intensive experimental search for it certainly seems warranted.

ACKNOWLEDGMENTS

We are grateful to CERN in Geneva for its kind hospitality during the summer of 1961 when this work was done. We are also much indebted to Professor T. D. Lee for many valuable comments and discussions and for his encouragement. We should also like to thank our colleagues at Brookhaven, CERN, and Columbia for many comments, and especially Dr. J. D. Bjorken who independently made the observation that the two-pion decay mode of the W could be computed exactly if the vector current of the weak interactions was conserved.

PHYSICAL REVIEW VOLUME 125, NUMBER 5 MARCH 1, 1962

Reaction, $\pi^-+p\rightarrow e^++e^-+n$, as a Means of Measuring the Electromagnetic Form Factor of the Charged Pion*

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(Received October 2, 1961)

The success of the single pion exchange approximation in explaining the experimental results for the reaction $\pi+n\rightarrow\pi+\pi+n$ in the region of the two-pion resonance suggests that the single pion exchange contribution to the reaction $\pi+n\rightarrow e^++e^-+n$ should be dominant for center-of-mass energies of the electron-positron system in the region of this resonance. A measurement of the rate of this reaction in this region should, therefore, provide direct information about the charged-pion electromagnetic form factor in the neighborhood of the two-pion resonance. We have calculated the differential cross section for the reaction $\pi^-+p\rightarrow e^++e^-+n$, assuming a counter experiment. Our estimates indicate that counting rates of the order of ten per hour are attainable by accelerators capable of producing negative-pion beams with an average flux of 10^6 pions per second.

I. INTRODUCTION

A RESONANCE in the two-pion $J=1$, $T=1$ state as a means of explaining the relatively rapid change of the nucleon electromagnetic form factors with momentum transfer has been proposed by Frazer and Fulco.[1] Since then there has been a considerable amount of experimental effort devoted to the search for this resonance by examining processes with at least two pions in the final state. Recently, Walker and his coworkers, and Pickup *et al.*[2] have obtained the strongest evidence for this resonance by a careful examination of the processes

$$\pi^-+p\rightarrow\begin{cases}\pi^++\pi^-+n\\ \pi^0+\pi^-+p.\end{cases}\qquad(1)$$

In analyzing their data, these groups find that for cases where the momentum transferred to the recoil nucleon is small, it is consistent to assume that the dominant contribution to process (1) in the resonant region comes from the single pion exchange graph shown in Fig. 1(a). A comparison of the relative rates of process (1) with each other and with the alternate reaction, $\pi^-+p\rightarrow\pi^0+\pi^0+n$ supports the notion that the strong two-pion interaction occurs in the $T=1$ state. The observed branching ratio for reaction (1) in the region of resonance also lends further support to the result that the single pion exchange graph gives the dominant contribution to this process.[3]

It is the purpose of this paper to point out that, in light of these results, the measurement of the process

$$\pi^-+p\rightarrow e^++e^-+n\qquad(2)$$

is now very much worth doing. The observed dominance of the single pion exchange graph [Fig. 1(a)] for pion production strongly suggests that the corresponding graph for electron pair production, shown in Fig. 1(b), would also be dominant, again in the region of small momentum transfer and provided we restrict ourselves to center-of-mass energies of the electron-positron system close to that of the two-pion resonant energy. If this is true, we have a means of measuring the charged-pion electromagnetic form factor for the most interesting values of the invariant four-momentum transfer. This could be done without recourse to a difficult extrapolation procedure,[4] making it practical to use counters to observe the electron pair.

* Work supported in part by the U. S. Atomic Energy Commission.

[1] W. R. Frazer and J. R. Fulco, Phys. Rev. Letters 2, 365 (1959); Phys. Rev. **117**, 1609 (1960).

[2] A. R. Erwin, R. March, W. D. Walker, and E. West, Phys. Rev. Letters **6**, 628 (1961); E. Pickup, D. K. Robinson, and E. O. Salant, *ibid.* **7**, 192 (1961).

