PHYSICAL REVIEW VOLUME 125, NUMBER 5 MARCH 1, 1962

Properties of Σ Decays and the Σ^0 Lifetime*

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(Received July 18, 1961)

The consequences of a possible nonconservation of parity in the electromagnetic decay, $\Sigma^0 \rightarrow \Lambda^0+\gamma$, are deduced and an experimental test of parity conservation is proposed. In order to devise a method to determine the Σ^0 lifetime, $\tau(\Sigma^0)$, a study is made of $\Lambda^0 \rightarrow \Sigma^0$ conversion in Λ^0–nucleus collisions and it is found that the $\Lambda^0 \rightarrow \Sigma^0$ conversion induced by the nuclear Coulomb field, with cross section proportional to $Z^2/\tau(\Sigma^0)$, is the dominant process at large Z for Σ^0 generation in approximately forward directions. In addition, the implications of a principle of "minimal weak coupling" (involving a conserved baryon-meson polar-vector current) are exhibited for $\Sigma^\pm \rightarrow \Lambda^0$ strangeness-conserving leptonic decays and are used to propose a further conceivable method to determine the Σ^0 lifetime. Throughout the discussion, the various cross sections and decay rates are given for the two possible relative intrinsic parities of the Λ^0 and Σ^0.

THE Σ^0 hyperon is the most short lived of all the known elementary particles other than the "excited baryons" or the "multi-mesons" associated with scattering resonances. Whereas the "excited baryons" or the "multi-mesons" are not universally considered to be elementary particles, the Σ^0 is customarily treated as such essentially because of the belief that it would be stable if all but the strong interactions were turned off. Since the lifetime of the Σ^0 is, according to theoretical estimates,[1,2] in the neighborhood of 10^{-19} sec, the usual methods for measuring lifetimes are not practical and the determination of the Σ^0 mean life becomes a challenging experimental problem.

It is also ordinarily assumed that the electromagnetic $\Sigma^0 \rightarrow \Lambda^0$ conversion involves a parity-conserving interaction (Sec. I). This assumption is as yet without direct experimental verification and, though we believe it true, we have nevertheless scrutinized the $\Sigma^0 \rightarrow \Lambda^0+\gamma$ decay for a method of testing the consequences of any possible parity nonconservation (Sec. II).

With the use of the general form of the electromagnetic $\Sigma^0-\Lambda^0$ vertex function the problem of $\Lambda^0 \rightarrow \Sigma^0$ conversion in the Coulomb field of a nucleus is studied (Sec. III), the results being used to propose an experiment for the determination of the Σ^0 lifetime. The method depends essentially on the observation that the strength of the $\Sigma^0-\Lambda^0$ electromagnetic vertex determines both the lifetime of the Σ^0 and the differential cross section for $\Lambda^0 \rightarrow \Sigma^0$ conversion in a Coulomb field.

Finally, the leptonic and electromagnetic decays of the Σ^+, Σ^-, Σ^0 multiplet into the Λ^0 are compared (Sec. IV). If "minimal weak coupling" together with a conserved baryon-meson polar-vector current is assumed to govern all strangeness-conserving weak interactions, the characteristics of the $\Sigma^\pm \rightarrow \Lambda^0+e^\pm+\nu$ process can, under favorable circumstances, also be used to determine the lifetime of the Σ^0.

I. PHENOMENOLOGY OF Σ^0 ELECTROMAGNETIC DECAY

First let us assume that the electromagnetic interactions always entail parity conservation. The principle of minimal electromagnetic coupling then determines the form of the Lagrangian which couples the baryons and mesons with the electromagnetic field:

$$\begin{aligned}\mathcal{L}^{(\mathrm{em})}(x)=e\Big\{&\bar{\psi}_N(x)i\gamma_\mu\left(\frac{1+\tau_3}{2}\right)\psi_N(x)\\&+\bar{\psi}_\Sigma(x)i\gamma_\mu t_3\psi_\Sigma(x)-\bar{\psi}_\Xi(x)i\gamma_\mu\left(\frac{1-\tau_3}{2}\right)\psi_\Xi(x)\\&+i\phi_\pi{}^\dagger(x)t_3(-\overleftrightarrow{\partial}_\mu)\phi_\pi(x)\\&+i\phi_K{}^\dagger(x)\left(\frac{1+\tau_3}{2}\right)2(-\overleftrightarrow{\partial}_\mu)\phi_K(x)\Big\}\mathfrak{A}_\mu(x)\\&+O(e^2)\equiv j_\mu(x)\mathfrak{A}_\mu(x)+O(e^2),\end{aligned}\tag{1}$$

$$\psi_N\equiv\begin{vmatrix}\psi_p\\\psi_n\end{vmatrix};\quad\psi_\Sigma\equiv\begin{vmatrix}(1/\sqrt{2})(\psi_{\Sigma^+}+\psi_{\Sigma^-})\\(i/\sqrt{2})(\psi_{\Sigma^+}-\psi_{\Sigma^-})\\\psi_{\Sigma^0}\end{vmatrix};\quad\psi_\Xi\equiv\begin{vmatrix}\psi_{\Xi^0}\\\psi_{\Xi^-}\end{vmatrix};$$

$$\phi_\pi\equiv\begin{vmatrix}(1/\sqrt{2})(\phi_{\pi^+}+\phi_{\pi^-})\\(i/\sqrt{2})(\phi_{\pi^+}-\phi_{\pi^-})\\\phi_{\pi^0}\end{vmatrix};\quad\phi_K\equiv\begin{vmatrix}\phi_{K^+}\\\phi_{K^0}\end{vmatrix},$$

$$(t_k)_{mn}\equiv-i\epsilon_{mnk};\quad A\overleftrightarrow{\partial}_\mu B\equiv\tfrac{1}{2}[A(\partial_\mu B)-(\partial_\mu A)B],$$

there being, according to Eq. (1), no *primitive* electromagnetic coupling whatsoever for π^0, K^0, n, Λ^0, Σ^0, Ξ^0. Certain matrix elements of the current $j_\mu(x)$ have been partially explored; in particular, considerable information is available about $\langle p'|j_\mu(x)|p\rangle$, $\langle n'|j_\mu(x)|n\rangle$ which describe the interaction of a proton and a neutron, respectively, with the electromagnetic field.[3] The

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[1] M. Gell-Mann, *Proceedings of the Annual International Conference on High-Energy Physics at CERN* (CERN Scientific Information Service, Geneva, 1958), p. 162.
[2] J. Dreitlein and B. Lee, Phys. Rev. **124**, 1274 (1961).
[3] R. Hofstadter, C. DeVries, and R. Herman, Phys. Rev. Letters **6**, 290 (1961); R. Hofstadter and R. Herman, *ibid.* **6**, 293 (1961); D. N. Olson, H. F. Schopper, and R. R. Wilson, *ibid.* **6**, 286 (1961).

current $j_\mu(x)$ also gives rise to a nonvanishing matrix element for Σ^0 electromagnetic decay, $\langle\Lambda^0|j_\mu(x)|\Sigma^0\rangle$, whose magnitude determines the lifetime of the Σ^0 and which can be written as

$$\langle\Lambda^0|j_\mu(x)|\Sigma^0\rangle=e^{ik_\rho x_\rho}\langle\Lambda^0|j_\mu(0)|\Sigma^0\rangle,\ k_\rho\equiv(p_\Sigma-p_\Lambda)_\rho;\quad k_\rho^2=|\mathbf{k}|^2-k_0^2=|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|^2-(E_\Sigma-E_\Lambda)^2. \tag{2}$$

It is clear that only the isovector part of $j_\mu(0)$ (cf. Eqs. (48) below) contributes to $\langle\Lambda^0|j_\mu(0)|\Sigma^0\rangle$ since $|\Sigma^0\rangle$, $|\Lambda^0\rangle$ are, respectively, isovector and isoscalar.

We proceed to investigate the general form of $\langle\Lambda^0|j_\mu(0)|\Sigma^0\rangle$ and begin by applying the restrictions of Lorentz, space-inversion and time-reversal invariance. We then have, for the two possible Λ^0, Σ^0 relative intrinsic parities,

$$P_\Lambda P_\Sigma=+1,$$
$$\langle\Lambda^0|j_\mu(0)|\Sigma^0\rangle=F_1^{(+)}(k_\rho^2)i(\bar{u}_\Lambda\sigma_{\mu\nu}u_\Sigma)k_\nu+F_2^{(+)}(k_\rho^2)i(\bar{u}_\Lambda\gamma_\mu u_\Sigma)+F_3^{(+)}(k_\rho^2)(\bar{u}_\Lambda u_\Sigma)(m_\Sigma-m_\Lambda)k_\mu, \tag{3a}$$
$$P_\Lambda P_\Sigma=-1,$$
$$\langle\Lambda^0|j_\mu(0)|\Sigma^0\rangle=F_1^{(-)}(k_\rho^2)i(\bar{u}_\Lambda\gamma_5\sigma_{\mu\nu}u_\Sigma)k_\nu+F_2^{(-)}(k_\rho^2)i(\bar{u}_\Lambda\gamma_5\gamma_\mu u_\Sigma)+F_3^{(-)}(k_\rho^2)(\bar{u}_\Lambda\gamma_5 u_\Sigma)(m_\Sigma+m_\Lambda)k_\mu, \tag{3b}$$

where u_Λ, u_Σ are appropriate spinors and the form factors $F_1^{(+)}$, $\cdots$, $F_3^{(-)}$ are real for $k_\rho^2>-(2m_\pi)^2$. In addition, gauge invariance of the second kind permits a further reduction in the number of independnet form factors by requiring a differential conservation law for $j_\mu(x)$, whence

$$k_\mu\langle\Lambda^0|j_\mu(0)|\Sigma^0\rangle=0.. \tag{4}$$

Equations (3) and (4) yield

$$P_\Lambda P_\Sigma=+1,$$
$$\langle\Lambda^0|j_\mu(0)|\Sigma^0\rangle=F_1^{(+)}(k_\rho^2)i(\bar{u}_\Lambda\sigma_{\mu\nu}u_\Sigma)k_\nu+F_3^{(+)}(k_\rho^2)i(\bar{u}_\Lambda\gamma_\mu u_\Sigma)k_\nu^2+F_3^{(+)}(k_\rho^2)(\bar{u}_\Lambda u_\Sigma)(m_\Sigma-m_\Lambda)k_\mu, \tag{5a}$$
$$P_\Lambda P_\Sigma=-1,$$
$$\langle\Lambda^0|j_\mu(0)|\Sigma^0\rangle=F_1^{(-)}(k_\rho^2)i(\bar{u}_\Lambda\gamma_5\sigma_{\mu\nu}u_\Sigma)k_\nu+F_3^{(-)}(k_\rho^2)i(\bar{u}_\Lambda\gamma_5\gamma_\mu u_\Sigma)k_\nu^2+F_3^{(-)}(k_\rho^2)(\bar{u}_\Lambda\gamma_5 u_\Sigma)(m_\Sigma+m_\Lambda)k_\mu, \tag{5b}$$

which together with the Fourier-transformed Maxwell equations,

$$k_\nu^2\mathcal{A}_\mu(k_\lambda)=4\pi J_\mu(k_\lambda), \tag{6a}$$
$$k_\mu\mathcal{A}_\mu(k_\lambda)=0, \tag{6b}$$

gives

$$P_\Lambda P_\Sigma=+1,$$
$$\langle\Lambda^0|j_\mu(0)|\Sigma^0\rangle\mathcal{A}_\mu(k_\lambda)=F_1^{(+)}(k_\rho^2)i(\bar{u}_\Lambda\sigma_{\mu\nu}u_\Sigma)k_\nu\mathcal{A}_\mu(k_\lambda)+F_3^{(+)}(k_\rho^2)i(\bar{u}_\Lambda\gamma_\mu u_\Sigma)(4\pi J_\mu(k_\lambda)), \tag{7a}$$
$$P_\Lambda P_\Sigma=-1,$$
$$\langle\Lambda^0|j_\mu(0)|\Sigma^0\rangle\mathcal{A}_\mu(k_\lambda)=F_1^{(-)}(k_\rho^2)i(\bar{u}_\Lambda\gamma_5\sigma_{\mu\nu}u_\Sigma)k_\nu\mathcal{A}_\mu(k_\lambda)+F_3^{(-)}(k_\rho^2)i(\bar{u}_\Lambda\gamma_5\gamma_\mu u_\Sigma)(4\pi J_\mu(k_\lambda)). \tag{7b}$$

In Eqs. (7) and (6a), $J_\mu(k_\lambda)$ is the current (in momentum space) of all the sources of the electromagnetic field which may be present, not inclusive of the $\Sigma^0-\Lambda^0$ current itself—e.g., $J_\mu(k_\lambda)$ contains the current of the Dalitz pair, of any nuclear Coulomb field, etc. Thus only the $F_1^{(\pm)}(k_\rho^2)$ terms contribute to the process $\Sigma^0\to\Lambda^0+\gamma$ while all the terms, with $\mathcal{A}_\mu(k_\lambda)$ replaced by $4\pi J_\mu(k_\lambda)/k_\nu^2$, contribute to a $\Sigma^0\leftrightarrow\Lambda^0$ process where the photon is virtual with a nonzero "mass" $(-k_\rho^2)^{\frac{1}{2}}$. In the limit of $k_\rho^2\to 0$, $F_1^{(\pm)}$ is the $\Sigma^0-\Lambda^0$ transition magnetic moment $f^{(\pm)}e/2m_N$ while[2] $F_3^{(\pm)}$ is $\approx f^{(\pm)}e(r_{\Sigma-\Lambda}{}^2/6)$ where

$$r_{\Sigma-\Lambda}{}^2\equiv-6\left[\frac{d}{d(k_\rho^2)}\ln F_1^{(\pm)}(k_\rho^2)\right]_{k_\rho^2=0}$$

is the root-mean-square radius of the $\Sigma^0-\Lambda^0$ transition magnetic moment distribution.

An effective Lagrangian can now be constructed which simulates the $\Sigma^0-\Lambda^0$ vertex function of Eqs. (7) for $\Sigma^0\to\Lambda^0+\gamma$, viz.,

$$P_\Lambda P_\Sigma=+1,$$
$$\mathcal{L}^{(+)}(x)=\tfrac{1}{2}f^{(+)}(e/2m_N)(\bar{\psi}_\Lambda(x)\sigma_{\mu\nu}\psi_\Sigma(x))F_{\mu\nu}(x)+\text{H.c.}=f^{(+)}(e/2m_N)[(\bar{\psi}_\Lambda\boldsymbol{\sigma}\psi_\Sigma)\cdot\mathbf{H}-i(\bar{\psi}_\Lambda\boldsymbol{\alpha}\psi_\Sigma)\cdot\mathbf{E}]+\text{H.c.},$$
$$f^{(+)}e/2m_N\equiv F_1^{(+)}(0), \tag{8a}$$
$$P_\Lambda P_\Sigma=-1,$$
$$\mathcal{L}^{(-)}(x)=\tfrac{1}{2}f^{(-)}(e/2m_N)(\bar{\psi}_\Lambda(x)\gamma_5\sigma_{\mu\nu}\psi_\Sigma(x))F_{\mu\nu}(x)+\text{H.c.}=f^{(-)}(e/2m_N)[-(\bar{\psi}_\Lambda\boldsymbol{\alpha}\psi_\Sigma)\cdot\mathbf{H}+i(\bar{\psi}_\Lambda\boldsymbol{\sigma}\psi_\Sigma)\cdot\mathbf{E}]+\text{H.c.},$$
$$f^{(-)}e/2m_N\equiv F_1^{(-)}(0), \tag{8b}$$

so that $\mathcal{L}^{(+)}(x)$, $\mathcal{L}^{(-)}(x)$ contains *only* an anomalous or Pauli moment of the same type as the strong interactions induce in nucleons. Thus, as already noted, the Σ^0 would not decay electromagnetically in the absence of strong interactions, just as the neutron would have no electromagnetic couplings if the strong interactions were turned off.[4]

[4] Of course, the processes $\Sigma^0\to\Lambda^0+\gamma$, $n\to n+\gamma$ would still be possible, even in the absence of strong interactions, through the intervention of the weak interactions, i.e.,

$$\Sigma^0\to_{\text{weak}}\Lambda^0+p+\bar{p}\to_{\text{el. mag.}}\Lambda^0+\gamma;$$
$$n\to_{\text{weak}}n+p+\bar{p}\to_{\text{el. mag.}}n+\gamma.$$

Up to this point we have assumed that electromagnetic $\Sigma^0 \to \Lambda^0$ transitions conserve parity. To explore possible parity-nonconservation effects in $\Sigma^0 \to \Lambda^0+\gamma$, we consider an effective $\Sigma^0 \leftrightarrow \Lambda^0$ Lagrangian which is not invariant under space-inversion, i.e.,

$$\mathcal{L}(x;\lambda)=\frac{1}{2}\frac{f}{(1+\lambda^2)^{\frac{1}{2}}}\left(\frac{e}{2m_N}\right)(\bar{\psi}_\Lambda(1+\lambda\gamma_5)\sigma_{\mu\nu}\psi_\Sigma)F_{\mu\nu} +\text{H.c.}\equiv i_\mu(x)\mathcal{A}_\mu(x)+\text{H.c.},$$
$$\frac{f}{(1+\lambda^2)^{\frac{1}{2}}}\equiv f^{(+)};\quad \frac{\lambda f}{(1+\lambda^2)^{\frac{1}{2}}}\equiv f^{(-)};\qquad(9)$$
$$\mathcal{L}(x,0)=\mathcal{L}^{(+)}(x);\quad \mathcal{L}(x,\infty)=\mathcal{L}^{(-)}(x),$$

whence, nonrelativistically,

$$\mathcal{L}(x,\lambda)\cong\frac{if(e/2m_N)}{(1+\lambda^2)^{\frac{1}{2}}}\left\{(\psi_\Lambda{}^\dagger\boldsymbol{\sigma}\psi_\Sigma)\cdot\left[\left(\frac{\lambda+1}{2}\right)(\mathbf{E}-i\mathbf{H})+\left(\frac{\lambda-1}{2}\right)(\mathbf{E}+i\mathbf{H})\right]\right\}+\text{H.c.}\quad(10)$$

We shall explore, in the next section, the consequences of Eqs. (9) and (10). Equation (10) already indicates the tendency for the photon emitted in $\Sigma^0 \to \Lambda^0+\gamma$ to have a net circular polarization if parity is not conserved in the decay interaction since the operators creating left and right circularly polarized photons ($\mathbf{E}+i\mathbf{H}$ and $\mathbf{E}-i\mathbf{H}$, respectively) appear with different coefficients $[(\lambda-1)/2,\ (\lambda+1)/2]$ in $\mathcal{L}(x;\lambda)$.

