5, and 6 are shown on the right.

two levels to be made. Analysis of several spectra obtained at angles near 0° and using targets varying slightly in thickness indicate rapidly varying differential cross sections for both states. These measurements suggest a ratio of $\sim 1:2$ for the relative feeding of the 5.63- and 5.80-Mev levels, respectively, for the conditions used in the coincidence studies. It is to be noted that since each of the 5.63- and 5.80-Mev levels shows finite yield at 0° they must each have parity equal to $(-1)^J$.¹⁷

Use was made of the 900-channel two-dimensional pulse-height analyzer¹⁹ in the coincidence measurements. Figure 3 contains part of the two-dimensional

spectrum for a coincidence run in which the movable counter was at 40° . For these measurements the analyzer was divided into its 9×100 mode of operation. The spectrum on the left shows the "A" spectrum (0° counter) on which are set 8 windows. The "B" (movable counter) in coincidence with each of these windows are displayed in 8 separate 100-channel analyzers. Those spectra in coincidence with windows 4, 5, and 6 are shown on the right of Fig. 3. The bottom spectrum, in coincidence with window 4, shows, as the intense group, alpha particles from the decay of the 7.19- and 7.23-Mev excited states of Ne^{20} and with less intensity alpha particles from the decay of the 6.75-Mev excited state: these latter appear in coincidence with window 4 because the tail of the corresponding group in the "A" spectrum extends into window 4. The middle spectrum on the right of Fig. 3 corresponds to window 5 and contains predominantly alpha particles from the decay of the 6.75-Mev excited state of Ne^{20} . The upper spectrum is in coincidence with window 6 and shows groups of alpha particles from the decay of the 5.63- and 5.80-Mev states of Ne^{20} . Since all "B" spectra share a single pulse-height-to-channel-number encoder it is possible to use the two coincidence spectra on the bottom of Fig. 3 to obtain an α -particle energy calibration ($1 \text{ Mev} \approx \text{channel } 25$). Using this internal calibration the alpha particles from the decay of the 5.63- and 5.80-Mev levels are predicted to appear at the positions of the two arrows in the top spectrum. With this information and with a knowledge of the line shapes it is possible to resolve the spectrum into two components as shown. Thus, use is made of the movable counter, detecting low-energy alpha particles, to resolve the 5.63- and 5.80-Mev levels. An automatic check on random events is provided by the 9×100 -channel analyzer since window 8, which contains only alpha-particle groups leading to the ground and first-excited states of Ne^{20} , contains only random events.

Figure 4 contains the "B" spectra coincident with window 6 from a number of runs for laboratory angles

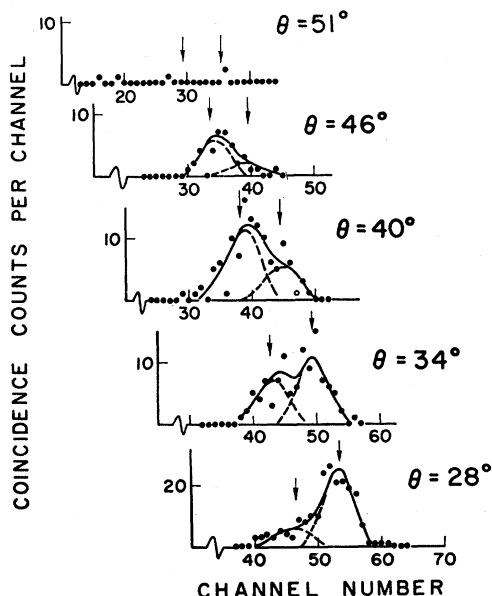


FIG. 4. Spectra obtained in the movable counter in coincidence with the group $\alpha_{4,5}$, at 0° . The arrows indicate the calculated positions of pulses due to alpha-particle decay of the 5.63- and 5.80-Mev states of Ne^{20} .