[3] In the resonance region, for $\Delta^2<10$, Pickup *et al.*[2] find $\sigma(\pi^0,\pi^-)/\sigma(\pi^+,\pi^-)=0.42\pm0.06$.[2] The ratio predicted by the single pion exchange model is one half. Another simple explanation for this observed value of the branching ratio is that this reaction proceeds predominantly through the state of total isotopic spin 3/2. This would predict, however, much too large a cross section for the reaction $\pi^++p\rightarrow\pi^0+\pi^++p$, i.e., $\sigma(\pi^0,\pi^+)=(9/2)\sigma(\pi^+,\pi^-)$.

[4] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959).

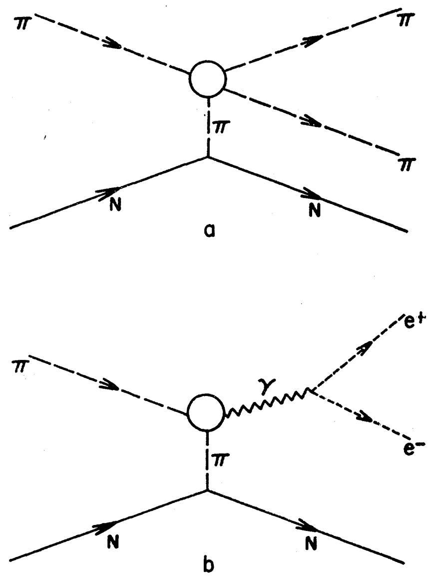

FIG. 1. (a) Single pion exchange graph for the reaction $\pi+N\rightarrow\pi+\pi+N$. (b) Single pion exchange graph for the reaction $\pi+N\rightarrow e^++e^-+N$.

This is by no means an original suggestion. The related processes, $e^-+p\rightarrow e^-+\pi^++(n+\cdots)$ have been proposed by Drell[5] and by Frazer[6] (the latter for "elastic production" $e^-+p\rightarrow e^-+\pi^++n$) as a means of measuring the charged-pion electromagnetic form factor. Unfortunately, this would be for spacelike values of the momentum transfer and hence far from the region of enhancement. Drell[5] has also suggested measuring process (2), or more generally more inelastic processes, in the region of enhancement. Unfortunately, at the very high energies considered, the expected counting rates appear to be too low.[7]

In this paper we relate the amplitude for process (1) to the amplitude for process (2) by means of a dispersion relation in the photon mass. This enables us to show that the dominance of the single pion-exchange graph for $\pi^-+p\rightarrow\pi^++\pi^-+n$ implies that the corresponding graph [Fig. 1(b)] should give the dominant contribution to $\pi^-+p\rightarrow e^++e^-+n$. We then give some numerical estimates for the differential cross section for the latter process, taking an incident pion energy of 1.4 Bev. These results indicate that workable counting rates can be obtained in this energy region using, possibly, existing accelerators and definitely using the accelerators with higher beam currents now nearing completion.

II. THE AMPLITUDE FOR $\pi^-+p\rightarrow e^++e^-+n$

The S matrix for reaction (2) can be written, to lowest order in the electromagnetic fine structure constant α,

$$S=-(2\pi)^4i[\delta_4(p+q-p'-p_1-p_2)mM/(2q_0p_0p_0'p_{10}p_{20})^{\frac{1}{2}}]T, \quad (3)$$

where

$$T=[4\pi\alpha i\bar{u}(p_1)\gamma_\mu v(p_2)/k^2]\langle p'\|J_\mu(0)\|pq+\rangle. \quad (4)$$

q is the four-momentum of the incident pion, p and p' the four-momenta of the incident and recoil nucleons, respectively, and p_1, p_2 the four-momenta of the electron and positron. $k^2=(p_1+p_2)^2=\mathbf{k}^2-k_0^2$. We take $\hbar=c=m_\pi=1$, so that m, the electron mass, and M, the nucleon mass, are measured in units of the pion mass. The symbol $\|q,\cdots\rangle$ denotes states with the invariant normalization, i.e., $\langle p'\|=(p_0'/M)^{\frac{1}{2}}\langle p'|$. The effects of the strong interaction are entirely contained in the covariant matrix element,