II. POSSIBLE PARITY NONCONSERVATION IN ELECTROMAGNETIC Σ^0 DECAY

The decay rate of $\Sigma^0 \to \Lambda^0+\gamma$, which we shall need below, follows from the usual procedure for computing transition probabilities on the basis of an effective Lagrangian such as that of Eq. (9). Thus,

$$\begin{aligned}1/\tau(\Sigma^0)&=\int 2\pi\delta(E_\gamma+E_\Lambda-m_\Sigma)\times\tfrac{1}{2}\sum_{S_\gamma S_\Lambda S_\Sigma}|\langle\Lambda^0|i_\mu(0)|\Sigma^0\rangle\mathcal{A}_\mu(k_\lambda)|^2\frac{d^3p_\gamma}{(2\pi)^3}\\&=\int 2\pi\delta(E_\gamma+E_\Lambda-m_\Sigma)\times\tfrac{1}{2}\sum_{S_\gamma S_\Lambda S_\Sigma}\left|\left[\frac{ife/2m_N}{(1+\lambda^2)^{\frac{1}{2}}}(\bar{u}_\Lambda(1+\lambda\gamma_5)\sigma_{\mu\nu}u_\Sigma)\times(p_\Sigma-p_\Lambda)_\nu\right]\left[\frac{(\epsilon_\gamma)_\mu}{(2E_\gamma)^{\frac{1}{2}}}\right](4\pi)^{\frac{1}{2}}\right|^2\frac{d^3p_\gamma}{(2\pi)^3}\\&=4(fe/2m_N)^2(E_\gamma)^3=f^2\times 5.0\times 10^{18}\ \text{sec}^{-1},\end{aligned}\quad(11a)$$

where

$$\begin{aligned}E_\gamma=m_\Sigma-E_\Lambda&=(m_\Sigma-m_\Lambda)(m_\Sigma+m_\Lambda)/2m_\Sigma\\&=(m_\Sigma-m_\Lambda)(0.97);\quad e^2=1/137.\end{aligned}\quad(11b)$$

The estimate given in reference 2 for $f^{(\pm)}$, the $\Sigma^0 \to \Lambda^0$ transition magnetic moment in units of $e/2m_N$,

$$f^{(\pm)}\approx 1.2(g_\Lambda{}^{(\pm)}g_\Sigma/g_N{}^2),\quad(11c)$$

with $g_\Lambda{}^{(\pm)}$, g_Σ, g_N the renormalized $\pi\Lambda\Sigma$, $\pi\Sigma\Sigma$, πNN coupling constants, yields in the parity-conserving case where $f^{(+)}=f(\lambda=0)$ or $f^{(-)}=f(\lambda=\infty)$:

$$1/\tau(\Sigma^0)\approx(g_\Lambda{}^{(\pm)}g_\Sigma/g_N{}^2)^2\times 10^{19}\ \text{sec}^{-1}\approx 10^{18-20}\ \text{sec}^{-1},\quad(11d)$$

which is an equation that will be used in what follows

We proceed to discuss the polarization properties of the γ and Λ^0. These are given by evaluating the matrix element

$$\langle\Lambda^0|i_\mu(0)|\Sigma^0\rangle\mathcal{A}_\mu(k_\lambda)=\langle\Lambda^0|i_\mu(0)|\Sigma^0\rangle\langle\gamma|\mathcal{A}_\mu(0)|0\rangle$$

for definite S_γ, S_Λ, S_Σ characterizing the states

$$|\gamma\rangle,\quad|\Lambda^0\rangle,\quad|\Sigma^0\rangle.$$

For the case in which the Σ^0 has a polarization $\mathbf{P}_\Sigma=\langle\boldsymbol{\sigma}_\Sigma\rangle$, the square of the transition matrix element

$$|\langle\Lambda^0|i_\mu(0)|\Sigma^0\rangle\langle\gamma|\mathcal{A}_\mu(0)|0\rangle|^2$$

is proportional to

$$\begin{aligned}W(\hat{s},\delta,\hat{k},\mathbf{P}_\Sigma)=[&\boldsymbol{\zeta}\cdot\boldsymbol{\zeta}^*(1-\mathbf{P}_\Sigma\cdot\hat{s})-i\boldsymbol{\zeta}\times\boldsymbol{\zeta}^*\cdot(\mathbf{P}_\Sigma-\hat{s})\\&+\boldsymbol{\zeta}\cdot\mathbf{P}_\Sigma\boldsymbol{\zeta}^*\cdot\hat{s}+\boldsymbol{\zeta}^*\cdot\mathbf{P}_\Sigma\boldsymbol{\zeta}\cdot\hat{s}],\end{aligned}$$

with

$$\boldsymbol{\zeta}\equiv\left[\left(\frac{1+\delta}{2}\right)\left(\frac{\lambda-1}{2}\right)\left(\frac{\boldsymbol{\varepsilon}+i\boldsymbol{\eta}}{\sqrt{2}}\right)+\left(\frac{1-\delta}{2}\right)\left(\frac{\lambda+1}{2}\right)\left(\frac{\boldsymbol{\varepsilon}-i\boldsymbol{\eta}}{\sqrt{2}}\right)\right]\Big/(1+\lambda^2)^{\frac{1}{2}};\quad(12)$$

$\boldsymbol{\varepsilon}$, $\boldsymbol{\eta}$ = unit vectors in direction of the emitted photon's electric, magnetic fields; $\hat{k}\equiv\boldsymbol{\varepsilon}\times\boldsymbol{\eta}=\hat{p}_\gamma=-\hat{p}_\Lambda$. Here W represents the relative probability for various final state configurations specified by given Σ^0 polarization $\mathbf{P}_\Sigma$, Λ^0 spin direction $\hat{s}$, photon polarization $\delta=1$, -1 for left, right circularly polarized photons, and photon, Λ^0 emission directions $\hat{k}$, $\hat{p}_\Lambda(=-\hat{k})$. It is to be noted that W is different for $\lambda=0$ and for $\lambda=\infty$ so that an investigation of the various correlations predicted by W is sufficient, in the parity conserving case, to determine the Λ^0, Σ^0 relative intrinsic parity.[5]

Specifically, the relative probability of emission of a

Such weak-interaction-mediated contributions to $\Sigma^0 \to \Lambda^0+\gamma$, $n \to n+\gamma$ are presumably actually present and adjoin negligibly small parity-nonconserving terms to the corresponding matrix elements.

[5] R. Gatto, Phys. Rev. **109**, 610 (1957); G. Feldman and T. Fulton, Nuclear Phys. **8**, 106 (1958); J. Sucher and G. Snow, Nuovo cimento **18**, 195 (1960); N. Byers and H. Burkhardt, Phys. Rev. **121**, 281 (1961); L. Michel and H. Rouhaninejad, *ibid.* **122**, 242 (1961).

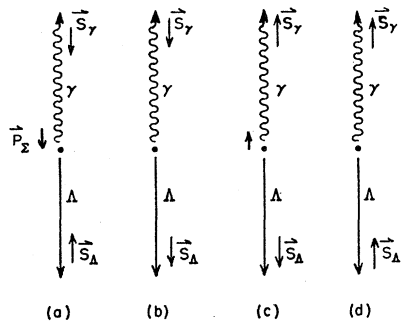

FIG. 1. Special configuration in the decay $\Sigma^0 \to \Lambda^0+\gamma$ illustrating the selection rules operative. A small arrow indicates the helicity of the particle. Configurations (a) and (c) are allowed; (b) and (d) are forbidden.

left, right circularly polarized photon is

$$\begin{aligned} w_1 &\equiv W(\hat{s},1,\hat{k},\mathbf{P}_\Sigma)=\left[\left(\frac{\lambda-1}{2}\right)^2\Big/(\lambda^2+1)\right] \\ &\qquad\times\{(1+\hat{s}\cdot\hat{k})(1-\mathbf{P}_\Sigma\cdot\hat{k})\}, \\ w_{-1} &\equiv W(\hat{s},-1,\hat{k},\mathbf{P}_\Sigma)=\left[\left(\frac{\lambda+1}{2}\right)^2\Big/(\lambda^2+1)\right] \\ &\qquad\times\{(1-\hat{s}\cdot\hat{k})(1+\mathbf{P}_\Sigma\cdot\hat{k})\}, \end{aligned} \tag{13}$$

and if the Λ^0 spin and momentum directions are summed over, an average net circular polarization results:

$$P_\gamma\equiv\frac{\langle w_{-1}\rangle-\langle w_1\rangle}{\langle w_{-1}\rangle+\langle w_1\rangle}=\frac{2\lambda}{\lambda^2+1}. \tag{14}$$

The physical significance of the factors $(1\pm\hat{s}\cdot\hat{k})$, $(1\pm\mathbf{P}_\Sigma\cdot\hat{k})$ in Eq. (13) is obvious. Thus in w_1 the factor $1+\hat{s}\cdot\hat{k}$ guarantees that $(\mathbf{S}_\gamma+\mathbf{S}_\Lambda)\cdot\hat{k}=\mathbf{S}_\Sigma\cdot\hat{k}$ is $\frac{1}{2}$ and not $\frac{3}{2}$. Similarly the factor $1-\mathbf{P}_\Sigma\cdot\hat{k}$ ensures that $(\mathbf{S}_\gamma+\mathbf{S}_\Lambda)\cdot\mathbf{P}_\Sigma=\mathbf{S}_\Sigma\cdot\mathbf{P}_\Sigma$ is never negative. The four diagrams in Fig. 1 illustrate the ideas graphically.

We next consider the polarization properties of the emitted Λ^0. Let us first compute the direction-polarization correlation of the Λ^0 by summing $W(\hat{s},\delta,\hat{k},\mathbf{P}_\Sigma)$ over the two possible photon polarizations:

$$\begin{aligned} w(\hat{s},\hat{p}_\Lambda,\mathbf{P}_\Sigma)&\equiv w_1+w_{-1} \\ &=\frac{1}{2}\left\{1-\mathbf{P}_\Sigma\cdot\hat{p}_\Lambda\hat{s}\cdot\hat{p}_\Lambda-\frac{2\lambda}{\lambda^2+1}\hat{p}_\Lambda\cdot(\mathbf{P}_\Sigma-\hat{s})\right\}, \end{aligned} \tag{15}$$

from which we obtain the polarization of the Λ^0, $\mathbf{P}_\Lambda$, by considering the value of $\hat{s}$ which maximizes:

$$\begin{aligned} &\frac{w(\hat{s},\hat{p}_\Lambda,\mathbf{P}_\Sigma)-w(-\hat{s},\hat{p}_\Lambda,\mathbf{P}_\Sigma)}{w(\hat{s},\hat{p}_\Lambda,\mathbf{P}_\Sigma)+w(-\hat{s},\hat{p}_\Lambda,\mathbf{P}_\Sigma)} \\ &\qquad=\left(\frac{[2\lambda/(\lambda^2+1)]-\mathbf{P}_\Sigma\cdot\hat{p}_\Lambda}{1-[2\lambda/(\lambda^2+1)]\mathbf{P}_\Sigma\cdot\hat{p}_\Lambda}\right)\hat{s}\cdot\hat{p}_\Lambda. \end{aligned} \tag{16}$$

Equation (16) shows that the Λ^0 is longitudinally polarized, with[6]

$$\mathbf{P}_\Lambda=\left(\frac{[2\lambda/(\lambda^2+1)]-\mathbf{P}_\Sigma\cdot\hat{p}_\Lambda}{1-[2\lambda/(\lambda^2+1)]\mathbf{P}_\Sigma\cdot\hat{p}_\Lambda}\right)\hat{p}_\Lambda, \tag{17}$$

so that a study of the variation of $\mathbf{P}_\Lambda$ with the angle θ_Λ between $\mathbf{P}_\Sigma$ and $\hat{p}_\Lambda$ will test whether or not parity is conserved in $\Sigma^0\to\Lambda^0+\gamma$. We also note that for $\lambda=\pm1$ the Λ^0 is completely longitudinally polarized whatever the value of $\mathbf{P}_\Sigma\cdot\hat{p}_\Lambda$ and that even if $\mathbf{P}_\Sigma=0$,[7]

$$\mathbf{P}_\Lambda]_0=\frac{2\lambda}{\lambda^2+1}\hat{p}_\Lambda. \tag{18}$$

Finally, if $\hat{p}_\Lambda$ is averaged over, the corresponding $\langle\mathbf{P}_\Lambda\rangle$ is necessarily along $\hat{P}_\Sigma$ and turns out to be

$$\begin{aligned} \langle\mathbf{P}_\Lambda\rangle&=\frac{\displaystyle\int_0^\pi\left\{\left(\frac{2\lambda}{\lambda^2+1}-|\mathbf{P}_\Sigma|\cos\theta_\Lambda\right)\cos\theta_\Lambda\right\}\sin\theta_\Lambda d\theta_\Lambda}{\displaystyle\int_0^\pi\left(1-\frac{2\lambda}{\lambda^2+1}|\mathbf{P}_\Sigma|\cos\theta_\Lambda\right)\sin\theta_\Lambda d\theta_\Lambda}\hat{P}_\Sigma \\ &=-\tfrac{1}{3}\mathbf{P}_\Sigma, \end{aligned} \tag{19}$$

which is independent of λ and so independent of whether or not parity is conserved in $\Sigma^0\to\Lambda^0+\gamma$.