¹⁹ Designed and built by the Electronics Branch, Atomic Energy of Canada Limited.

of the movable counter between 28° and 51° . Again the arrows indicate the predicted positions of pulses due to alpha-particle decay of the 5.63- and 5.80-Mev states of Ne^{20} . The analyses of these spectra into two components is indicated by the dashed lines. Notice that at $\theta = 51^\circ$ (corresponding to a center-of-mass angle $\sim 90^\circ$) both groups disappear, immediately requiring odd spin for both states. These spectra are only approximately normalized to equal numbers of alpha particles at 0° but the changing ratio of 5.63- to 5.80-Mev state decay alpha particles is apparent.

Figure 5 shows the results of the analysis of these coincidence spectra normalized to the unresolved alpha-particle group feeding these states detected at 0° . At the top is the angular correlation for the 5.80-Mev level and at the bottom that for the 5.63-Mev level. Since the solid angle of the movable counter is known the normalization of the theoretical curves is not arbitrary. The solid curves through the points have been normalized in such a way that the total number of coincidences from both groups integrate to unity over the sphere. A small correction has been applied to take into account the fact that Γ_γ/Γ for the 5.63-Mev state has been shown²⁰ to be 0.07. For the 5.80-Mev state Γ_γ/Γ has been shown²⁰ to be <0.006 .

As discussed above, the angular correlation reduces to $[(2J+1)/4\pi][P_J(\cos\theta)]^2$ only for a vanishingly small solid-angle counter at 0° . In practice, since the 0° counter has a finite solid angle the angular correlation may contain a small term due to $P(m=\pm 1)$, with magnitude of order α^2 , where α is the 0° counter half angle ($\alpha \sim 0.1$ radian). Contributions due to $P(m>1)$ are expected to be of order α^4 and would be negligible. To investigate these possibilities the angular correlations have been fitted by least squares to the expression given in Eq. (2) treating the coefficients $P(m)$ as adjustable parameters.²¹ If a fit produces the unphysical result that some $P(m)$ is negative a new fit is obtained while holding that $P(m)=0$. Fits have been attempted for all $J < 6$.

Figure 6 contains the two best fits obtained for the 5.80-Mev state angular correlation. The data have been transferred to angles less than 90° for convenience (the angular correlation must necessarily be symmetric about 90°). Apart from the poor agreement at 48° with the curve for $J=2$ it is extremely unlikely that $P(m=1)$ should be so large and $P(m=0)$ so small [the ratio $P(m=1)/P(m=0)$ would be expected to be of order 0.01]; the restriction which requires $P(m=0)=0$ for states of parity equal to $(-1)^{J+1}$ in a reaction such as this¹⁷ does not apply since it is demonstrated by the decay into $\text{O}^{16} + \text{He}^4$ that the 5.80-Mev level has parity equal to $(-1)^J$. One concludes, therefore, that the $J=1$ fit is unique.

²⁰ J. A. Kuehner and E. Almqvist (to be published).

²¹ The author is indebted to Dr. J. M. Kennedy and the staff of the Datatron computer for preparation of the fitting program.

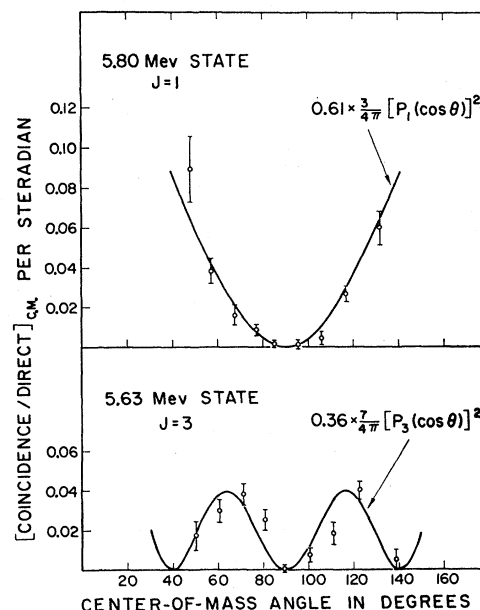


FIG. 5. At the top is the angular correlation for the 5.80-Mev level and at the bottom that for the 5.63-Mev level. The theoretical expressions have been normalized in such a way that the total number of decays of both groups integrate to unity over the sphere.