$$T_\mu(W^2,\Delta^2;-k^2)=\langle p'\|J_\mu(0)\|pq+\rangle, \quad (5)$$

eJ_μ is the electromagnetic current operator due to only strongly interacting particles. $W^2=-(p+q)^2$ and $\Delta^2=(p'-p)^2$. For $k^2=0$, T_μ is the matrix element for pion photoproduction ($-k^2$ is the mass of the virtual photon). For $k^2>0$, or k^2 space-like, T_μ is the matrix element for pion electroproduction. We are interested here in T_μ for timelike values of k^2 and, in particular, for values $-k^2\approx t_r$. t_r is the square of the center-of-mass energy of the two-pion system at resonance.

In what follows, we shall take for T_μ only the contribution arising from the single pion exchange graph shown in Fig. 1(b). Before doing so, however, we would like to partly justify in a more formal way how the corresponding assumption for process (1) implies the validity of our assumption here. This discussion is not essential to the rest of the paper and is presented, therefore, in small print.

To show this, we must assume that the invariant amplitudes, into which T_μ decomposes, satisfy dispersion relations in the variable k^2. Since only a limited number of these invariant functions shall enter this discussion, we postpone the explicit decomposition of T_μ. Hence, let us represent the dispersion relations satisfied by these functions schematically by writing down a relation for T_μ

$$T_\mu(W^2,\Delta^2;-k^2)=\frac{1}{\pi}\int_4^\infty\frac{\mathrm{Im}T_\mu(W^2,\Delta^2;\rho^2)}{\rho^2+k^2-i\epsilon}d\rho^2, \quad (6)$$

and

$$\mathrm{Im}T_\mu=\tfrac{1}{2}(2\pi)^4\textstyle\sum_n\langle 0|J_\mu(0)|n\rangle\langle n|\bar{u}(p')f(0)\|pq+\rangle\delta_4(k-p_n). \quad (7)$$

$f(x)$ is the nucleon current operator. We do not concern ourselves here with the question of subtractions. The object is to find T_μ from (6) in the region $-k^2\approx t_r$ where T_μ, and hence $\mathrm{Im}T_\mu$ becomes strongly peaked. As long as we subtract at a point far from $-k^2=t_r$, the dominant contribution to T_μ will come from the integral in Eq. (6), largely from the region of integration near $\rho^2=t_r$. Consequently, let us approximate $\mathrm{Im}T_\mu$ given by Eq. (7) by taking only the contribution due to the two-pion intermediate state:

$$\mathrm{Im}T_\mu\cong\tfrac{1}{2}(2\pi)^{-2}\int\delta_+(q_1^2+1)\delta_+(q_2^2+1)\delta_4(k-q_1-q_2)$$
$$\times(q_1-q_2)_\mu F_\pi^*(-k^2)\langle q_1,q_2-\|\bar{u}(p')f(0)\|pq+\rangle d^4q_1d^4q_2. \quad (8)$$

F_π is the electromagnetic form factor of the charged pion. $\langle q_1,q_2-\|\bar{u}(p')f(0)\|pq+\rangle$ is the invariant amplitude for $\pi^-+p\rightarrow\pi^++\pi^-+n$. Let us take for this amplitude, for $-(q_1+q_2)^2\approx t_r$, the contribution coming from the single pion exchange graph in

[5] S. D. Drell, Revs. Modern Phys. **33**, 458 (1961).
[6] W. R. Frazer, Phys. Rev. **115**, 1763 (1959).
[7] See reference 5, footnote 27.

Fig. 1(a). This assumption appears to have been justified by recent experiments.[2] It must be remembered that this approximation is only valid for Δ^2 small, i.e., $\Delta^2<10$.