We proceed to discuss the directional asymmetries which are expected when parity is not conserved, in particular the up-down asymmetry in the Λ^0 or γ emission relative to the Σ^0 production plane. Since the Σ^0 is produced in a parity conserving strong interaction, e.g., $\pi^-+p\to\Sigma^0+K^0$, $\mathbf{P}_\Sigma$ is parallel or antiparallel to the normal to this production plane so that the up-down Λ^0 emission asymmetry is proportional to $|\mathbf{P}_\Sigma|$. In fact, summing the $w(\hat{s},\hat{p}_\Lambda,\mathbf{P}_\Sigma)$ of Eq. (15) over the Λ^0 spin direction $\hat{s}$, we have for the correlation function, $f(\cos\theta_\Lambda)$, between the Λ^0 direction $\hat{p}_\Lambda$ and the polarization $\mathbf{P}_\Sigma$:

$$f(\cos\theta_\Lambda)\equiv\sum_{\hat{s}}w(\hat{s},\hat{p}_\Lambda,\mathbf{P}_\Sigma)=1-\frac{2\lambda}{\lambda^2+1}|\mathbf{P}_\Sigma|\cos\theta_\Lambda, \tag{20}$$

so that the up-down Λ^0 emission asymmetry, α, is

$$\begin{aligned} \alpha&=\frac{\displaystyle\int_0^{\pi/2}f(\cos\theta_\Lambda)\sin\theta_\Lambda d\theta_\Lambda-\int_{\pi/2}^{\pi}f(\cos\theta_\Lambda)\sin\theta_\Lambda d\theta_\Lambda}{\displaystyle\int_0^\pi f(\cos\theta_\Lambda)\sin\theta_\Lambda d\theta_\Lambda} \\ &=-\frac{2\lambda}{\lambda^2+1}\frac{|\mathbf{P}_\Sigma|}{2}. \end{aligned} \tag{21}$$

[6] In the parity conserving case: $\lambda=0,\infty$, Eq. (17) becomes $\mathbf{P}_\Lambda=-(\mathbf{P}_\Sigma\cdot\hat{p}_\Lambda)\hat{p}_\Lambda$, a relation obtained previously by the authors of reference 5.

[7] For $\lambda=1$, Eq. (18) gives $\mathbf{P}_\Lambda]_0=\hat{p}_\Lambda$, a result which follows immediately from Eq. (10) since

$$\begin{aligned} (u_\Lambda{}^\dagger\boldsymbol{\sigma}u_\Sigma)\cdot(\boldsymbol{\varepsilon}-i\boldsymbol{\eta})&=[u_\Lambda{}^\dagger(\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}+i\boldsymbol{\sigma}\cdot\hat{p}_\Lambda\times\boldsymbol{\varepsilon})u_\Sigma] \\ &=[u_\Lambda{}^\dagger(1+\boldsymbol{\sigma}\cdot\hat{p}_\Lambda)(\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon})u_\Sigma]. \end{aligned}$$

α is a convenient experimental quantity to measure since its determination involves only the difference between the numbers of Λ^0 emitted into the upper and into the lower hemispheres the distinction between the upper $(0\leqq\theta_\Lambda\leqq\pi/2)$ and the lower $(\pi/2<\theta_\Lambda\leqq\pi)$ being independent of the Σ^0 momentum. If the momentum of the Σ^0 is not well defined we need only replace $|\mathbf{P}_\Sigma|$ in Eqs. (20) and (21) by a suitable average over the momenta of the accepted Σ^0.

We thus see that an experimental study of the $\mathbf{P}_\Lambda$, α [Eqs. (17) and (21)], with $\mathbf{P}_\Sigma$ calculated from observation of $\langle\mathbf{P}_\Lambda\rangle$ [Eq. (19)], should be sufficient for the determination of the parity-nonconservation parameter: $2\lambda/(\lambda^2+1)$. It may also be mentioned that considerations similar to those in Eqs. (9)–(21) can be given for a study of parity-nonconservation effects in $\Lambda^0\rightarrow n+\gamma$, $\Sigma^+\rightarrow p+\gamma$, $\Xi^{-,0}\rightarrow\Sigma^{-,0}+\gamma$,[8] $\mu^\pm\rightarrow e^\pm+\gamma$ (if this last reaction is not absolutely forbidden),[8] etc.

III. DETERMINATION OF THE Σ^0 LIFETIME FROM $\Lambda^0\rightarrow\Sigma^0$ CONVERSION IN A NUCLEAR COULOMB FIELD

As we have previously mentioned, the lifetime of the Σ^0, $\tau(\Sigma^0)$, is very short compared to the lifetimes of all other elementary particles, but it is still very long compared to the lifetimes of the "excited baryons" or of (most of) the "multi-mesons". Hence, the experimental methods normally used for finding the lifetimes of elementary particles or "excited baryons" and "multi-mesons" should be essentially inapplicable to the Σ^0. In fact, the usual experimental methods for finding a particle's mean life fall into two classes:

(a) *Individual Lifespan Measurements.* The flight distance from birth to death of an individual unstable particle in, e.g., an emulsion, gives its rest-frame lifespan if the energy of the particle is known. Taking the minimum resolvable distance in an emulsion as $0.1\mu=10^{-5}$ cm and assuming that $\tau(\Sigma^0)=10^{-19}$ sec (Sec. II), the Σ^0 energy required to produce a flight distance of 10^{-5} cm, is

$$E_\Sigma=m_\Sigma/(1-v_\Sigma^2)^{\frac{1}{2}}=\left(\frac{10^{-5}\text{ cm}/v_\Sigma}{10^{-19}}\right)m_\Sigma=3\times10^3 m_\Sigma$$
$$=3.6\times10^3\text{ Bev.}\qquad(22)$$

This method (a) is then out of question for the Σ^0 at the present time.

(b) *Mass Spread Measurements.* The spread in mass in a collection of identical unstable particles is related to the particle lifetime by the uncertainty principle,

$$\Delta m=1/\tau.\qquad(23)$$

In the case of the Σ^0, with $\tau(\Sigma^0)=10^{-19}$ sec, Eq. (23) yields

$$\Delta m_\Sigma=6\text{ kev};\quad \Delta m_\Sigma/(m_\Sigma-m_\Lambda)=10^{-4},\qquad(24)$$

so that the relatively short Σ^0 lifetime induces only a 0.01% spread in the mass difference. Thus method (b) is also quite inapplicable to the Σ^0 with presently available techniques.

The method for determination of $\tau(\Sigma^0)$ which we shall develop in this section, namely the deduction of $\tau(\Sigma^0)$ from observations on the differential cross section for $\Lambda^0\rightarrow\Sigma^0$ conversion in a nuclear Coulomb field, $d\sigma_{\text{Coul}}(\Lambda^0\rightarrow\Sigma^0)/d\Omega$, is reminiscent of a procedure wherein the lifetime of an excited state of nucleus is deduced from observations on the differential cross section for excitation of that state by the Coulomb field of, say, an incident proton. This last deduction depends on the fact that both the excitation cross section and the excited state reciprocal lifetime are essentially proportional to the square of the same nuclear matrix element; analogously, both $d\sigma_{\text{Coul}}(\Lambda^0\rightarrow\Sigma^0)/d\Omega$ and $1/\tau(\Sigma^0)$ are essentially proportional to the square of the form factor $|F_1^{(\pm)}(0)|^2=(f^{(\pm)}e/2m_N)^2$ defined above, so that $1/\tau(\Sigma^0)$ can be expressed as a product of $d\sigma_{\text{Coul}}(\Lambda^0\rightarrow\Sigma^0)/d\Omega$ and immediately calculable quantities. Thus $1/\tau(\Sigma^0)$ can be obtained from an experimental study of $\Lambda^0\rightarrow\Sigma^0$ conversion in a Λ^0-nucleus collision provided that $d\sigma_{\text{Coul}}(\Lambda^0\rightarrow\Sigma^0)/d\Omega$ can be satisfactorily extracted from the directly observed $d\sigma_{\Lambda-\text{nucl}}(\Lambda^0\rightarrow\Sigma^0)/d\Omega$ which also contains a contribution to the $\Lambda^0\rightarrow\Sigma^0$ conversion process induced solely by the strong interactions. We shall show, however, that for small $|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|$:

$$\text{(a)}\quad \frac{d\sigma_{\text{Coul}}(\Lambda^0\rightarrow\Sigma^0)}{d\Omega}\gtrapprox\frac{d\sigma_{\text{strong}}(\Lambda^0\rightarrow\Sigma^0)}{d\Omega}$$
$$\equiv\left\{\frac{d\sigma_{\Lambda-\text{nucl}}(\Lambda^0\rightarrow\Sigma^0)}{d\Omega}-\frac{d\sigma_{\text{Coul}}(\Lambda^0\rightarrow\Sigma^0)}{d\Omega}\right\}$$

and

$$\text{(b)}\quad \frac{d\sigma_{\text{Coul}}(\Lambda^0\rightarrow\Sigma^0)}{d\Omega}\quad\text{and}\quad\frac{d\sigma_{\text{strong}}(\Lambda^0\rightarrow\Sigma^0)}{d\Omega}$$

have very different dependence on Z and on $|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|$, so that for small $|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|$ and high Z the nuclear Coulomb field contribution to the $\Lambda^0\rightarrow\Sigma^0$ conversion in a Λ^0-nucleus collision can be fairly readily identified and used to calculate $d\sigma_{\text{Coul}}(\Lambda^0\rightarrow\Sigma^0)/d\Omega$ and eventually $\tau(\Sigma^0)$.

We proceed to write down the matrix element for $\Lambda^0\rightarrow\Sigma^0$ conversion in a nuclear Coulomb field. From Eqs. (7) and (6a) we have, with

$$\begin{aligned}
&k_\rho\equiv(p_\Sigma-p_\Lambda)_\rho,\\
&k_\rho^2=|\mathbf{k}|^2-k_0^2=|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|^2-(E_\Sigma-E_\Lambda)^2,\\
&P_\Lambda P_\Sigma=+1,\\
&\langle\Sigma^0|j_\mu(0)|\Lambda^0\rangle\mathcal{A}_\mu(k_\lambda)\\
&\quad=\left[F_1^{(+)}(k_\rho^2)(\bar{u}_\Sigma\sigma_{\mu\nu}u_\Lambda)\frac{k_\nu}{k_\rho^2}\right.\\
&\qquad\left.+F_3^{(+)}(k_\rho^2)(\bar{u}_\Sigma\gamma_\mu u_\Lambda)\right]i4\pi J_\mu(k_\lambda),
\end{aligned}\qquad(25a)$$

[8] R. Behrends, Phys. Rev. 111, 1691 (1958).

$P_\Lambda P_\Sigma = -1,$

$$\langle\Sigma^0|j_\mu(0)|\Lambda^0\rangle\mathfrak{A}_\mu(k_\lambda) = \left[F_1^{(-)}(k_\rho^2)(\bar{u}_\Sigma\gamma_5\sigma_{\mu\nu}u_\Lambda)\frac{k_\nu}{k_\rho^2} + F_3^{(-)}(k_\rho^2)(\bar{u}_\Sigma\gamma_5\gamma_\mu u_\Lambda)\right]i4\pi J_\mu(k_\lambda), \quad (25b)$$

where

$$J_\mu(k_\lambda) = \delta_{\mu 4} iZe\int\rho(|\mathbf{x}|/R)e^{-i\mathbf{k}\cdot\mathbf{x}}d^3x = \delta_{\mu 4}iZe\rho(|\mathbf{k}|R), \quad (26)$$

$\rho(|\mathbf{x}|/R)$ being the normalized-to-unity ground-state distribution of charge within the nucleus of radius R, and $\rho(|\mathbf{k}|R) = \rho(|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|R)$ the corresponding nuclear form factor. With neglect of the kinetic energy of the recoiling nucleus so that $E_\Sigma = E_\Lambda$, we have

$$(k_\rho^2)^{\frac{1}{2}} = |\mathbf{p}_\Sigma - \mathbf{p}_\Lambda| = p_\Lambda\{\delta^2 + (1-\delta)[2\sin(\theta/2)]^2\}^{\frac{1}{2}} \cong p_\Lambda\{\delta^2+\theta^2\}^{\frac{1}{2}}, \quad (27a)$$

with

$$\frac{p_\Lambda - p_\Sigma}{p_\Lambda} \equiv \delta = 1 - \left[1 - \frac{(m_\Sigma^2 - m_\Lambda^2)}{p_\Lambda^2}\right]^{\frac{1}{2}} \cong \frac{1}{2}\frac{m_\Sigma^2 - m_\Lambda^2}{p_\Lambda^2}; \quad (27b)$$

$$\theta \equiv \cos^{-1}(\hat{p}_\Sigma\cdot\hat{p}_\Lambda).$$

Equations (27) show that the momentum transfer for $\Lambda^0 \to \Sigma^0$ conversion in a nuclear Coulomb field, $|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda| \cong p_\Lambda\{\delta^2+\theta^2\}^{\frac{1}{2}}$, is small compared to m_π for $0 \leqq \theta \lessapprox \delta$ and $\delta \cong \frac{1}{2}(m_\Sigma^2 - m_\Lambda^2)/p_\Lambda^2 \ll m_\pi/p_\Lambda$, the last condition always being satisfied for sufficiently large[9] p_Λ, e.g., for

$$p_\Lambda = 4m_\Lambda = 4.4 \text{ Bev}/c, \quad \delta \cong 0.0043, \quad m_\pi/p_\Lambda = 0.030.$$

Further, since[2] for small k_ρ^2,

$$F_1^{(\pm)}(k_\rho^2) \cong f^{(\pm)}(e/2m_N)[1 - (\tfrac{1}{6}r^2_{\Sigma-\Lambda})k_\rho^2],$$
$$F_3^{(\pm)}(k_\rho^2) \approx f^{(\pm)}e(r^2_{\Sigma-\Lambda}/6),$$

and since[2] $r^2_{\Sigma-\Lambda} \approx 1/2m_\pi^2$, we have

$$\frac{F_3^{(\pm)}(k_\rho^2)}{F_1^{(\pm)}(k_\rho^2)/(k_\rho^2)^{\frac{1}{2}}} \approx (\tfrac{1}{3}r^2_{\Sigma-\Lambda})\frac{(m_N|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|)}{(1 - \frac{1}{6}r^2_{\Sigma-\Lambda}|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|^2)} \approx 1.2(|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|/m_\pi), \quad (28)$$

and, as just seen, $|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|/m_\pi$ is small compared to unity for $0 \leqq \theta \lessapprox \delta$ and $p_\Lambda \gtrapprox 4m_\Lambda$. Thus under conditions of approximately forward $\Lambda^0 \to \Sigma^0$ conversion with high-energy incident Λ^0, we can write Eqs. (25) as

$P_\Lambda P_\Sigma = +1,$

$$\langle\Sigma^0|j_\mu(0)|\Lambda^0\rangle\mathfrak{A}_\mu(k_\lambda) \cong f^{(+)}\frac{e}{2m_N}(\bar{u}_\Sigma\boldsymbol{\alpha}u_\Lambda)\cdot\frac{(\mathbf{p}_\Sigma - \mathbf{p}_\Lambda)}{|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|^2}4\pi Ze\rho(|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|R), \quad (29a)$$

$P_\Lambda P_\Sigma = -1,$

$$\langle\Sigma^0|j_\mu(0)|\Lambda^0\rangle\mathfrak{A}_\mu(k_\lambda) \cong -f^{(-)}\frac{e}{2m_N}(\bar{u}_\Sigma\boldsymbol{\sigma}u_\Lambda)\cdot\frac{(\mathbf{p}_\Sigma - \mathbf{p}_\Lambda)}{|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|^2}4\pi Ze\rho(|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|R), \quad (29b)$$

where the importance of $\langle\Sigma^0|j_\mu(0)|\Lambda^0\rangle$ at small $\Lambda^0 \to \Sigma^0$ momentum transfers, through its proportionality to the photon propagator $1/k_\rho^2 = 1/|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|^2$, is explicitly shown. Physically, this importance is a consequence of the long range character of the nuclear Coulomb field which permits peripheral collisions and which for small $|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|$ is composed coherently from the Coulomb fields of the individual nuclear protons $[4\pi Ze\rho(|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|R) \cong 4\pi Ze$ for $|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|R = |\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|0.8A^{\frac{1}{3}}/m_\pi \ll 1]$.

For larger θ, where $|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|$ becomes $\gtrapprox m_\pi$ and the Λ^0-nucleus collisions are definitely nonperipheral, the terms in $F_3^{(\pm)}$ must be included in Eqs. (29) and the dependence of $F_1^{(\pm)}$, $F_3^{(\pm)}$ on k_ρ^2 considered. However for such large $|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|$ the strong interactions dominate the $\Lambda^0 \to \Sigma^0$ conversion and the contribution of the nuclear Coulomb field matrix element $\langle\Sigma^0|j_\mu(0)|\Lambda^0\rangle\mathfrak{A}_\mu(k_\lambda)$ is, in any case, relatively negligible.