Figure 7 contains the two best fits obtained for the 5.63-Mev state angular correlation. Again the data have been transferred to angles less than 90° . As before, the fit for $J=4$ must be ruled out because of lack of agreement with the data and because of the

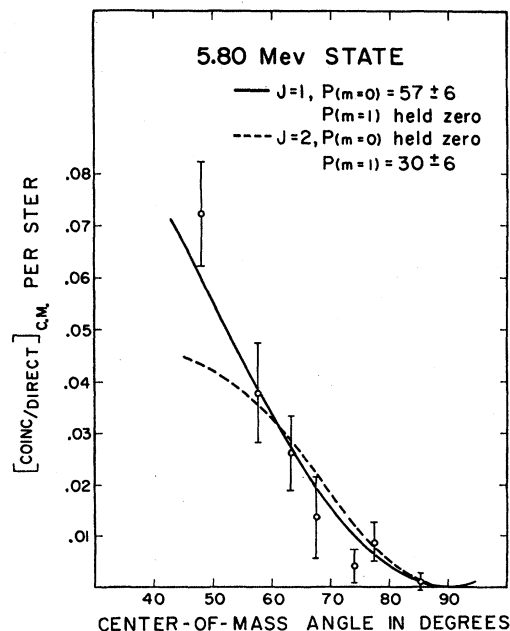


FIG. 6. Least-squares fits of the expression given in Eq. (2) are shown for the 5.80-Mev state angular correlation for values of $J=1$ and 2. Values of the population parameters required are shown. No other values of $J < 6$ produced satisfactory fits. The data have been transferred to angles less than 90° for convenience.

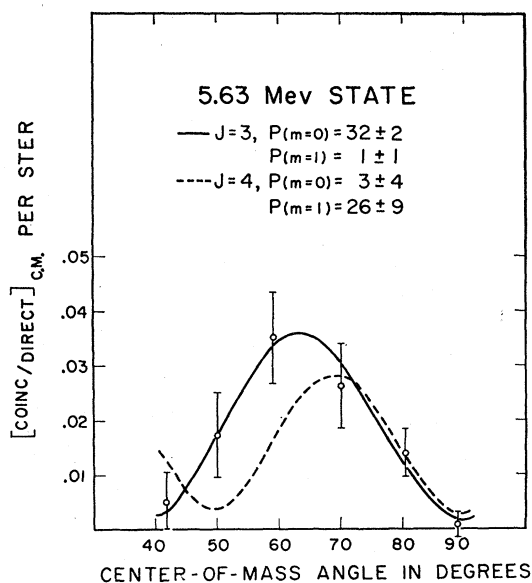


FIG. 7. Least-squares fits of the expression given in Eq. (2) are shown for the 5.63-Mev state angular correlation for values of $J=3$ and 4. Values of the population parameters required are shown. No other values of $J < 6$ produced satisfactory fits. The data have been transferred to angles less than 90° for convenience.

large ratio of $P(m=1)$ to $P(m=0)$. This leaves the $J=3$ fit as the only satisfactory one for the 5.63-Mev state angular correlation.