We have, then,

$$\langle q_1, q_2-\|\bar{u}(p')f(0)\|pq+\rangle = -\sqrt{2}gi\bar{u}(p')\gamma_5 u(p) \times T_{\pi\pi}(-(q_1+q_2)^2, \cos\theta)/(\Delta^2+1). \quad (9)$$

$T_{\pi\pi}(-(q_1+q_2)^2, \cos\theta)$ is the $\pi^+\pi^-$ scattering amplitude for center-of-mass energy squared $-(q_1+q_2)^2$. θ is defined as the angle between $\mathbf{q}_1$ and $\mathbf{q}$ in the center-of-mass system of the two pions q_1, q_2. When Eq. (9) is substituted into the expression for $\mathrm{Im}T_\mu$ it is seen, as expected, that only the P-wave part of $T_{\pi\pi}$ can contribute. Furthermore, this approximation will yield contributions to the imaginary part of only two of the invariant functions appearing in T_μ. Expressing T_μ in terms of these invariants, we write

$$T_\mu = i\bar{u}(p')\gamma_5 u(p)[Ak_\mu+Bq_\mu]+\cdots. \quad (10)$$

With our approximations, only ImA and ImB are nonzero. After some straightforward algebra, we find, setting $\rho^2=-k^2$,

$$\mathrm{Im}B(W^2,\Delta^2;\rho^2)=\frac{2\sqrt{2}g}{\Delta^2+1}\eta(\rho^2,\Delta^2)F_\pi^*(\rho^2)e^{i\delta}\sin[\delta(\rho^2)], \quad \rho^2\approx t_r.$$

$\delta(\rho^2)$ is the $J=1$, $T=1$ pion-pion phase shift as a function of the square of the total center-of-mass energy, ρ^2. $\eta(\rho^2,\Delta^2)$ is the magnitude of the ratio of the pion momentum with energy $\frac{1}{2}(\rho^2)^{\frac{1}{2}}$ to the momentum of the incident pion $\mathbf{q}$ evaluated in the photon rest system, i.e., $\mathbf{k}=0$.

$$\eta(\rho^2,\Delta^2)=\left[\frac{(\rho^2)^2-4\rho^2}{(\rho^2+\Delta^2+1)^2-4\rho^2}\right]^{\frac{1}{2}}.$$

Remember that for $\rho^2\approx t_r$,

$$\mathrm{Im}F_\pi(\rho^2)=F_\pi^*(\rho^2)e^{i\delta}\sin[\delta(\rho^2)],$$

so that we write

$$\mathrm{Im}B=\frac{2\sqrt{2}g}{\Delta^2+1}\eta(\rho^2,\Delta^2)\,\mathrm{Im}F_\pi(\rho^2), \quad \rho^2\approx t_r. \quad (11)$$

Electromagnetic current conservation requires

$$k_\mu T_\mu=0$$

This condition is explicitly satisfied by $\mathrm{Im}T_\mu$ given by Eq. (7) or (8), so that ImA is simply related to ImB,

$$\mathrm{Im}A=-(kq/k^2)\,\mathrm{Im}B=-\tfrac{1}{2}[(\rho^2+\Delta^2+1)/\rho^2]\,\mathrm{Im}B.$$

Since $\bar{u}(p_1)\gamma k v(p_2)=0$ for $k=p_1+p_2$, A does not contribute anything to T, defined by Eq. (4).

$\mathrm{Im}F_\pi(\rho^2)$ is strongly peaked at $\rho^2=t_r$. Hence we can make the approximation,

$$\mathrm{Im}B=\frac{2\sqrt{2}g\eta(t_r,\Delta^2)}{\Delta^2+1}\,\mathrm{Im}F_\pi(\rho^2), \quad \rho^2\approx t_r.$$

Substituting this into the dispersion relation for B and comparing this with the dispersion relation for F_π,

$$F_\pi(-k^2)=\frac{1}{\pi}\int_4^\infty \frac{\mathrm{Im}F_\pi(\rho^2)}{\rho^2+k^2-i\epsilon}d\rho^2,$$
$$B=\frac{2\sqrt{2}g\eta(t_r,\Delta^2)}{\Delta^2+1}F_\pi(\rho^2), \quad \rho^2\approx t_r. \quad (12)$$