We now use the matrix elements of Eqs. (29) to calculate $d\sigma_{\text{Coul}}(\Lambda^0 \to \Sigma^0)/d\Omega$. Analogously to Eq. (11a) we have

$$\frac{d\sigma^{(\pm)}{}_{\text{Coul}}(\Lambda^0 \to \Sigma^0)}{d\Omega} = \int 2\pi\delta(E_\Sigma - E_\Lambda)\tfrac{1}{2} \times \sum_{s_\Sigma s_\Lambda}|\langle\Sigma^0|j_\mu(0)|\Lambda^0\rangle\mathfrak{A}_\mu(k_\lambda)|^2\frac{d^3p_\Sigma}{(2\pi)^3}\bigg/\left(\frac{p_\Lambda}{E_\Lambda}\right), \quad (30)$$

so that, substituting Eqs. (29) into Eq. (30) and performing the indicated operations:

$$\frac{d\sigma^{(\pm)}{}_{\text{Coul}}(\Lambda^0 \to \Sigma^0)}{d\Omega} = 4Z^2e^2\left(\frac{f^{(\pm)}e}{2m_N}\right)^2\left[\frac{(\mathbf{p}_\Sigma\times\mathbf{p}_\Lambda)^2}{|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|^4} + \frac{\frac{1}{4}(m_\Sigma \mp m_\Lambda)^2}{|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|^2} + \tfrac{1}{4}\right] \times\left(\frac{p_\Sigma}{p_\Lambda}\right)[\rho(|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|R)]^2, \quad (31a)$$

[9] It is worth mentioning explicitly that $|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|_{\theta=0} = p_\Lambda\delta \cong \frac{1}{2}[(m_\Sigma^2 - m_\Lambda^2)/p_\Lambda]$ approaches zero at large p_Λ as $(p_\Lambda)^{-1}$.

or, using Eqs. (27),

$$
\begin{aligned}
\frac{d\sigma^{(\pm)}{}_{\mathrm{Coul}}(\Lambda^0\to\Sigma^0)}{d\Omega}
&=4Z^2e^2\left(\frac{f^{(\pm)}e}{2m_N}\right)^2\left[\frac{(1-\delta)^2(\sin\theta)^2}{[\delta^2+(1-\delta)(2\sin\frac{1}{2}\theta)^2]^2}\right.\\
&\quad\left.+\frac{\frac{1}{4}(m_\Sigma\mp m_\Lambda)^2/p_\Lambda^2}{[\delta^2+(1-\delta)(2\sin\frac{1}{2}\theta)^2]}+\frac{1}{4}\right]\\
&\qquad\times(1-\delta)[\rho(|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|R)]^2\\
&\cong 4Z^2e^2\left(\frac{f^{(\pm)}e}{2m_N}\right)^2\\
&\quad\times\left[\frac{\theta^2}{(\delta^2+\theta^2)^2}+\tfrac{1}{2}\delta\left(\frac{m_\Sigma-m_\Lambda}{m_\Sigma+m_\Lambda}\right)^{\pm1}\Big/(\delta^2+\theta^2)\right]\\
&\qquad\times[\rho(|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|R)]^2, \qquad (31\text{b})
\end{aligned}
$$

the $\pm$ signs referring to the two possible Λ^0, Σ^0 relative intrinsic parities: $P_\Lambda P_\Sigma=\pm1$. Expressing, via Eqs. (11), the $\Lambda^0\to\Sigma^0$ transition moment $(f^{(\pm)}e/2m_N)$ in terms of the Σ^0 decay reciprocal lifetime, $1/\tau(\Sigma^0)$, Eq. (31b) becomes

$$
\begin{aligned}
\frac{d\sigma^{(\pm)}{}_{\mathrm{Coul}}(\Lambda^0\to\Sigma^0)}{d\Omega}
&\cong\left(\frac{Z^2e^2}{(m_\Sigma-m_\Lambda)^2}\,\frac{1/\tau(\Sigma^0)}{(m_\Sigma-m_\Lambda)}\right)\\
&\quad\times\left[\frac{\theta^2}{(\delta^2+\theta^2)^2}+\tfrac{1}{2}\delta\left(\frac{m_\Sigma-m_\Lambda}{m_\Sigma+m_\Lambda}\right)^{\pm1}\Big/(\delta^2+\theta^2)\right]\\
&\qquad\times[\rho(|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|R)]^2, \qquad (32)
\end{aligned}
$$

the proportionality of $d\sigma^{(\pm)}{}_{\mathrm{Coul}}(\Lambda^0\to\Sigma^0)/d\Omega$ to $1/\tau(\Sigma^0)$ being explicitly shown. It is to be emphasized that $d\sigma^{(\pm)}{}_{\mathrm{Coul}}(\Lambda^0\to\Sigma^0)/d\Omega$ peaks strongly in the approximately forward directions, i.e., as a function of θ, $d\sigma^{(\pm)}{}_{\mathrm{Coul}}(\Lambda^0\to\Sigma^0)/d\Omega$ has a sharp maximum at[10]

$$
\theta=\delta\left\{\left[1-\tfrac{1}{2}\delta\left(\frac{m_\Sigma-m_\Lambda}{m_\Sigma+m_\Lambda}\right)^{\pm1}\right]\Big/\left[1+\tfrac{1}{2}\delta\left(\frac{m_\Sigma-m_\Lambda}{m_\Sigma+m_\Lambda}\right)^{\pm1}\right]\right\}\lesssim\delta\ll1,
$$

and is relatively very small at $\theta\gg\delta$. It is amusing to mention that in the strictly forward direction Eq. (32) yields:

$$
\left\{\frac{d\sigma^{(-)}{}_{\mathrm{Coul}}(\Lambda^0\to\Sigma^0)/d\Omega}{d\sigma^{(+)}{}_{\mathrm{Coul}}(\Lambda^0\to\Sigma^0)/d\Omega}\right\}_{\theta=0}\cong\left(\frac{m_\Sigma+m_\Lambda}{m_\Sigma-m_\Lambda}\right)^2=950, \qquad (33)
$$

thus offering an enormous discrimination between results expected for negative and for positive Λ^0, Σ^0 relative intrinsic parities.

Let us now assume that

$$
\begin{aligned}
&\rho(|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|R)\\
&\quad\approx\frac{\sin(|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|R)}{(|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|R)}\cong\frac{\sin[p_\Lambda\delta R(1+\theta^2/\delta^2)^{\frac{1}{2}}]}{[p_\Lambda\delta R(1+\theta^2/\delta^2)^{\frac{1}{2}}]}\\
&\quad\approx\frac{\sin(p_\Lambda\delta R\sqrt{2})}{(p_\Lambda\delta R\sqrt{2})};\quad 0\leqq\theta/\delta\leqq\pi/p_\Lambda\delta R;\quad R\cong0.8A^{\frac{1}{3}}/m_\pi\\
&\quad\approx0;\qquad\qquad \pi/p_\Lambda\delta R<\theta/\delta, \qquad (34)
\end{aligned}
$$

so that we are using the nuclear form factor appropriate to **E** vanishing within the nucleus and hence essentially excluding any Coulomb field $\Lambda^0\to\Sigma^0$ conversion in which the Λ^0 is born inside the nucleus. Substituting Eq. (34) into Eq. (32) we obtain, using Eq. (27b) for δ,

$$
\begin{aligned}
\frac{d\sigma^{(\pm)}{}_{\mathrm{Coul}}(\Lambda^0\to\Sigma^0)}{d\Omega}
&\cong\left[\left(\frac{70\text{ mb}}{\text{sr}}\right)\left(\frac{Z}{82}\right)^2\left(\frac{10^{-19}\text{ sec}}{\tau(\Sigma^0)}\right)x^4\right]\\
&\quad\times\left[\frac{(\theta/\delta)^2}{[1+(\theta/\delta)^2]^2}+\frac{\frac{1}{2}(0.07/x^2)(1/30)^{\pm1}}{[1+(\theta/\delta)^2]}\right]\\
&\quad\times\left[\frac{\sin(0.65A^{\frac{1}{3}}/x)}{(0.65A^{\frac{1}{3}}/x)}\right]^2;\quad 0\leqq\theta/\delta\leqq6.8x/A^{\frac{1}{3}}\\
&\cong0;\qquad\qquad 6.8x/A^{\frac{1}{3}}<\theta/\delta, \qquad (35\text{a})
\end{aligned}
$$

where

$$
x\equiv p_\Lambda/m_\Lambda;\quad \delta\cong0.07/x^2, \qquad (35\text{b})
$$

so that the peak differential cross section for nuclear Coulomb field $\Lambda^0\to\Sigma^0$ conversion is, taking[11] $Z=82$, $A^{\frac{1}{3}}=6$, $\tau(\Sigma^0)=(g_\Lambda{}^{(\pm)}g_\Sigma/g_N{}^2)^{-2}$ 10^{-19} sec [Eq. (11d)], $x=4$:

$$
\left\{\frac{d\sigma^{(+)}{}_{\mathrm{Coul}}(\Lambda^0\to\Sigma^0)}{d\Omega}\right\}_{\theta=\delta\cong0.004}\cong\left(\frac{g_\Lambda{}^{(+)}g_\Sigma}{g_N{}^2}\right)^2 3.4\text{ barns/sr}, \qquad (36\text{a})
$$

$$
\left\{\frac{d\sigma^{(-)}{}_{\mathrm{Coul}}(\Lambda^0\to\Sigma^0)}{d\Omega}\right\}_{\theta=\delta\cong0.004}\cong\left(\frac{g_\Lambda{}^{(-)}g_\Sigma}{g_N{}^2}\right)^2 4.2\text{ barns/sr}. \qquad (36\text{b})
$$

[10] In working out the position of this maximum, $\rho(|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|R)=\rho[p_\Lambda(\delta^2+\theta^2)^{\frac{1}{2}}R]$ can be treated as independent of θ since its actual rate of variation with θ is considerably slower than that of the other θ-dependent factor in Eq. (32) for $d\sigma^{(\pm)}{}_{\mathrm{Coul}}(\Lambda^0\to\Sigma^0)/d\Omega$.

[11] The choice of $Z=82$ in our numerical illustrations is motivated by the possibility of use of, e.g., a spark chamber with lead plates for the observation of the $\Lambda^0\to\Sigma^0$ conversion. The mean flight path before decay of a Λ^0 with $x=4$ is about 30 cm.

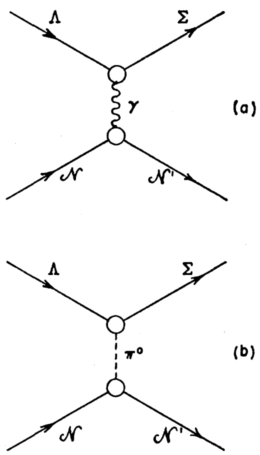

FIG. 2. Production of a Σ^0 from a Λ^0 in a Λ^0-nucleus collision. (a) is the Coulomb, (b) the competing strong-interaction-induced process.

Finally, the Eqs. (35) show that the total cross section for nuclear Coulomb field $\Lambda^0 \to \Sigma^0$ conversion is essentially all contained in a forward cone of opening angle

$$\theta_c \approx \delta(6.8x/A^{\frac{1}{3}}) \cong (0.5/xA^{\frac{1}{3}}) \qquad (=1.2° \text{ for } x=4,\ A^{\frac{1}{3}}=6), \quad (37a)$$

and is given numerically by

$$\sigma^{(\pm)}{}_{\mathrm{Coul}}(\Lambda^0 \to \Sigma^0) \cong \int_0^{0.5/xA^{\frac{1}{3}}} [d\sigma^{(\pm)}{}_{\mathrm{Coul}}(\Lambda^0\to\Sigma^0)/d\Omega] 2\pi\theta d\theta$$
$$= \left\{(2.3\ \mathrm{mb})\left(\frac{Z}{82}\right)^2\left(\frac{10^{-19}\ \mathrm{sec}}{\tau(\Sigma^0)}\right)\left[\frac{\sin(0.65A^{\frac{1}{3}}/x)}{(0.65A^{\frac{1}{3}}/x)}\right]^2\right\}$$
$$\times\left\{\left[1+\frac{1}{2}\left(\frac{0.07}{x^2}\right)\left(\frac{1}{30}\right)^{\pm 1}\right]\ln\left[1+\left(\frac{6.8x}{A^{\frac{1}{3}}}\right)^2\right]^{\frac{1}{2}} - \frac{1}{2}\frac{(6.8x/A^{\frac{1}{3}})^2}{(6.8x/A^{\frac{1}{3}})^2+1}\right\}, \quad (37b)$$

which for $Z=82$, $A^{\frac{1}{3}}=6$, $\tau(\Sigma^0)=(g_\Lambda{}^{(\pm)}g_\Sigma/g_N{}^2)^{-2}\times 10^{-19}$ sec [Eq. (11d)], $x=4$, is

$$\sigma_{\mathrm{Coul}}{}^{(+)}(\Lambda^0\to\Sigma^0) \cong (g_\Lambda{}^{(+)}g_\Sigma/g_N{}^2)^2(1.8\ \mathrm{mb});$$
$$\sigma_{\mathrm{Coul}}{}^{(-)}(\Lambda^0\to\Sigma^0) \cong (g_\Lambda{}^{(-)}g_\Sigma/g_N{}^2)^2(2.0\ \mathrm{mb}). \quad (37c)$$

Equations (35)–(37) are the basis of our expectation that the $\Lambda^0 \to \Sigma^0$ conversion process in a high-Z nuclear Coulomb field is governed by cross sections of the respectable order of magnitude of millibarns. Results comparable to those in Eqs. (25)–(37) have also been obtained by Pomeranchuk and Shmushkevitch,[12] by Williams,[12] and by Valuev[12] in publications received after the present paper was completed.

[12] I. Ya Pomeranchuk and I. M. Shmushkevitch, Nuclear Phys. **23**, 452 (1961); W. S. C. Williams, Nuovo cimento **19**, 1278 (1961); B. Valuev, Dubna preprint.

We now consider the question of the size of the background to the Coulomb field $\Lambda^0 \to \Sigma^0$ conversion, viz., the $\Lambda^0 \to \Sigma^0$ conversion induced by the strong interactions between the incident Λ^0 and the target nucleus. The particular strong interaction which is most important to the $\Lambda^0 \to \Sigma^0$ conversion at small $|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|$ is, in all likelihood, the exchange of a single virtual π^0 between the Λ^0 and the nucleus (see Fig. 2); this follows since small $|\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|$ correspond to large Λ^0-nucleus impact parameters, i.e., to peripheral collisions, for which the relatively long range of the virtual π^0 should prove decisive. More mathematically, one can say that for small real $(k_\rho{}^2)^{\frac{1}{2}}$ the strong interaction induced matrix element is dominated by the pole at $k_\rho{}^2 = -m_\pi{}^2$ in the π^0 propagator $1/(k_\rho{}^2+m_\pi{}^2)$ so that the dependence of this matrix element on $(k_\rho{}^2)^{\frac{1}{2}} = |\mathbf{k}| = |\mathbf{p}_\Sigma - \mathbf{p}_\Lambda|$ is essentially determined.