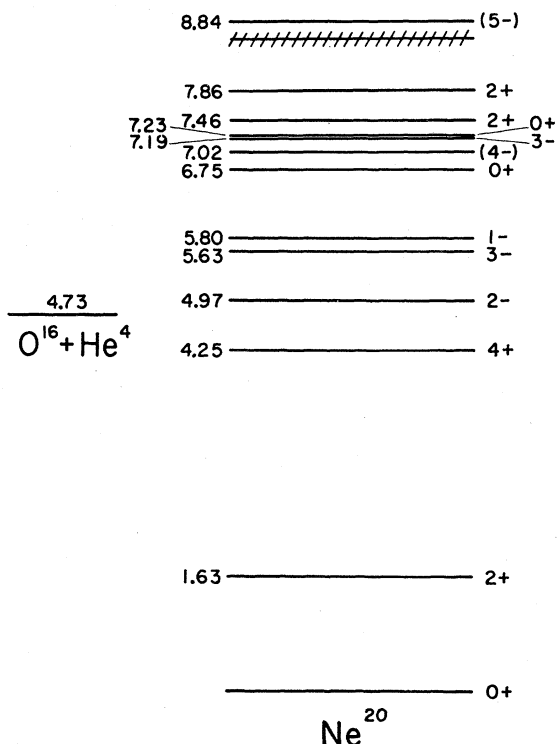


FIG. 8. Level diagram of Ne^{20} including the latest spin and parity assignments. The alpha-particle binding energy is shown on the left.

The measurements thus establish spins of 1 and 3, respectively, for the 5.80- and 5.63-Mev states of Ne^{20} . Since each of these states decays by alpha-particle emission to the $0+$ ground state of O^{16} they must each have parity equal to $(-1)^J$, i.e., negative in each case.

DISCUSSION

Figure 8 is a level diagram of Ne^{20} including the latest spin and parity assignments. In addition to the assignments taken from the latest compilation²² of Ajzenberg and Lauritsen, assignments of $4+$ to the 4.25-Mev level,²³ $2-$ to the 4.97-Mev,^{23,9} $3-$ and $1-$ to the 5.63- and 5.80-Mev levels, respectively, from the present work, and $(4-)$ to the 7.02-Mev level⁴ are included in the figure. The binding energy of $\text{O}^{16} + \text{He}^4$ is shown on the left.

As mentioned above, the lowest level above the alpha-particle binding energy, at 4.97-Mev excitation and which has long been assumed to play a dominant role in Ne^{20} production in stars,⁶⁻⁸ has been shown⁹ to be $2-$ and hence not to participate in thermonuclear reactions involving $\text{O}^{16} + \text{He}^4$. As a consequence of this the properties of the 5.63- and 5.80-Mev levels assume a greater importance in astrophysics. As demonstrated by these experiments, both of these levels can be formed by alpha-particle capture on O^{16} .

A measurement²⁴ of the gamma-ray yield in the reaction $\text{O}^{16}(\alpha, \gamma)\text{Ne}^{20}$ has yielded a preliminary value of 0.003 ± 0.002 ev for the quantity $(2l+1)\Gamma_\gamma\Gamma_\alpha/\Gamma$ involving the 5.63-Mev level of Ne^{20} . As yet no such measurement has been made for the 5.80-Mev level. As discussed recently⁹ the above value is such as to permit the 5.63-Mev level to contribute significantly in the formation of Ne^{20} in stars providing a sufficiently high temperature exists. Cameron¹⁰ has shown that the $\text{O}^{16}(\alpha, \gamma)\text{Ne}^{20}$ reaction through the 5.63-Mev level becomes much faster than the $3\alpha \rightarrow \text{C}^{12}$ and $\text{C}^{12}(\alpha, \gamma)\text{O}^{16}$ reactions for temperatures above 5×10^8 °K. At this higher temperature the helium reactions are greatly speeded up and the situation corresponds to a thermonuclear explosion. Thus it is likely that most of the Ne^{20} in the galaxy has been formed in this way in supernova explosions.

A comparison of the properties of levels of Ne^{20} with the predictions of the collective model has been made.¹² In this comparison five bands of levels are suggested. The lowest band, based on the $0+$ ground state, includes $2+$ and $4+$ levels at 1.63 and 4.25 Mev, respectively. Such a band, extending to $J=8$, is also predicted by a four-particle shell-model calculation¹³ including configuration mixing and gives close agreement with measured level energies of the $0+$, $2+$, and $4+$ levels; the $6+$ and $8+$ levels have not yet been identified.

²² F. Ajzenberg-Selove and T. Lauritsen, Nuclear Phys. 2, 1 (1959).