For

$$t_r\gg\Delta^2, \quad \eta(t_r,\Delta^2)=1+O(\Delta^2/t_r).$$

Because of the approximations involved in obtaining B, it is consistent to replace η by 1. Hence, we find for T_μ, restricting Δ^2 to small values, and $-k^2$ to values near t_r,

$$T_\mu=\frac{2\sqrt{2}gi\bar{u}(p')\gamma_5 u(p)}{\Delta^2+1}\left[q_\mu-\frac{kq}{k^2}k_\mu\right]F_\mu(-k^2), \quad -k^2\approx t_r. \quad (13)$$

The term proportional to q_μ in Eq. (13) is just the result we would have obtained from the single pion exchange graph in Fig. 1(b). This result is not surprising. It must be stressed, however, that it follows from the assumption, made in Eq. (9), that the single pion exchange graph gives the dominant contribution to $\pi^-+p\to\pi^++\pi^-+n$ for Δ^2 small and $-k^2\sim t_r$. It is reassuring that we find a gauge-invariant solution, by this method, which gives the same contribution to T as is obtained from the single pion exchange graph. Since the latter result is not gauge invariant we could not be sure that gauge invariance would require other contributions to T_μ from other graphs which do contribute to T. This is indeed the case for $k^2=0$, or $k^2>0$, where one must introduce all the possible invariants to find a consistent gauge-invariant solution.[8] Note that for $k^2=0$, Eq. (13) is meaningless. At this point $F_\pi(0)=1$ and the terms in (13) no longer become large so that the many invariant functions we have neglected become important.

We see then, that a study of reaction (2) in the region $-k^2\approx t_r$ provides the greatest hope for extracting information about the electromagnetic structure of the charged pion, for it is in this region of the photon mass, provided $\Delta^2<10$, that the amplitude for this process, T_μ, is most simply related to F_π.

It remains to see whether or not the rate predicted for this reaction is large enough to make the experiment feasible in the near future. In the succeeding section we give numerical estimates for this rate, taking for T_μ the results of Eq. (13).

III. DIFFERENTIAL CROSS SECTION

We take for the invariant scattering amplitude T for reaction (2), $\pi^-+p\to e^++e^-+n$, the single pion-exchange contribution:

$$T=\sqrt{2}8\pi g\alpha\frac{i\bar{u}(p_1)\gamma q v(p_2)}{k^2}\frac{i\bar{u}(p')\gamma_5 u(p)}{\Delta^2+1}F_\pi(-k^2). \quad (14)$$

Using this expression in the S matrix we easily obtain for the differential cross section in an arbitrary coordinate system,[9]

$$d\sigma(\pi^-+p\to e^++e^-+n)\equiv d\sigma(e^+,e^-)=\frac{8g^2\alpha^2}{(2\pi)^3}\left|\frac{F_\pi(-k^2)}{k^2}\right|^2\frac{\Delta^2}{(\Delta^2+1)^2} \times\frac{[2(qp_1)(qp_2)+\frac{1}{2}k^2]\delta_+[(p+q-p_1-p_2)^2+M^2]d^3p_1d^3p_2}{p_0q_0p_{10}p_{20}|(\mathbf{q}/q_0)-(\mathbf{p}/p_0)|}, \quad (15)$$

[8] J. S. Ball, Phys. Rev. **124**, 2014 (1961); P. Dennery, *ibid.* **124**, 2000 (1961).

[9] The differential cross section, $\partial^2\sigma(e^+,e^-)/\partial k^2\partial\Delta^2$, expressed in terms of the invariant variables, takes on a much simpler form. For a counter experiment where the electron pair is detected at fixed angles, this simpler expression is not directly applicable.

g, the pion-nucleon coupling constant, satisfies the usual relation, $(g^2/4\pi)(1/2M)^2 \equiv f^2 \cong 0.08$. The notation used in Eqs. (14) and (15) is defined at the beginning of Sec. II. We may expect Eq. (15) to be a good approximation provided

$$-k^2 \approx t_r \approx 30 \quad \text{and} \quad \Delta^2 < 10,$$

for incident π^- energies, in the laboratory system, between 1 and 2 Bev.[2]

To estimate the counting rate that Eq. (15) predicts, let us take the laboratory system where the proton is at rest. Consider an experiment in which the electron counters are placed in a continuous annular ring about the π^- incident beam which passes through the center. We can then count both the electron and positron at any azimuthal angles but at the same angle θ with respect to the π^- direction. In the laboratory system, denote the pion four-momentum by $(\mathbf{q}_L,\omega_L)$ and express p_1 and p_2 in spherical coordinates with $\mathbf{q}_L$ in the direction of the z axis, i.e.,

$$p_i = (E_i,\theta_i,\phi_i;E_i), \quad i=1,2.$$

Since we restrict ourselves to values of $-(p_1+p_2)^2 \approx t_r \gg m^2$, we can neglect the electron mass.