Granting the validity of the above arguments, it becomes necessary to estimate the matrix element for $\Lambda^0 \to \Sigma^0$ conversion due to the exchange of a single virtual π^0 between the colliding Λ^0 and nucleus. Analogously to the procedure of Eqs. (1)–(7), (25), (26) and (29), we may write:

$$P_\Lambda P_\Sigma = +1,$$
$$\langle \Sigma^0, \Psi_f | T_{\mathrm{strong}} | \Lambda^0, \Psi_i \rangle \approx g_\Lambda{}^{(+)}(\bar{u}_\Sigma \gamma_5 u_\Lambda)\phi(k_\lambda) = g_\Lambda{}^{(+)}(\bar{u}_\Sigma\gamma_5 u_\Lambda)\frac{4\pi J(\mathbf{k})}{|\mathbf{k}|^2+m_\pi{}^2}, \quad (38a)$$
$$P_\Lambda P_\Sigma = -1,$$
$$\langle \Sigma^0, \Psi_f | T_{\mathrm{strong}} | \Lambda^0, \Psi_i \rangle \approx g_\Lambda{}^{(-)}(\bar{u}_\Sigma u_\Lambda)\phi(k_\lambda) = g_\Lambda{}^{(-)}(\bar{u}_\Sigma u_\Lambda)\frac{4\pi J(\mathbf{k})}{|\mathbf{k}|^2+m_\pi{}^2}, \quad (38b)$$

with

$$J(k_\lambda) = J(\mathbf{k}) = \int J(\mathbf{x})e^{-i\mathbf{k}\cdot\mathbf{x}}d^3x$$
$$= g_N\int\left\{\int \Psi_f{}^\dagger\left[\sum_{l=1}^{A}\delta(\mathbf{x}-\mathbf{x}_l)(\gamma_4\gamma_5\tau_3)_l\right] \times \Psi_i d^3x_1\cdots d^3x_A\right\}e^{-i\mathbf{k}\cdot\mathbf{x}}d^3x$$
$$\cong g_N\frac{\mathbf{k}}{2m_N}\cdot\int\left\{\int \Psi_f{}^\dagger\left[\sum_{l=1}^{A}\delta(\mathbf{x}-\mathbf{x}_l)(\boldsymbol{\sigma}\tau_3)_l\right] \times \Psi_i d^3x_1\cdots d^3x_A\right\}e^{-i\mathbf{k}\cdot\mathbf{x}}d^3x$$
$$= g_N\frac{\mathbf{k}}{2m_N}\cdot\int \psi_0{}^\dagger\left[\sum_{l=1}^{A} e^{-i\mathbf{k}\cdot\mathbf{y}_l}(\boldsymbol{\sigma}\tau_3)_l\right] \times \psi_0\delta\left(\sum_{l=1}^{A}\mathbf{y}_l\right)d^3y_1\cdots d^3y_A, \quad (38c)$$

where T_{strong} is the transition operator of the strong interactions; $\phi(k_\lambda)$, $J(k_\lambda)$ is the amplitude and source function (in momentum space) of the π^0 field due to the target nucleus; $\Psi_i=\psi_0$, $\Psi_f=\psi_0\exp[-i\mathbf{k}\cdot\sum_{l=1}^A \mathbf{x}_l/A]$ are ground-state wavefunctions of the target nucleus ($\mathbf{y}_m\equiv\mathbf{x}_m-\sum_{l=1}^A\mathbf{x}_l/A$) with center of mass initially at rest and finally recoiling with momentum $[-\mathbf{k}]=[-(\mathbf{p}_\Sigma-\mathbf{p}_\Lambda)]$. From Eq. (38c) we have

$$J(\mathbf{k})=0:\ \text{target nucleus isospin and/or spin}=0,\quad (38d)$$

$$J(\mathbf{k})\approx\frac{g_N|\mathbf{k}|}{2m_N}\{\tfrac{1}{3}\langle(2\mathbf{S}_{\text{prot}}-2\mathbf{S}_{\text{neut}})^2\rangle\}^{\frac{1}{2}}\rho(|\mathbf{k}|R)$$
$$\approx\frac{g_N|\mathbf{k}|}{2m_N}\rho(|\mathbf{k}|R):$$
$$\text{target nucleus isospin and spin}\neq0,\quad (38e)$$

where $\rho(|\mathbf{k}|R)$ is given in Eqs. (26) and (34). The estimate of Eq. (38e) is justified if one remembers that $\sum_{l=1}^A(\boldsymbol{\sigma}\tau_3)_l=2\mathbf{S}_{\text{prot}}-2\mathbf{S}_{\text{neut}}$, where $\mathbf{S}_{\text{prot}}$ ($\mathbf{S}_{\text{neut}}$) is the resultant spin of all the target nucleus's protons (neutrons), and that the distributions in space of the target nucleus's protons and neutrons are very similar. Thus, combining Eqs. (38a), (38b), and (38e), we can describe the most unfavorable background situation by the matrix elements:

$$P_\Lambda P_\Sigma=+1,$$
$$\langle\Sigma^0,\Psi_f|T_{\text{strong}}|\Psi_i,\Lambda^0\rangle\approx\left[\frac{g_\Lambda{}^{(+)}g_N|\mathbf{k}|}{2m_N}\right]\times(\bar{u}_\Sigma\gamma_5u_\Lambda)\frac{4\pi\rho(|\mathbf{k}|R)}{|\mathbf{k}|^2+m_\pi{}^2},\quad (39a)$$

$$P_\Lambda P_\Sigma=-1,$$
$$\langle\Sigma^0,\Psi_f|T_{\text{strong}}|\Psi_i,\Lambda^0\rangle\approx\left[\frac{g_\Lambda{}^{(-)}g_N|\mathbf{k}|}{2m_N}\right]\times(\bar{u}_\Sigma u_\Lambda)\frac{4\pi\rho(|\mathbf{k}|R)}{|\mathbf{k}|^2+m_\pi{}^2}.\quad (39b)$$

Equations (39) yield, following a procedure analogous to that of Eqs. (29)–(31), and with $\delta\cong\frac{1}{2}(m_\Sigma{}^2-m_\Lambda{}^2)/p_\Lambda{}^2$, $0\leqq\theta\lessapprox\delta\ll m_\pi/p_\Lambda$ [Eq. (27b) *et seq.*]:

$$\frac{d\sigma_{\text{strong}}{}^{(+)}(\Lambda^0\to\Sigma^0)}{d\Omega}\approx\left(\frac{g_\Lambda{}^{(+)}}{g_N}\right)^2\left(\frac{g_N{}^4}{m_N{}^2}\right)\left[\frac{(\delta^2+\theta^2)\theta^2/4}{(\delta^2+\theta^2+m_\pi{}^2/p_\Lambda{}^2)^2}\right]\times[\rho(|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|R)]^2,\quad (40a)$$

$$\frac{d\sigma_{\text{strong}}{}^{(-)}(\Lambda^0\to\Sigma^0)}{d\Omega}\approx\left(\frac{g_\Lambda{}^{(-)}}{g_N}\right)^2\left(\frac{g_N{}^4}{m_N{}^2}\right)\left(\frac{m_\Lambda}{p_\Lambda}\right)^2\left[\frac{\delta^2+\theta^2}{(\delta^2+\theta^2+m_\pi{}^2/p_\Lambda{}^2)^2}\right]\times[\rho(|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|R)]^2,\quad (40b)$$

with $\rho(|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|)$ given by Eqs. (34) and (35b). It is to be noted that on the basis of Eqs. (29) and (39):

$$\sum_{s_\Sigma s_\Lambda}\mathfrak{A}_\mu(k_\lambda)\langle\Sigma^0|j_\mu(0)|\Lambda^0\rangle\langle\Sigma^0,\Psi_f|T_{\text{strong}}|\Psi_i,\Lambda^0\rangle^*=0,\quad (41)$$

so that

$$\frac{d\sigma_{\Lambda-\text{nucl}}{}^{(\pm)}(\Lambda^0\to\Sigma^0)}{d\Omega}=\frac{d\sigma_{\text{Coul}}{}^{(\pm)}(\Lambda^0\to\Sigma^0)}{d\Omega}+\frac{d\sigma_{\text{strong}}{}^{(\pm)}(\Lambda^0\to\Sigma^0)}{d\Omega},\quad (42)$$

without any interference term. Comparison of Eqs. (40) with Eq. (31b) or Eq. (32) shows that $d\sigma_{\text{strong}}{}^{(\pm)}(\Lambda^0\to\Sigma^0)/d\Omega$ has a very different dependence on θ for $0\leqq\theta\ll m_\pi/p_\Lambda$ than does $d\sigma_{\text{Coul}}{}^{(\pm)}(\Lambda^0\to\Sigma^0)/d\Omega$, the latter peaking much more sharply than the former in the approximately forward directions essentially because of the difference between the photon and π^0 propagators: $1/p_\Lambda{}^2(\delta^2+\theta^2)$ and $1/[p_\Lambda{}^2(\delta^2+\theta^2)+m_\pi{}^2]$. It also follows from Eqs. (40) and Eq. (37a) that for $0\leqq\theta\lessapprox\theta_c$:

$$\left(\frac{d\sigma_{\text{strong}}{}^{(+)}(\Lambda^0\to\Sigma^0)}{d\Omega}\bigg/\frac{d\sigma_{\text{strong}}{}^{(-)}(\Lambda^0\to\Sigma^0)}{d\Omega}\right)$$
$$\approx(g_\Lambda{}^{(+)}/g_\Lambda{}^{(-)})^2(p_\Lambda/m_\Lambda)^2(\theta^2/4)$$
$$\lessapprox(g_\Lambda{}^{(+)}/g_\Lambda{}^{(-)})^2(0.06)/A^{\frac{2}{3}}$$
$$=2\times10^{-3}(g_\Lambda{}^{(+)}/g_\Lambda{}^{(-)})^2,\quad (A^{\frac{1}{3}}=6).\quad (43a)$$

Since according to Eqs. (35)–(37):

$$\left(\frac{d\sigma_{\text{Coul}}{}^{(+)}(\Lambda^0\to\Sigma^0)}{d\Omega}\bigg/\frac{d\sigma_{\text{Coul}}{}^{(-)}(\Lambda^0\to\Sigma^0)}{d\Omega}\right)\cong(g_\Lambda{}^{(+)}/g_\Lambda{}^{(-)})^2,\quad (43b)$$

it is clear that the strong interaction background to the Coulomb field $\Lambda^0\to\Sigma^0$ conversion is far more serious in the case $P_\Lambda P_\Sigma=-1$ than in the case $P_\Lambda P_\Sigma=+1$.

We proceed to calculate the total cross section for strong-interaction-induced $\Lambda^0\to\Sigma^0$ conversion. On the basis of Eqs. (40) and (34), this, like the Coulomb field $\Lambda^0\to\Sigma^0$ conversion, is essentially all confined by the cutoff property of the nuclear form factor $\rho(|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|R)$ in a forward cone of opening angle

$\theta_c \approx \delta(6.8x/A^{\frac{1}{3}}) \cong 0.5/(xA^{\frac{1}{3}})$ ($=1.2°$ for $x=4$, $A^{\frac{1}{3}}=6$). [See Eqs. (37a) and (35b).] We obtain, with $g_N{}^2=14$:

$$\sigma_{\rm strong}{}^{(\pm)}(\Lambda^0 \to \Sigma^0) \cong \int_0^{0.5/xA^{\frac{1}{3}}} \frac{d\sigma_{\rm strong}{}^{(\pm)}(\Lambda^0\to\Sigma^0)}{d\Omega} 2\pi\theta d\theta$$

$$\approx (1\times10^{-2}\,{\rm mb})\left(\frac{g_\Lambda{}^{(+)}}{g_N}\right)^2\left(\frac{1}{x}\right)^2\left(\frac{6.8}{A^{\frac{1}{3}}}\right)^6 \times\left[\frac{\sin(0.65A^{\frac{1}{3}}/x)}{(0.65A^{\frac{1}{3}}/x)}\right]^2 \quad (44a)$$

$$\approx (6\,{\rm mb})\left(\frac{g_\Lambda{}^{(-)}}{g_N}\right)^2\left(\frac{1}{x}\right)^2\left(\frac{6.8}{A^{\frac{1}{3}}}\right)^4 \times\left[\frac{\sin(0.65A^{\frac{1}{3}}/x)}{(0.65A^{\frac{1}{3}}/x)}\right]^2, \quad (44b)$$

$$[\sigma_{\rm strong}{}^{(+)}(\Lambda^0\to\Sigma^0)/\sigma_{\rm strong}{}^{(-)}(\Lambda^0\to\Sigma^0)] \approx 2\times10^{-3}(g_\Lambda{}^{(+)}/g_\Lambda{}^{(-)})^2(6.8/A^{\frac{1}{3}})^2, \quad (44c)$$

whence for $x=4$, $A^{\frac{1}{3}}=6$:

$$\sigma_{\rm strong}{}^{(+)}(\Lambda^0\to\Sigma^0)\approx(1.0\times10^{-3}\,{\rm mb})(g_\Lambda{}^{(+)}/g_N)^2, \quad (44d)$$

$$\sigma_{\rm strong}{}^{(-)}(\Lambda^0\to\Sigma^0)\approx(0.5\,{\rm mb})(g_\Lambda{}^{(-)}/g_N)^2. \quad (44e)$$

Thus, comparison with Eq. (37c) yields, if one takes $g_\Sigma=g_N$ ($Z=82$, $A^{\frac{1}{3}}=6$, $x=4$):

$$[\sigma_{\rm Coul}{}^{(+)}(\Lambda^0\to\Sigma^0)/\sigma_{\rm strong}{}^{(+)}(\Lambda^0\to\Sigma^0)]\approx1800, \quad (44f)$$

$$[\sigma_{\rm Coul}{}^{(-)}(\Lambda^0\to\Sigma^0)/\sigma_{\rm strong}{}^{(-)}(\Lambda^0\to\Sigma^0)]\approx4, \quad (44g)$$

so that the strong interaction induced $\Lambda^0\to\Sigma^0$ conversion is quite unimportant if $P_\Lambda P_\Sigma=+1$ and is far from dominant even if $P_\Lambda P_\Sigma=-1$. It must also be emphasized that the $\Lambda^0\to\Sigma^0$ conversion experiment should be performed with targets of different Z, and incident Λ^0 of different[13] $p_\Lambda(\equiv xm_\Lambda)$, in order to identify properly the contributions of $\sigma_{\rm Coul}{}^{(+)}(\Lambda^0\to\Sigma^0)$ and $\sigma_{\rm strong}{}^{(+)}(\Lambda^0\to\Sigma^0)$ or of $\sigma_{\rm Coul}{}^{(-)}(\Lambda^0\to\Sigma^0)$ and $\sigma_{\rm strong}{}^{(-)}(\Lambda^0\to\Sigma^0)$. Finally, we should mention that at least in principle, one will be able to decide on the basis of the variation with p_Λ whether the expression for

$$\sigma_{\Lambda-{\rm nucl}}{}^{(+)}(\Lambda^0\to\Sigma^0)=\sigma_{\rm Coul}{}^{(+)}(\Lambda^0\to\Sigma^0)+\sigma_{\rm strong}{}^{(+)}(\Lambda^0\to\Sigma^0)$$

or for

$$\sigma_{\Lambda-{\rm nucl}}{}^{(-)}(\Lambda^0\to\Sigma^0)=\sigma_{\rm Coul}{}^{(-)}(\Lambda^0\to\Sigma^0)+\sigma_{\rm strong}{}^{(-)}(\Lambda^0\to\Sigma^0)$$

agrees better with the data, i.e., decide on the basis of the variation with p_Λ of the experimental $\sigma_{\Lambda-{\rm nucl}}(\Lambda^0\to\Sigma^0)$ whether $P_\Sigma P_\Lambda=+1$ or -1.

The previous discussion dealt with the strong interaction induced $\Lambda^0\to\Sigma^0$ conversion in which the Λ^0 interacts with the target nucleus as a whole, the latter recoiling in its ground state with relatively negligible kinetic energy, i.e., dealt with *coherent* strong interaction induced $\Lambda^0\to\Sigma^0$ conversion. We must now estimate the order of magnitude of the *incoherent* strong interaction induced $\Lambda^0\to\Sigma^0$ conversion which contributes to the background of our Coulomb field $\Lambda^0\to\Sigma^0$ conversion, i.e., we must estimate the strong-interaction-induced differential cross section in approximately forward directions of the reaction: Λ^0+nucleus in ground state $\psi_0\to\Lambda^0$+nucleus in excited state ψ_n. From Eqs. (38a)–(38c) this cross section is proportional to

$$|M|^2\equiv\sum_n\left|\frac{g_N(\mathbf{k}/2m_N)}{|\mathbf{k}|^2+m_\pi{}^2}\cdot\int\psi_n{}^\dagger\{\sum_{l=1}^A e^{-i\mathbf{k}\cdot\mathbf{y}_l}(\boldsymbol{\sigma}\tau_3)_l\}\psi_0 \times\delta(\sum_{l=1}^A\mathbf{y}_l)d^3y_1\cdots d^3y_A\right|^2(1-\delta_{n0}), \quad (45a)$$

where the sum over n runs over all energetically accessible states and may, without appreciable error, be extended to run over all states; also we have neglected the effect of the excitation of the nucleus on $\mathbf{p}_\Sigma-\mathbf{p}_\Lambda$ and on $E_\Sigma-E_\Lambda$, so that we are still taking[14]

$$|\mathbf{k}|^2=|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|^2\cong p_\Lambda{}^2(\delta^2+\theta^2),\quad k_0{}^2=(E_\Sigma-E_\Lambda)^2\cong0.$$

Use of closure over the ψ_n then yields

$$|M|^2\cong\left[\left(\frac{g_N}{2m_N}\right)^2\Big/(|\mathbf{k}|^2+m_\pi{}^2)^2\right] \times\Big\{\int\psi_0{}^\dagger[\sum_{l,m=1}^A e^{-i\mathbf{k}\cdot\mathbf{y}_{lm}}(\boldsymbol{\sigma}\tau_3)_l\cdot\mathbf{k}(\boldsymbol{\sigma}\tau_3)_m\cdot\mathbf{k}]\psi_0 \times\delta(\sum_{l=1}^A\mathbf{y}_l)d^3y_1\cdots d^3y_A - \left|\int\psi_0{}^\dagger[\sum_{l=1}^A e^{-i\mathbf{k}\cdot\mathbf{y}_l}(\boldsymbol{\sigma}\tau_3)_l\cdot\mathbf{k}]\psi_0 \times\delta(\sum_{l=1}^A\mathbf{y}_l)d^3y_1\cdots d^3y_A\right|^2\Big\}, \quad (45b)$$

[13] It is worth mentioning that one will be able to determine x by kinematic reconstruction, since in each particular $\Lambda^0\to\Sigma^0$ conversion with $\Sigma^0\to(\Lambda^0)'+\gamma$, the energy of the γ and of the p, π^- decay products of the $(\Lambda^0)'$ will presumably be available, i.e., $[(xm_\Lambda)^2+(m_\Lambda)^2]^{\frac{1}{2}}=E(\Sigma^0)=E[(\Lambda^0)']+E(\gamma)=E(p)+E(\pi^-)+E(\gamma)$.