²³ C. Broude and H. E. Gove, Bull. Am. Phys. Soc. 6, 37 (1961).

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Two additional positive-parity bands are suggested by the energy separations of the well-known 0^+ and 2^+ levels at 6.75 and 7.46 Mev and at 7.23 and 7.86 Mev, respectively.

The presence of a 3^- level at 5.63 Mev, together with the recently assigned⁹ 2^- level at 4.97 Mev, and a possible 5^- level at 8.84 Mev, has led to the discovery⁴ of a new level at 7.02 Mev whose properties^{3,4} are consistent with it being the third member of a $K=2$ negative-parity band. Similarly, the 1^- level at 5.80 Mev may form a band with the 3^- level at 7.19 Mev. It is interesting to note that if a rotational band is associated with these levels no evidence for a 2^- member near 6.3 Mev is present. The absence of 2^- and 4^- members of this band requires a K quantum number of 0 and implies an octopole shape deformation or vibration of the nucleus. Such octopole motion giving rise to $K=0$ and 2 negative-parity bands has recently been considered by Lipas and Davidson.²⁵ Similar low-lying negative-parity bands are also expected on the shell model when particle excitation

to $2p$ and $1f$ orbits as well as core excitation are included.²⁶

CONCLUSIONS

The alpha-particle angular correlation measurements described above allow unambiguous assignments of 3^- and 1^- , respectively, to be made to the 5.63- and 5.80-Mev levels of Ne^{20} . The value of $(2l+1)\Gamma_\gamma\Gamma_\alpha/\Gamma$ for the 5.63-Mev level is known to be sufficiently large for this level to contribute significantly to Ne^{20} formation in supernova explosions via the $\text{O}^{16}(\alpha,\gamma)\text{Ne}^{20}$ reaction. The observed assignments together with other data suggest the presence of negative-parity rotational bands in Ne^{20} and thus imply an octopole vibration of this nucleus.

ACKNOWLEDGMENTS

The author is indebted to Dr. E. Almqvist and Dr. A. E. Litherland for stimulating discussions concerning these measurements.

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²⁵ P. O. Lipas and J. P. Davidson, Nuclear Phys. **26**, 80 (1961).

Level Structure of Cr^{52+}

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The low excited states of Cr^{52} have been investigated by studying the decay of Mn^{52} using scintillation spectrometers and a double-focusing beta-ray spectrometer. In addition to the three strong lines at 0.74, 0.94, and 1.43 Mev, a number of weak transitions have been observed that require the addition of at least one new level at 3.614 Mev and which yield information on the spins and parities of the various levels. The following gamma rays have been observed: 1.434 Mev (100%), 1.332 Mev (5.7%), 1.246 Mev (5.8%), 1.214 Mev (2.9%), 0.935 Mev (83.9%), 0.847 Mev (2.6%), 0.744 Mev (81.9%), and 0.346 Mev (0.9%), in addition to several weaker and more uncertain lines. A decay scheme has been constructed which consists of levels at 1.434 Mev (2^+), 2.369 Mev (4^+), 2.648 Mev, 2.766 Mev (4^+), 3.112 Mev (6^+), 3.161 Mev ($1,2,3$), 3.614 Mev (5^+ , 6^+), and 3.832 Mev (5^+ , 6^+). A comparison has been made between the experimentally determined level structure and several theoretical calculations. Mn^{54} (290 day) was present in the Mn^{52} . The energy of the Mn^{54} gamma-ray transition was determined to be 834.9 ± 1.1 kev.

I. INTRODUCTION

FOR the past several years a program has been underway at this laboratory to investigate the level structure of even-even nuclei in the vicinity of the $f_{7/2}$ shell (N or Z from 20 to 28 nucleons). There have been several calculations made for nuclei in this region,

based on both the independent-particle model¹⁻⁴ and the collective model,⁵⁻⁷ and each seems to offer only partial success. It is clear, however, that more complete experimental information is needed for a satisfactory

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