If we eliminate the δ function in (15) by performing the ϕ_1 and ϕ_2 integrations, and introduce the new variables,

$$\eta = E_1+E_2, \quad \rho = E_1E_2, \tag{16}$$

we obtain the useful result,

$$\left(\frac{\partial^3\sigma(e^+,e^-)}{\partial\theta_1\partial\theta_2\partial\eta}\right)_{\theta_1=\theta_2=\theta} = 8f^2\alpha^2\frac{M}{\omega_L}\sin\theta\frac{\Delta^2}{(\Delta^2+1)^2}|F_\pi(Q^2)|^2\Sigma(\eta),$$

with

$$\begin{aligned}\Sigma(\eta) &= (Q^2)^{-\frac{3}{2}}\{\omega_L^2(1-\cos\theta)^2[3(a^2+b^2)-2ab] \\ &\qquad -Q^2(a+b)\},\\ a &= Q^2/(4\sin^2\theta), \quad b=\tfrac{1}{4}\eta^2, \\ Q^2 &= -k^2 = 2[M+\omega_L(1-\cos\theta)]\eta - 2\omega_L M - 1, \\ \Delta^2 &= 2M(\omega_L-\eta).\end{aligned} \tag{17}$$

We have set $q_L=\omega_L$ in Eq. (17). $\eta=E_1+E_2$, can lie in the range

$$\eta_1 \leqslant \eta \leqslant \eta_2.$$

η_2 is determined simply by the kinematics which requires $b \geqslant a$;

$$\eta_2 = \frac{\xi - [\xi^2-(2\omega_L M+1)\cos\theta]^{\frac{1}{2}}}{\sin^2\theta}, \tag{18}$$

$$\xi = M+\omega_L(1-\cos\theta).$$

The lower limit η_1 depends on the range of values of E_1, E_2 and $|\phi_1-\phi_2|$, that the counters select. Let us suppose that the electron counters select only those events satisfying the conditions,

$$E_1, E_2 \geqslant E_0; \quad \beta \leqslant |\phi_1-\phi_2| \leqslant \pi, \tag{19}$$

we then obtain for η_1,

$$\eta_1 = \frac{2\omega_L M + 1 - 4E_0^2\sin^2\theta\sin^2(\beta/2)}{2\xi - 4E_0\sin^2\theta\sin^2(\beta/2)}. \tag{20}$$

Ideally, an experiment should try to determine the quantity $(\partial^3\sigma/\partial\theta_1\partial\theta_2\partial\eta)_{\theta_1=\theta_2=\theta}$ as a function of η and, therefore, Q^2. This would determine $|F_\pi(Q^2)|^2$ as a function of Q^2; provided, of course, the single pion exchange approximation is valid. The angle θ should be chosen so that Δ^2 takes on its minimum value when $Q^2\approx t_r$. In practice, measuring the differential cross section as a function of η may prove too difficult. Information about $|F_\pi|^2$ can still be obtained by measuring $(\partial^2\sigma/\partial\theta_1\partial\theta_2)_{\theta_1=\theta_2=\theta}$ for various values of θ. To estimate the counting rate, we have calculated $(\partial^2\sigma/\partial\theta_1\partial\theta_2)_{\theta_1=\theta_2=\theta}$ for several values of θ. The numerical integrations over η were performed graphically, taking as limits η_2 and η_1, obtained by imposing conditions (19) with $\beta=2\pi/3$ and for $E_0=3$ and $E_0=2$. For simplicity, we took for $|F_\pi|^2$ a Breit-Wigner form,

$$|F_\pi|^2 = \frac{t_r^2}{(Q^2-t_r)^2+\Gamma^2}, \quad t_r=30, \Gamma=6.6,$$

suggested by the results of Walker *et al.*[2] Table I shows the results obtained for an incident pion energy, $\omega_L=10$.