[14] We actually have, taking the excitation energy of the nucleus ϵ_n into account:
$$k_\rho{}^2=|\mathbf{k}|^2-k_0{}^2=|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|^2-(E_\Sigma-E_\Lambda)^2 \cong[p_\Lambda{}^2\delta^2(1+2\epsilon_n/p_\Lambda\delta)+\epsilon_n{}^2+p_\Lambda{}^2\theta^2]-\epsilon_n{}^2=p_\Lambda{}^2\delta^2(1+2\epsilon_n/p_\Lambda\delta)+p_\Lambda{}^2\theta^2.$$
[compare Eqs. (27)]. The correction factor to the value of $(k_\rho{}^2)_{\theta=0}$: $1+2\epsilon_n/p_\Lambda\delta=1+(2\epsilon_n/m_\Lambda)[x/(7\times10^{-2})]$ [*cf.* Eq. (35b)] is, numerically, equal to 1.5 for $\epsilon_n=5$ Mev, $x=4$; inclusion of this correction does not change the order of magnitude of our estimate for $|M|^2$ in Eqs. (45b) and (45c).

and making the same kind of estimate as that in Eqs. (38c)–(38e), we have

$$|M|^2\approx\left[\left(\frac{g_N}{2m_N}|\mathbf{k}|\right)^2\Big/(|\mathbf{k}|^2+m_\pi{}^2)^2\right]$$
$$\times\tfrac{1}{3}\langle(2\mathbf{S}_{\mathrm{prot}}-2\mathbf{S}_{\mathrm{neut}})^2\rangle\{1-[\rho(|\mathbf{k}|R)]^2\}$$
$$\approx\left[\left(\frac{g_N}{2m_N}|\mathbf{k}|\right)^2\Big/(|\mathbf{k}|^2+m_\pi{}^2)^2\right]$$
$$\times\{1-[\rho(|\mathbf{k}|R)]^2\}. \quad (45c)$$

Hence, comparing Eq. (45c) with Eqs. (39) and considering the general procedure of Eqs. (38)–(40), we obtain for the total incoherent cross-section within the forward cone of opening angle $\theta_c[\approx\delta(6.8x/A^{\frac{1}{3}})\cong 0.5/xA^{\frac{1}{3}}$—Eq. (37a)]

$$\{[\sigma_{\mathrm{strong}}{}^{(\pm)}(\Lambda^0\to\Sigma^0)]_{\mathrm{incoher}}/[\sigma_{\mathrm{strong}}{}^{(\pm)}(\Lambda^0\to\Sigma^0)]_{\mathrm{coher}}\}$$
$$\approx\frac{1-[\rho(|\mathbf{k}|R)]^2}{[\rho(|\mathbf{k}|R)]^2}$$
$$\approx\frac{1-[\sin(0.65A^{\frac{1}{3}}/x)/(0.65A^{\frac{1}{3}}/x)]^2}{[\sin(0.65A^{\frac{1}{3}}/x)/(0.65A^{\frac{1}{3}}/x)]^2}$$
$$\cong\tfrac{1}{3}[x=4,\ A^{\frac{1}{3}}=6\text{—Eqs. (34), (35)}], \quad (46)$$

with a very similar estimate being valid for

$$\{[\sigma_{\mathrm{Coul}}{}^{(\pm)}(\Lambda^0\to\Sigma^0)]_{\mathrm{incoher}}/[\sigma_{\mathrm{Coul}}{}^{(\pm)}(\Lambda^0\to\Sigma^0)]_{\mathrm{coher}}\}.$$

Therefore, Eqs. (44f) and (44g) for the ratio of the Coulomb field to the strong interaction induced $\Lambda^0\to\Sigma^0$ conversion cross sections can be left unaltered, the therein contained

$$\sigma_{\mathrm{Coul}}{}^{(\pm)}(\Lambda^0\to\Sigma^0)\quad\text{and}\quad\sigma_{\mathrm{strong}}{}^{(\pm)}(\Lambda^0\to\Sigma^0)$$

being interpretable as

$$[\sigma_{\mathrm{Coul}}{}^{(\pm)}(\Lambda^0\to\Sigma^0)]_{\mathrm{coher+incoher}}$$

and

$$[\sigma_{\mathrm{strong}}{}^{(\pm)}(\Lambda^0\to\Sigma^0)]_{\mathrm{coher+incoher}}.$$

We thus conclude that for high Z and sufficiently large p_Λ the conversion process possesses a relatively dominant Coulomb field component within the forward cone of opening angle θ_c [Eq. (37a)]. Hence the Σ^0 lifetime should be calculable from the observation of the cross sections in Eqs. (35)–(37), (42), (40) and (44), provided that the incoherent, mostly strong interaction induced, $\Lambda^0\to\Sigma^0$ conversion *outside* this forward cone can be ***kinematically distinguished*** from $\Lambda^0\to\Sigma^0$ conversion *within* the cone. While it is clear that such a kinematic distinction will be difficult to make in practice[15] encouragement regarding the ultimate possibilities of our method can be taken from the fact that a generally similar method has recently proved itself capable of determining the lifetime of the π^0.[16]

Finally, it should be mentioned that the general method of the present section can be extended to determine the total or partial lifetime of any other possibly existing elementary particle or of any actually existing "excited baryon" or "multi-meson" if this should decay wholly or partly through an electromagnetic channel. Thus, one can conceivably determine the electromagnetic decay rate of a $\rho^\pm$ with $J=1$ ($\rho^\pm\to\pi^\pm+\gamma$ in competition with a predominant $\rho^\pm\to\pi^0+\pi^\pm$) by a study of $\pi^\pm\to\rho^\pm$ conversion in $\pi^\pm$—high Z nucleus peripheral collisions. In fact, this general method has already been applied by Bég, DeCelles, and Marr[17] to relate the rate of the electromagnetic decay of a K^* with $J=1$ ($K^*\to K+\gamma$ in competition with a predominant $K^*\to K+\pi$) to the cross section for $K\to K^*$ conversion in a nuclear Coulomb field.

IV. RELATIONS BETWEEN LEPTONIC AND ELECTROMAGNETIC Σ DECAY

Consider the strangeness-conserving decays of Fig. 3:

$$\Sigma^-\to\Lambda^0+e^-+\bar{\nu},\quad \Sigma^+\to\Lambda^0+e^++\nu,\quad \Sigma^0\to\Lambda^0+\gamma,$$

which are reminiscent of a $T=1$ nuclear isobaric triplet exhibiting leptonic and electromagnetic decays to a

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[15] Incoherent, mostly strong-interaction-induced, $\Lambda^0\to\Sigma^0$ conversion appreciably outside the forward cone will involve a sizeable momentum and energy transfer from the Λ^0 to the target nucleus—or effectively to a nucleon in the target nucleus: Λ^0+nucleon $\to\Sigma^0$+nucleon—so that $E_\Sigma=(E_\Lambda)'+E_\gamma$ will be noticeably less than E_Λ, thus affording a basis for kinematic distinction over and above that arising from an experimental estimate of the angle θ; also this incoherent, strong-interaction-induced $\Lambda^0\to\Sigma^0$ conversion with sizeable momentum and energy transfer will quite often result in a Σ^0 accompanied by one or more pions thus affording a further distinction vis-à-vis a Σ^0 produced in a coherent Coulomb field $\Lambda^0\to\Sigma^0$ conversion process. As an (empirical) estimate of the cross section associated with the incoherent strong-interaction-induced $\Lambda^0\to\Sigma^0$ conversion with such large momentum and energy transfer that θ lies between, say, $5\theta_c=6.0°$ (19.0°: c.m.) and $\theta_c=1.2°$ (3.8°: c.m.) [Eq. (37a) with $x=4$, $A^{\frac{1}{3}}=6$] we may quote [see G. Alexander *et al.*, Phys. Rev. Letters **7**, 341 (1961)]:

$$[(8.5\pm4.9)\text{ mb}/4\pi]A^{\frac{2}{3}}[\pi(19/57)^2-\pi(3.8/57)^2]=(8\pm4)\text{ mb},$$

which is to be compared with the $\sigma^{(\pm)}{}_{\mathrm{Coul}}(\Lambda^0\to\Sigma^0)$ of Eq. (37c). It should also be mentioned that Λ^0+nucleus $\to\Sigma^0$+nucleus; $\Sigma^0\to\Lambda^0+\gamma$ is kinematically distinguishable from a background Λ^0 bremsstrahlung (Λ^0+nucleus $\to\Lambda^0+\gamma$+nucleus) since in the former case $(\mathbf{p}_\Lambda+\mathbf{p}_\gamma)^2-(E_\Lambda+E_\gamma)^2=-(m_\Sigma)^2$ for each final state Λ^0, γ. In addition rough theoretical estimates indicate that Λ^0 bremsstrahlung has a much smaller cross section than Coulomb field $\Lambda^0\to\Sigma^0$ conversion [by a factor (Λ^0 magnetic moment/$\Lambda^0-\Sigma^0$ transition moment)^4 $(f^{(\pm)}e)^2\ll1$)].

[16] *Theory:* H. Primakoff, Phys. Rev. **81**, 899 (1951); V. Glaser and R. A. Ferrell, *ibid.* **121**, 886 (1961); C. Chiuderi and G. Morpurgo, Nuovo cimento **19**, 497 (1961). *Experiment:* A. V. Tollestrup, S. Berman, R. Gomez, and H. Ruderman, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 27; H. Ruderman, S. Berman, R. Gomez, A. V. Tollestrup, and R. Talman, Bull. Am. Phys. Soc. **5**, 508 (1960).

[17] M. A. B. Bég, P. C. DeCelles, and R. B. Marr, Phys. Rev. **124**, 622 (1961).

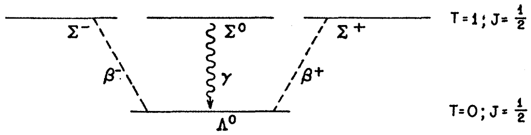

FIG. 3. The Σ-Λ multiplet structure.

$T=0$ nuclear isobaric singlet, e.g., ${}_5B^{12}\rightarrow {}_6C^{12}+e^-+\bar{\nu}$; ${}_7N^{12}\rightarrow {}_6C^{12}+e^++\nu$; ${}_6(C^{12})^*\rightarrow {}_6C^{12}+\gamma$. In the latter case, as pointed out by Gell-Mann[18] on the basis of the conserved vector current hypothesis,[19] the γ-decay rate of $({}_6C^{12})^*$ is related to the $e^\mp$ momentum spectra of ${}_5B^{12}$, ${}_7N^{12}$; analogously, we may anticipate that the decay rate of $\Sigma^0\rightarrow\Lambda^0+\gamma$ is related to the $e^\mp$ momentum spectra and decay rates of $\Sigma^\mp\rightarrow\Lambda^0+e^\mp+\nu$. We proceed to investigate various aspects of this relationship and shall establish such a connection between the leptonic and the electromagnetic Σ decays that under favorable circumstances an observation of, e.g., the $e^\mp$ momentum spectrum of $\Sigma^\mp\rightarrow\Lambda^0+e^\mp+\nu$ should be sufficient to determine the lifetime of $\Sigma^0\rightarrow\Lambda^0+\gamma$.

We begin by writing down the weak interaction Lagrangian, $\mathcal{L}^{(\mathrm{weak})}(x)$, which describes the primitive coupling of the baryons and mesons to the leptons in *strangeness conserving* transitions. We shall postulate a specific form for $\mathcal{L}^{(\mathrm{weak})}(x)$ and attempt to motivate our choice by analogical arguments relative to the form of $\mathcal{L}^{(\mathrm{em})}(x)$ in Eq. (1).

The form we assume for $\mathcal{L}^{(\mathrm{weak})}(x)$ is determined by a principle of "minimal weak coupling," viz.:

$$\begin{aligned}\mathcal{L}^{(\mathrm{weak})}(x)&\\ &=j_\mu^{(\mathrm{weak})}(x)L_\mu(x)+\mathrm{H.c.}\\ &\equiv j_\mu^{(\mathrm{weak})}(x)[-i\bar{\psi}_e(x)\gamma_\mu(1+\gamma_5)\psi_\nu(x)]+\mathrm{H.c.},\end{aligned}\tag{47a}$$

with a charge exchange baryon-meson "weak" current:

$$\begin{aligned}j_\mu^{(\mathrm{weak})}(x)\equiv\frac{G}{\sqrt{2}}\bigg\{&\bar{\chi}_N(x)i\gamma_\mu\frac{\tau_+}{2}\chi_N(x)\\ &+\bar{\chi}_\Sigma(x)i\gamma_\mu t_+\chi_\Sigma(x)+\bar{\chi}_\Xi(x)i\gamma_\mu\frac{\tau_+}{2}\chi_\Xi(x)\\ &+i\phi_\pi^\dagger(x)t_+(-\overleftrightarrow{\partial}_\mu)\phi_\pi(x)\\ &+i\phi_K^\dagger(x)\left(\frac{\tau_+}{2}\right)2(-\overleftrightarrow{\partial}_\mu)\phi_K(x)\bigg\},\end{aligned}\tag{47b}$$

where

$$\begin{aligned}\chi_N&\equiv\left(\frac{1+\gamma_5}{\sqrt{2}}\right)\psi_N,\\ \bar{\chi}_N&\equiv\left[\frac{1+\gamma_5}{\sqrt{2}}\psi_N\right]^\dagger\gamma_4=\bar{\psi}_N\frac{(1-\gamma_5)}{\sqrt{2}},\ \text{etc.},\\ \tau_+&\equiv\tau_1+i\tau_2;\quad t_+\equiv t_1+it_2,\end{aligned}\tag{47c}$$

so that

$$j_\mu^{(\mathrm{weak})}(x)=j_\mu^{(V)}(x)+j_\mu^{(A)}(x),\tag{47d}$$

with

$$\begin{aligned}j_\mu^{(V)}(x)=(G/\sqrt{2})\{&\bar{\psi}_N(x)i\gamma_\mu(\tau_+/2)\psi_N(x)\\ &+\bar{\psi}_\Sigma(x)i\gamma_\mu t_+\psi_\Sigma(x)+\bar{\psi}_\Xi(x)i\gamma_\mu(\tau_+/2)\psi_\Xi(x)\\ &+i\phi_\pi^\dagger(x)t_+(-\overleftrightarrow{\partial}_\mu)\phi_\pi(x)\\ &+i\phi_K^\dagger(x)(\tau_+/2)2(-\overleftrightarrow{\partial}_\mu)\phi_K(x)\},\end{aligned}\tag{47e}$$

$$\begin{aligned}j_\mu^{(A)}(x)=(G/\sqrt{2})\{&\bar{\psi}_N(x)i\gamma_\mu\gamma_5(\tau_+/2)\psi_N(x)\\ &+\bar{\psi}_\Sigma(x)i\gamma_\mu\gamma_5 t_+\psi_\Sigma(x)\\ &+\bar{\psi}_\Xi(x)i\gamma_\mu\gamma_5(\tau_+/2)\psi_\Xi(x)\}.\end{aligned}\tag{47f}$$