Table I. $(\partial^2\sigma(e^+,e^-)/\partial\theta_1\partial\theta_2)_{\theta_1=\theta_2=\theta}$, in cm²/rad² calculated in the one-pion exchange approximation, for several values of θ.[a] $\omega_L=10$; $\beta=2\pi/3$.

		$\theta_0=\cos^{-1}0.83$ (33.9°)	$\cos^{-1}0.80$ (36.9°)	$\cos^{-1}0.77$ (39.6°)
$\partial^2\sigma(e^+,e^-)/\partial\theta_1\partial\theta_2$,	$E_0=2$	4.3×10^{-31}	5.3×10^{-31}	5.1×10^{-31}
	$E_0=3$	3.9×10^{-31}	4.7×10^{-31}	4.1×10^{-31}

[a] E_0 and β are defined in Eq. (19).

The minimum angle taken, $\theta_0=34°$, was chosen by the condition that $Q^2=t_r$ when Δ^2 is a minimum, at the point $\eta=\eta_2$. For $\theta<\theta_0$, $Q^2<t_r$, for $\theta>\theta_0$ the minimum value of Δ^2 occurs for $Q^2>t_r$, so that $Q^2=t_r$ occurs at increasing values of Δ^2 as we increase θ above θ_0. To restrict ourselves to small values of Δ^2, θ must be chosen close to the value θ_0. As we increase E_0, the lower bound on the electron or positron energy, η_1 increases, thereby reducing the maximum value of Δ^2 that can be obtained. As seen from the table, $(\partial^2\sigma/\partial\theta_1\partial\theta_2)_{\theta_1=\theta_2=\theta}$ decreases slowly with increasing E_0. Consequently, even larger values of E_0 may be desirable.

Further calculations indicate that the differential cross section is only weakly dependent on the incident energy in the energy range between 1 and 2 Bev. This is true provided one always takes values of θ close to

the minimum angle θ_0 that arises at each particular energy. The dependence of Δ^2 on energy is more significant. The minimum value of Δ^2, permitted by the kinematics, decreases with increasing energy. Unfortunately, the rate of increase of Δ^2 with increasing θ becomes greater at the same time. These two opposing tendencies make beam energies in the neighborhood of 1.4 Bev the most desirable.

To estimate the expected counting rate let us assume that the counters restrict θ to the range $(\theta_0, \theta_0+\Delta\theta)$ with $\Delta\theta/\theta_0\ll 1$. It is easy to verify that

$$\sigma(e^+,e^-)=\left(\frac{\partial^2\sigma(e^+,e^-)}{\partial\theta_1\partial\theta_2}\right)_{\theta_1=\theta_2=\theta\approx\theta_0}(\Delta\theta)^2+O[(\Delta\theta)^4],$$

so that

$$\begin{gathered}\sigma=4.3(\Delta\theta)^2\times10^{-31}\ \mathrm{cm}^2,\\ \omega_L=10,\qquad E_0=3,\\ \beta=2\pi/3,\quad \theta_0=34°.\end{gathered}\tag{21}$$

Taking $\Delta\theta=0.1$ radian, a target of 10^{24} protons/cm^2, we have the result that a beam of $10^6\ \pi^-$/sec yields a rate of 15 counts per hour.