Decomposing the electromagnetic current $j_\mu(x)$ of Eq. (1) into an isoscalar part $ej_\mu^{(0,0)}(x)$ and an isovector part $ej_\mu^{(1,3)}(x)$:

$$j_\mu(x)=e[j_\mu^{(0,0)}(x)+j_\mu^{(1,3)}(x)],\tag{48a}$$

$$\begin{aligned}j_\mu^{(0,0)}(x)=\{&\bar{\psi}_N(x)i\gamma_\mu\tfrac{1}{2}\psi_N(x)-\bar{\psi}_\Xi(x)i\gamma_\mu\tfrac{1}{2}\psi_\Xi(x)\\ &+i\phi_K^\dagger(x)(\tfrac{1}{2})2(-\overleftrightarrow{\partial}_\mu)\phi_K(x)\},\end{aligned}\tag{48b}$$

$$\begin{aligned}j_\mu^{(1,3)}(x)=\{&\bar{\psi}_N(x)i\gamma_\mu(\tau_3/2)\psi_N(x)+\bar{\psi}_\Sigma(x)i\gamma_\mu t_3\psi_\Sigma(x)\\ &+\bar{\psi}_\Xi(x)i\gamma_\mu(\tau_3/2)\psi_\Xi\\ &+i\phi_\pi^\dagger(x)t_3(-\overleftrightarrow{\partial}_\mu)\phi_\pi(x)\\ &+i\phi_K^\dagger(\tau_3/2)2(-\overleftrightarrow{\partial}_\mu)\phi_K\},\end{aligned}\tag{48c}$$

we note that the "weak" current of Eqs. (47b) and (47c) is obtained from the isovector part of the electromagnetic current of Eqs. (48a) and (48c) by replacing the field amplitudes ψ_N, ψ_Σ, ψ_Ξ by their positive chirality components χ_N, χ_Σ, χ_Ξ, the isobaric spin operators τ_3, t_3 by the isobaric spin operators τ_+, t_+, and the electromagnetic coupling constant e by the weak-coupling constant $G/\sqrt{2}$. These replacements, together with that of $\mathcal{A}_\mu(x)$ by $L_\mu(x)$ constitute the analogy between the principles of minimal electromagnetic coupling and minimal weak coupling which motivates our choice of the form of $\mathcal{L}^{(\mathrm{weak})}(x)$ given that of $\mathcal{L}^{(\mathrm{em})}(x)$. We also note, on the basis of Eqs. (48c), (47e) and (47c), that

$$j_\mu^{(V)}(x)=\frac{G}{\sqrt{2}}[j_\mu^{(1,1)}(x)+ij_\mu^{(1,2)}(x)],\tag{48d}$$

and that the space integrals of the isovector components $j_4^{(1,m)}(x)$ $(m=1,2,3)$ give the corresponding components of the total isobaric spin of the system of baryons and mesons:

$$i^{-1}\int d^3x\,j_4^{(1,m)}(x)=T_m,\tag{48e}$$

We now consider the differential conservation laws satisfied by the various currents of Eqs. (47) and (48), this consideration affording some further support for

[18] M. Gell-Mann, Phys. Rev. **111**, 362 (1958).
[19] R. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

our choice of $\mathcal{L}^{(\text{weak})}(x)$. We have[20]:

$$\partial j_\mu(x)/\partial x_\mu=0, \tag{49a}$$

$$\partial j_\mu^{(1,m)}(x)/\partial x_\mu=0;\quad \partial j_\mu^{(V)}(x)/\partial x_\mu=0: \quad \text{conservation of } T_m;\quad \text{of } T_1+iT_2. \tag{49b}$$

the second relation in Eq. (49b) being the differential conservation law for the baryon-meson polar-vector current. It is to be recalled that the conservation law of Eq. (49b) for **T** holds only if one neglects in the baryon-meson Lagrangian the mass differences among members of a given isomultiplet and assumes that the various baryon-meson strong interactions are strictly invariant under rotations in isospace; it is also to be recalled that, in the absence of strong interaction symmetries higher than isospin rotational invariance, **T** is the only such conserved quantity and the $j_\mu^{(V)}(x)$ of Eq. (47e) the only corresponding conserved charge exchange baryon-meson polar-vector current. Thus, if no higher strong interaction symmetries are present, our choice for $\mathcal{L}^{(\text{weak})}(x)=[j_\mu^{(V)}(x)+j_\mu^{(A)}(x)]L_\mu(x)+\text{H.c.}$ contains the only available conserved charge exchange baryon-meson polar-vector current. If, on the other hand, such higher symmetries are present, there exist quasi-conserved charge exchange baryon-meson polar-vector currents $i_\mu^{(V)}(x)$, different from $j_\mu^{(V)}(x)$, and one might argue that terms

$$\sim[i_\mu^{(V)}(x)L_\mu(x)+\text{H.c.}]$$

should also be present in $\mathcal{L}^{(\text{weak})}(x)$; for example, for an existing doublet symmetry, we have[21]:

$$\partial i_\mu^{(V)}(x)/\partial x_\mu=0, \tag{50a}$$

with

$$\begin{aligned} i_\mu^{(V)}(x)&\equiv j_\mu^{(\text{doub})}(x)\\ &=\frac{G}{\sqrt{2}}\{\bar\psi_{\Lambda^0}(x)i\gamma_\mu\psi_{\Sigma^-}(x)+\bar\psi_{\Sigma^+}(x)i\gamma_\mu\psi_{\Lambda^0}(x)\\ &\quad-\bar\psi_{\Sigma^0}(x)i\gamma_\mu\psi_{\Sigma^-}(x)+\bar\psi_{\Sigma^+}(x)i\gamma_\mu\psi_{\Sigma^0}(x)\\ &\quad-i\phi^+{}_{K^+}2(-\overleftrightarrow{\partial}_\mu)\phi_{K^0}\}. \end{aligned} \tag{50b}$$

We shall however assume that such higher symmetries are in fact absent and employ the

$$\begin{aligned}\mathcal{L}^{(\text{weak})}(x)&=j_\mu^{(\text{weak})}(x)L_\mu(x)+\text{H.c.}\\ &=(j_\mu^{(V)}(x)+j_\mu^{(A)}(x))L_\mu(x)+\text{H.c.}\end{aligned}$$

of Eqs. (47)–(49) to describe the strangeness conserving leptonic decays of the baryons and mesons; it is important to mention that our

$$\mathcal{L}_\mu^{(\text{weak})}(x)=[j_\mu^{(V)}(x)+j_\mu^{(A)}(x)]L_\mu(x)+\text{H.c.}$$

does not contain any *primitive* $\Sigma-\Lambda-e\nu$ coupling while an $\mathcal{L}^{(\text{weak})}(x)$ containing a term

$$\sim[j_\mu^{(\text{doub})}(x)L_\mu(x)+\text{H.c.}]$$

would contain such a *primitive* coupling. Thus, with our $\mathcal{L}^{(\text{weak})}(x)$, $\Sigma^\pm\rightarrow\Lambda^0+e^\pm+\nu$ would *not* occur, just as with our $\mathcal{L}^{(\text{em})}(x)$, $\Sigma^0\rightarrow\Lambda^0+\gamma$ would *not* occur, if the strong interactions were turned off.

We now consider the general form of the $\Sigma^-\rightarrow\Lambda^0$ leptonic decay matrix element $\langle\Lambda^0|j_\mu^{(\text{weak})}(0)|\Sigma^-\rangle=\langle\Lambda^0|j_\mu^{(V)}(0)|\Sigma^-\rangle+\langle\Lambda^0|j_\mu^{(A)}(0)|\Sigma^-\rangle$ and begin, as in the corresponding discussion of $\langle\Lambda^0|j_\mu(0)|\Sigma^0\rangle$ in Eqs. (3)–(5), by applying the restrictions of Lorentz, space inversion and time-reversal invariance. We then have, with $(\gamma_5')\equiv1$ or γ_5, referring to $P_\Lambda P_\Sigma=+1$ or -1, and with $k_\mu\equiv(p_\Sigma-p_\Lambda)_\mu=(p_e+p_{\bar\nu})_\mu$:

$$\begin{aligned}\langle\Lambda^0|j_\mu^{(V)}(0)|\Sigma^-\rangle &=G_1^{(\pm)}(k_\rho^2)i(\bar u_\Lambda(\gamma_5')\sigma_{\mu\nu}u_\Sigma)k_\nu\\ &\quad+G_2^{(\pm)}(k_\rho^2)i(\bar u_\Lambda(\gamma_5')\gamma_\mu u_\Sigma)\\ &\quad+G_3^{(\pm)}(k_\rho^2)(\bar u_\Lambda(\gamma_5')u_\Sigma)(m_\Sigma\mp m_\Lambda)k_\mu,\end{aligned} \tag{51a}$$

$$\begin{aligned}\langle\Lambda^0|j_\mu^{(A)}(0)|\Sigma^-\rangle &=H_1^{(\pm)}(k_\rho^2)i(\bar u_\Lambda(\gamma_5')\sigma_{\mu\nu}\gamma_5u_\Sigma)k_\nu\\ &\quad+H_2^{(\pm)}(k_\rho^2)i(\bar u_\Lambda(\gamma_5')\gamma_\mu\gamma_5u_\Sigma)\\ &\quad+H_3^{(\pm)}(k_\rho^2)(\bar u_\Lambda(\gamma_5')\gamma_5u_\Sigma)(m_\Sigma\pm m_\Lambda)k_\mu,\end{aligned} \tag{51b}$$

where the weak interaction form factors $G_1^{(\pm)}(k_\rho^2),\cdots;$ $H_1^{(\pm)}(k_\rho^2),\cdots$ are real for

$$k_\rho^2=|\mathbf{p}_\Sigma-\mathbf{p}_\Lambda|^2-(E_\Sigma-E_\Lambda)^2=(\mathbf{p}_e+\mathbf{p}_{\bar\nu})^2-(E_e+E_{\bar\nu})^2>-(2m_\pi)^2;\ -(m_\pi)^2.$$

Further, from Eqs. (48d) and (48a), and remembering the isovector character of $j_\mu^{(1,m)}(x)$ and the isoscalar character of

$$j_\mu^{(0,0)}(x)\,(\text{i.e. }\langle\Lambda^0\colon T=0|j_\mu^{(0,0)}(0)|\Sigma^0\colon T=1\rangle=0):$$

$$\begin{aligned}\langle\Lambda^0|j_\mu^{(V)}(0)|\Sigma^-\rangle &=\frac{G}{\sqrt{2}}\{\langle\Lambda^0|j_\mu^{(1,1)}(0)|\Sigma^-\rangle+i\langle\Lambda^0|j_\mu^{(1,2)}(0)|\Sigma^-\rangle\}\\ &=\frac{2}{\sqrt{2}}\left(\frac{G}{\sqrt{2}}\right)\langle\Lambda^0|j_\mu^{(1,3)}(0)|\Sigma^0\rangle\\ &=\frac{2}{\sqrt{2}}\left[\frac{G}{\sqrt{2}}\Big/e\right]\langle\Lambda^0|j_\mu(0)|\Sigma^0\rangle,\end{aligned} \tag{51c}$$

which in view of Eqs. (3) and (51a) establishes the proportionality of the weak interaction form factors $G_i^{(\pm)}(k_\rho^2)$ to the electromagnetic form factors $F_i^{(\pm)}(k_\rho^2)$;

[20] It is to be noted that the simultaneous validity of $\partial j_\mu(x)/\partial x_\mu=0$ and $\partial j_\mu^{(1,3)}/\partial x_\mu=0$ implies, using Eq. (48a), that $\partial j_\mu^{(0,0)}/\partial x_\mu=0$, i.e., implies the conservation of total hypercharge $Y=S+B=2(Q-T_3)$.

[21] R. Behrends and A. Sirlin, Phys. Rev. **121**, 324 (1961). We are indebted to Dr. Behrends for pointing out to us this explicit possibility.

in fact, using Eqs. (51c) and (5), we have:

$$\langle\Lambda^0|j_\mu^{(V)}(0)|\Sigma^-\rangle = \frac{2}{\sqrt{2}}\left[\frac{G}{\sqrt{2}}\Big/e\right]\{F_1^{(\pm)}(k_\rho^2)i(\bar{u}_\Lambda(\gamma_5')\sigma_{\mu\nu}u_\Sigma)k_\nu + F_3^{(\pm)}(k_\rho^2)i(\bar{u}_\Lambda(\gamma_5')\gamma_\mu u_\Sigma)k_\nu^2 + F_3^{(\pm)}(k_\rho^2)(\bar{u}_\Lambda(\gamma_5')u_\Sigma)(m_\Sigma\mp m_\Lambda)k_\mu\}. \quad (52)$$

It is to be noted that Eq. (52) predicts that $\langle\Lambda^0|j_\mu^{(V)}(0)|\Sigma^-\rangle\to 0$ as $k_\nu\to 0$; this also follows directly from the fact that as $k_\nu\to 0$,

$$\int d^3x\, j_\mu^{(V)}(\mathbf{x},0)L_\mu(\mathbf{x},0)\to\left[\int d^3x\, j_4^{(V)}(\mathbf{x},0)\right]L_4(0) = (G/\sqrt{2})(T_1+iT_2)iL_4(0)\quad [\text{Eqs. (48d), (48e)}]$$

so that as

$k_\nu\to 0$,

$$\left\langle\Lambda^0\text{: }T=0\left|\int d^3x\, j_\mu^{(V)}(\mathbf{x},0)L_\mu(\mathbf{x},0)\right|\Sigma^-\text{: }T=1\right\rangle\to \frac{G}{\sqrt{2}}iL_4(0)\langle\Lambda^0\text{: }T=0|T_1+iT_2|\Sigma^-\text{: }T=1\rangle=0.$$

We now recall, from the discussion just preceding Eq. (28), that for small k_ρ^2:

$$F_1^{(\pm)}(k_\rho^2)\cong\left(f^{(\pm)}\frac{e}{2m_N}\right)\left(1-\frac{1}{12m_\pi^2}k_\rho^2\right);$$
$$F_3^{(\pm)}(k_\rho^2)\cong f^{(\pm)}e\left(\frac{1}{12m_\pi^2}\right), \quad (53a)$$

where $f^{(\pm)}$, the $\Sigma^0-\Lambda^0$ transition magnetic moment in units of $e/2m_N$, is related to the Σ^0 lifetime, $\tau(\Sigma^0)$, by Eq. (11a):

$$f^{(\pm)}=[2.0\times 10^{-19}\text{ sec}/\tau(\Sigma^0)]^{\frac{1}{2}}, \quad (53b)$$

while

$$k_\mu L_\mu(0)=(p_e+p_{\bar{\nu}})_\mu[-i\bar{\psi}_e(0)\gamma_\mu(1+\gamma_5)\psi_\nu(0)] = im_e[-i\bar{\psi}_e(0)(1+\gamma_5)\psi_\nu(0)]. \quad (53c)$$

Thus, combining Eqs. (52) and (51b) with Eqs. (53a) and (53c):

$$\langle\Lambda^0|j_\mu^{(V)}(0)|\Sigma^-\rangle L_\mu(0) \cong i\left(\frac{G}{\sqrt{2}}\frac{f^{(\pm)}}{\sqrt{2}}\right)\left[\left\{(\bar{u}_\Lambda(\gamma_5')\sigma_{\mu\nu}u_\Sigma)\frac{k_\nu}{m_N}\left(1-\frac{k_\rho^2}{12m_\pi^2}\right) + (\bar{u}_\Lambda(\gamma_5')\gamma_\mu u_\Sigma)\frac{k_\rho^2}{6m_\pi^2}\right\}L_\mu(0) + \left\{(\bar{u}_\Lambda(\gamma_5')u_\Sigma)\left(\frac{(m_\Sigma\mp m_\Lambda)m_e}{6m_\pi^2}\right)\right\}\times(-i\bar{\psi}_e(0)(1+\gamma_5)\psi_\nu(0))\right], \quad (54a)$$

$$\langle\Lambda^0|j_\mu^{(A)}(0)|\Sigma^-\rangle L_\mu(0) = i\left(\frac{G}{\sqrt{2}}\right)\left[\left\{K_1^{(\pm)}(k_\rho^2)(\bar{u}_\Lambda(\gamma_5')\sigma_{\mu\nu}\gamma_5 u_\Sigma)\frac{k_\nu}{m_N} + K_2^{(\pm)}(k_\rho^2)(\bar{u}_\Lambda(\gamma_5')\gamma_\mu\gamma_5 u_\Sigma)\right\}L_\mu(0) + \left\{K_3^{(\pm)}(k_\rho^2)(\bar{u}_\Lambda(\gamma_5')\gamma_5 u_\Sigma)\frac{(m_\Sigma\pm m_\Lambda)m_e}{m_\pi^2}\right\}\times(-i\bar{\psi}_e(0)(1+\gamma_5)\psi_\nu(0))\right], \quad (54b)$$

where

$$K_1^{(\pm)}(k_\rho^2)\equiv\frac{H_1^{(\pm)}(k_\rho^2)m_N}{(G/\sqrt{2})};$$
$$K_2^{(\pm)}(k_\rho^2)\equiv\frac{H_2^{(\pm)}(k_\rho^2)}{(G/\sqrt{2})}; \quad (54c)$$
$$K_3^{(\pm)}(k_\rho^2)\equiv\frac{H_3^{(\pm)}(k_\rho^2)m_\pi^2}{(G/\sqrt{2})}.$$

Also, since

$$|(k_\rho^2)_{\max}|\cong(m_\Sigma-m_\Lambda)^2=0.3m_\pi^2, \quad (54d)$$

and since for relatively small $|(k_\rho^2)_{\max}|$ we should have

$$K_1^{(\pm)}(k_\rho^2)\approx K_3^{(\pm)}(k_\rho^2) \approx K_2^{(\pm)}(k_\rho^2)\approx K_2^{(\pm)}(0)\equiv\kappa_A^{(\pm)}, \quad (54e)$$

we finally obtain from the Eqs. (54):

$$\langle\Lambda^0|j_\mu^{(V)}(0)|\Sigma^-\rangle L_\mu(0) \cong i\left(\frac{G}{\sqrt{2}}\frac{f^{(\pm)}}{\sqrt{2}}\right)\left\{(\bar{u}_\Lambda(\gamma_5')\sigma_{\mu\nu}u_\Sigma)\frac{k_\nu}{m_N}\right\}L_\mu(0), \quad (55a)$$

$$\langle\Lambda^0|j_\mu^{(A)}(0)|\Sigma^-\rangle L_\mu(0) \cong i\left(\frac{G}{\sqrt{2}}\kappa_A^{(\pm)}\right)\{(\bar{u}_\Lambda(\gamma_5')\gamma_\mu\gamma_5 u_\Sigma)\}L_\mu(0), \quad (55b)$$

wherein it is seen that $(G/\sqrt{2})(f^{(\pm)}/\sqrt{2})$ and $(G/\sqrt{2})\kappa_A{}^{(\pm)}$ play the roles of weak magnetism and axial vector effective coupling constants, respectively.

Equations (55) may now be used for the prediction of the $\Sigma^- \to \Lambda^0$ leptonic decay rate and e^- momentum spectrum from which quantities

$$f^{(\pm)} = [2.0\times10^{-19}\ \text{sec}/\tau(\Sigma^0)]^{\frac{1}{2}} \quad [\text{Eq. (53b)}]$$

can be determined if

$$\kappa_A{}^{(\pm)} \lesssim f^{(\pm)}(|k_\nu|_{\max}/m_N) \cong f^{(\pm)}(m_\Sigma - m_\Lambda)/m_N$$
$$= 0.086 f^{(\pm)} \approx 0.1(g_\Lambda{}^{(\pm)} g_\Sigma/g_N{}^2) \quad [\text{Eq. (11c)}].$$

Such a small value of $\kappa_A{}^{(\pm)}$:

$$\kappa_A{}^{(\pm)} \lesssim (m_\pi/m_N)(g_\Lambda{}^{(\pm)}/g_N) \approx (g_N m_\pi/2m_N)^2 (g_\Lambda{}^{(\pm)}/g_N),$$

may perhaps be anticipated with our $\mathcal{L}^{(\text{weak})}(x)$ which *provides no primitive coupling* for[22] $\Sigma^\mp \to \Lambda^0 + e^\mp + \nu$ and which, from the point of view of perturbation theory, ascribes the actual nonvanishing value of the term proportional to $K_2{}^{(\pm)}(k_\rho{}^2)$ in $\langle\Lambda^0|j_\mu{}^{(A)}(0)|\Sigma^-\rangle L_\mu(0)$ and so the actual nonvanishing value of $\kappa_A{}^{(\pm)}$ to a sequence of virtual reactions such as[23]:

$$\Sigma^- \to_{\text{strong}} \bar{p} + n + \Lambda^0;\quad \bar{p} + n \to_{\text{weak}} e^- + \bar{\nu}.$$

We proceed to calculate the $\Sigma^- \to \Lambda^0$ leptonic decay rate $\Gamma(\Sigma^- \to \Lambda^0 + e^- + \bar{\nu})$ and the corresponding e^- momentum spectrum $\mathfrak{N}(p_e)$. Using the matrix elements of the Eqs. (55) and remembering Eqs. (47d) and (47a) a standard calculation gives, with $p_m \equiv m_\Sigma - m_\Lambda = 0.086 m_N$; $G \equiv$ universal weak coupling constant $= 1.0\times10^{-5}/(m_N)^2$; $f^{(\pm)}$ as in Eq. (53b):

$P_\Lambda P_\Sigma = +1,$

$$\mathfrak{N}(p_e) = \frac{G^2}{2\pi^3}\left[3(\kappa_A{}^{(+)})^2 - 4\left(\kappa_A{}^{(+)}\frac{f^{(+)}}{\sqrt{2}}\right)\left(\frac{p_m - 2p_e}{m_N}\right) + (f^{(+)})^2\left(\frac{p_m{}^2 - (10/3)p_m p_e + (10/3)p_e{}^2}{m_N{}^2}\right)\right] \times [p_e{}^2(p_m - p_e)^2], \tag{56a}$$

$$\Gamma(\Sigma^- \to \Lambda^0 + e^- + \bar{\nu}) = \frac{G^2}{2\pi^3}\left[\frac{3(\kappa_A{}^{(+)})^2}{30} + \frac{(f^{(+)})^2}{105}\left(\frac{p_m}{m_N}\right)^2\right] p_m{}^5$$
$$= \left[1.1\times10^6(\kappa_A{}^{(+)})^2 + 1.7\times10^3\left(\frac{10^{-19}\ \text{sec}}{\tau(\Sigma^0)}\right)\right]\text{sec}^{-1}, \tag{56b}$$

$P_\Lambda P_\Sigma = -1,$

$$\mathfrak{N}(p_e) = \frac{G^2}{2\pi^3}\left[(\kappa_A{}^{(-)})^2 + (f^{(-)})^2\left(\frac{p_m{}^2 - \frac{2}{3}p_m p_e + \frac{2}{3}p_e{}^2}{m_N{}^2}\right)\right] \times [p_e{}^2(p_m - p_e)^2], \tag{56c}$$

$$\Gamma(\Sigma^- \to \Lambda^0 + e^- + \bar{\nu}) = \frac{G^2}{2\pi^3}\left[\frac{(\kappa_A{}^{(-)})^2}{30} + \frac{3(f^{(-)})^2}{105}\left(\frac{p_m}{m_N}\right)^2\right] p_m{}^5$$
$$= \left[3.7\times10^5(\kappa_A{}^{(-)})^2 + 5.1\times10^3\left(\frac{10^{-19}\ \text{sec}}{\tau(\Sigma^0)}\right)\right]\text{sec}^{-1}, \tag{56d}$$

while, as a comparison, we have from experiment:

$$\Gamma(\Sigma^- \to n + \pi^-) = 6.3\times10^9\ \text{sec}^{-1}, \tag{57}$$

Equations (56) show explicitly that, as expected from our previous discussion, $\kappa_A{}^{(\pm)}$ must be fairly small if the terms in $f^{(\pm)}$ or $1/\tau(\Sigma^0)$ are to be more or less dominant in, and so determinable from, the expressions for $\mathfrak{N}(p_e)$, $\Gamma(\Sigma^- \to \Lambda^0 + e^- + \bar{\nu})$. Thus, for example, in the case $P_\Lambda P_\Sigma = -1$ and with $\tau(\Sigma^0) = 5\times10^{-19}$ sec [i.e. $(g_\Lambda{}^{(-)})^2 = \frac{1}{5}g_N{}^2 = 3$, $g_\Sigma{}^2 = g_N{}^2$ in Eq. (11d)] the axial vector and weak magnetism terms in Eq. (56d) are about equal numerically for $\kappa_A{}^{(-)} = 1/15$ [i.e. $\kappa_A{}^{(-)} \approx (m_\pi/m_N)(g_\Lambda{}^{(-)}/g_N) = 0.15/\sqrt{5}$]. However, even if the term in $\kappa_A{}^{(\pm)}$ is much the larger,[24] eventual observation and analysis of the e^- momentum spectrum and leptonic decay rate of $\Sigma^- \to \Lambda^0 + e^- + \bar{\nu}$ should, at a minimum, ultimately confirm or deny the general validity of the principle of "minimal weak coupling" in strangeness-conserving leptonic decays.[25]

CONCLUSION

Although we share the general belief that the radiative decay, $\Sigma^0 \to \Lambda^0 + \gamma$, is subject to the conservation of parity, we have nevertheless included a discussion of the consequences of a possible breakdown

[22] It will be noted that our $\mathcal{L}^{(\text{weak})}(x)$ *does* of course provide a primitive coupling for

$$n \to p + e^- + \bar{\nu},\quad \Sigma^- \to \Sigma^0 + e^- + \bar{\nu},\quad \Sigma^0 \to \Sigma^+ + e^- + \bar{\nu},\quad \Xi^- \to \Xi^0 + e^- + \bar{\nu}.$$

[23] In this connection one should recall that the strong-interaction induced renormalization of the axial vector coupling constant in $n \to p + e^- + \bar{\nu}$, which is conceivably associated with virtual reactions such as $n \to_{\text{strong}} p + \bar{p} + n$; $\bar{p} + n \to_{\text{weak}} e^- + \bar{\nu}$, amounts empirically only to about 20%.

[24] J. Bernstein and R. Oehme, Phys. Rev. Letters **6**, 639 (1961), have considered a theory of $\Sigma^\pm \to \Lambda^0 + e^\pm + \nu$ with $P_\Lambda P_\Sigma = -1$ which predicts $\kappa_A{}^{(-)} \approx 6\times1.2 = 7.2$. This estimate, and an analogous one which can be made for the case $P_\Lambda P_\Sigma = +1$, are based on a Goldberger-Treiman type relationship:

$$\kappa_A{}^{(\pm)}/1.2 \approx [2m_N/(m_\Sigma \pm m_\Lambda)](g_\Lambda{}^{(\pm)}/g_N)$$

[see J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo cimento **17**, 757 (1960)] and always predict relatively large $\kappa_A{}^{(\pm)}$.

[25] It is worth commenting that on the basis of this principle (and with neglect of the Σ^+, Σ^- mass difference), the e^+ momentum spectrum and the leptonic decay rate in $\Sigma^+ \to \Lambda^0 + e^+ + \nu$ are the same, except for the sign of the term proportional to $\kappa_A{}^{(+)} f^{(+)}$ in the momentum spectrum, as the corresponding e^- momentum spectrum, and leptonic decay rate in $\Sigma^- \to \Lambda^0 + e^- + \bar{\nu}$. In addition, if the $\Delta S = \Delta Q$ rule is valid in hyperon leptonic decay, all e^+ appearing in Σ^+ leptonic decay must be ascribed to $\Sigma^+ \to \Lambda^0 + e^+ + \nu$. It is also worth mentioning that a further test of our "minimal" $\mathcal{L}^{(\text{weak})}(x)$ in $\Sigma^\pm \to \Lambda^0 + e^\pm + \nu$ decays can be obtained from a study of the Λ^0 polarization and of the correlation of the $e^\pm$ momentum with the $\Sigma^\pm$ spin.

of parity conservation in this decay. We merely note here that, at the moment, no direct evidence is available for or against parity conservation in high-energy electromagnetic interactions of strange particles.

Our main interest in this paper has been to devise a means of determination of the Σ^0 lifetime, $\tau(\Sigma^0)$, which remains the only unknown elementary particle lifetime. We conclude that the study of approximately forward $\Lambda^0 \rightarrow \Sigma^0$ conversion in a high-Z nuclear Coulomb field may well provide a feasible experimental procedure of finding $\tau(\Sigma^0)$.

We also suggest that the Σ^0 lifetime may be determined from an analysis of $\Sigma^\pm \rightarrow \Lambda^0$ strangeness-conserving leptonic decays if these decays are subject to a principle of "minimal weak coupling" and the associated assumption of a conserved baryon-meson polar-vector current, and if the effective axial vector coupling is not dominant. We emphasize that the advent of machines of high beam intensity should eventually permit a fairly detailed study of $\Sigma^\pm \rightarrow \Lambda^0+e^\pm+\nu$ and, in view of the expected "anomalous" features of these decays, point out that the corresponding decay rates and electron momentum spectra will surely provide important weak-interaction data even if they should prove incapable of yielding the value[26] of $\tau(\Sigma^0)$.

[26] For the sake of completeness we mention that one can write the weak-electromagnetic branching ratio

$$B \equiv \text{Rate}[\Sigma^0 \rightarrow p+\pi^-]/\text{Rate}[\Sigma^0 \rightarrow \Lambda^0+\gamma]$$
$$= \text{Rate}[\Sigma^+ \rightarrow p+\pi^0]/\text{Rate}[\Sigma^0 \rightarrow \Lambda^0+\gamma],$$

the second equality following upon use of the $\Delta T=\frac{1}{2}$ rule. Thus any future measurement of B together with already available empirical information about $\text{Rate}[\Sigma^+ \rightarrow p+\pi^0]=6\times10^9$ sec^{-1} is sufficient to determine $\text{Rate}[\Sigma^0 \rightarrow \Lambda^0+\gamma]=1/\tau(\Sigma^0)$. This weak-electromagnetic branching ratio method will be the least impractical method for determination of $\tau(\Sigma^0)$ if $\tau(\Sigma^0)$ is appreciably greater than the $10^{-18}-10^{-20}$ sec now anticipated [Eq. (11d)] since in this case the methods discussed in Secs. III and IV will become prohibitively difficult.

PHYSICAL REVIEW VOLUME 125, NUMBER 5 MARCH 1, 1962

Precision Measurement of the μ^+ Lifetime*†

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(Received September 28, 1961)

A new precision measurement of the mean lifetime of the positive muon, $\tau(\mu^+)$, is described. On the basis of 12 internally consistent, independent determinations performed under a variety of experimental conditions we obtain $\tau(\mu^+)=(2.203\pm0.004)$ μsec. These determinations were performed by means of an improved version of Swanson's "digitron"—a digital electronic time interval measuring device—embodying a continuous wave (CW) oscillator and several protective features. The available experimental conditions, in particular concerning the percentage of background events, were greatly superior to those available in earlier experiments here and elsewhere. Particular attention was paid to sources of systematic error, such as time dependence of background events and rate dependence of the over-all measuring process. All sources of background have been localized, their effects quantitatively exhibited and, in general, suppressed electronically during actual lifetime determinations.

The prediction of the conserved vector-current theory, using $ft(O^{14})=(3069\pm13)$ sec and $m_\mu=206.76m_e$, is (including radiative corrections) $\tau(\mu^+)=2.298\pm0.05$ μsec, and thus in disagreement with the experimental value reported here. This discrepancy is even greater than with the somewhat higher values obtained for $\tau(\mu^+)$ by other workers; these values are briefly reviewed.

A full discussion of the logical operation of the digitron, and of time-interval measuring devices in general, is presented. In Appendix I (by R. Winston) a general formalism is given which establishes the connection between arbitrary interval distributions presented at the input of such devices and their corresponding output interval distributions.

I. INTRODUCTION

A PRECISION measurement of the mean lifetime, $\tau(\mu^+)$, of muons is of great theoretical interest for several rather well-known reasons. This lifetime is essentially given by the rate for the decay process

$$\mu^+ \rightarrow e^+ + \nu + \bar{\nu}, \tag{1}$$

in which *only* weakly interacting leptons (as opposed, say, to the baryons in ordinary beta decay) participate. Hence one can extract from $\tau(\mu^+)$ valuable information concerning the absolute magnitudes of the "true" coupling constants, i.e., their magnitudes with the "renormalizing" effect of strong interactions switched off. All present-day evidence[1] concerning process (1) is compatible with the assumption that it is described by the $(V-A)$ interaction, i.e., that only two types of coupling (V,A) intervene in the relevant Fermi interaction and that the absolute magnitudes of the corresponding coupling constants are equal: $|C_V|=|C_A|=G$. Thus, there appears to be only one fundamental coupling constant, G, for weak interactions and its measure is given by $\tau(\mu^+)$.

* Research supported by a joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

† Based on a thesis submitted to the Faculty of the Department of Physics, the University of Chicago, in partial fulfillment of the requirements for the Ph.D. degree.

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[1] V. L. Telegdi, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 713.