With what appears to be a workable counting rate, the serious problem of background becomes the crucial issue. It would seem pertinent, therefore, to make a few preliminary remarks on this subject before closing this section. The reaction, $\pi^-+p\rightarrow\pi^++\pi^-+n$, will produce roughly 10^5 as many π^+,π^- pairs, (or their μ-decay product) which enter the counters in coincidence, as the reaction $\pi^-+p\rightarrow e^++e^-+n$ will produce electron pairs. The electron counters must therefore discriminate against charged pions and muons so that roughly only one in a thousand of these particles is counted. Neutral pions coming from the reactions, $\pi^-+p\rightarrow\pi^0+\pi^-+p$, $\pi^-+p\rightarrow 2\pi^0+n$ can also contribute real coincidences. The electron counters must discriminate against gamma rays which will convert into electron pairs in the counter. Only a fraction of the gamma rays, coming from the π^0 decay, will enter the counters and some of those which enter will have much reduced energies. Consequently, if the counters reject all but 1% of the incident gamma rays of sufficiently high energies, this source of background should present no difficulties. There is no way of discriminating against high energy electrons coming from the alternate decay mode $\pi^0\rightarrow\gamma+e^++e^-$, which occurs approximately 1/80 of the time.[10] The same arguments we applied to the gamma ray background also apply to the background due to these Dalitz pairs. The factor of 1/80 takes the place of the 1% gamma ray detection efficiency. The rejection of lower energy electrons is a very important factor in reducing the background due to both these sources. We see then that choosing as large as possible the lower bound on the electron energies that the counters accept, limits the size of Δ^2, restricts Q^2 more closely to values near t_r and helps considerably to reduce the background.

Another major source of background comes from accidental coincidences. The accidental rate depends on the incident pion flux during a pulse and, therefore, upon the properties of the particular accelerator being used. To get an idea of the conditions that need to be imposed on the flux, we estimated the accidental rate due to the processes $\pi^-+p\rightarrow\pi^0+n$ and $\pi^-+p\rightarrow 2\pi^0+n$. Taking recent data for the total cross sections for these two reactions combined,[11] we find the ratio of accidental coincidences to true events to be much less than one for beam fluxes much less than $10^{11}\ \pi^-$ per sec during the pulse. We have assumed a target of 10^{24} protons/cm^2 as earlier and a coincidence circuit resolving time of less than 5×10^{-9} sec. For proton synchrotrons with duty cycles of several seconds, an average beam flux of $10^6\ \pi^-$/sec means between 10^6 to $10^7\ \pi^-$ per pulse. For pulses spread out over a millisecond or longer duration, the accidental counting rate due to these two reactions should be negligible.

It must be emphasized that no detailed calculations of rates due to background have been made. The estimates made above are only rough ones and the various sources of background have not been exhausted. Nevertheless, from the discussion above, the conclusion seems plausible that the problems connected with background are surmountable. This is true provided electron counters satisfying the properties outlined above can be designed.

IV. CONCLUSIONS

It is unfortunate that this paper should appear so complicated when the ideas used here and the results obtained are so simple. We have calculated differential cross sections for the reaction $\pi^-+p\rightarrow e^++e^-+n$ by taking only the contribution from the single pion exchange graph shown in Fig. 1(b). We can only justify this approximation by appealing to the experimental results for the process[2] $\pi^-+p\rightarrow\pi^++\pi^-+n$. The dispersion relation presented (in small print) is merely a restatement of this fact in more formal language. The formalism does show, however, how the requirement of electromagnetic current conservation is satisfied while retaining the single pion exchange approximation. The differential cross section, given by Eq. (17), is tailored to fit a counter experiment and is therefore much more complicated than the simple result that is obtained for the differential cross section expressed in terms of invariant variables.

The rest of the paper has been concerned with numbers, a necessary evil to any theoretician who wishes to suggest an experiment. The numbers obtained seem encouraging. For an accelerator capable of attaining a beam of the order of $10^6\ \pi^-$ per sec, a counting rate of

[10] An electron and positron from the same Dalitz pair cannot make an acceptable event. The virtual photon, giving rise to the Dalitz pair, must have a mass less than one. Low-mass events are easily rejected by the electron counters.

[11] J. C. Brisson, J. D. Detoeuf, P. Falk-Vairant, L. Van Rossum, and G. Valladas, Nuovo cimento **19**, 210 (1961).

several an hour is to be expected. There is a serious problem of background but our rough estimates indicate that it is a solvable problem.

It is not absolutely certain, though quite probable, that the pion-pion resonance recently observed[2] will show up in electron pair production. If it does, the evidence would be overwhelming that this is indeed a $J=1$, $T=1$ resonance and plays an important role in the electromagnetic structure of nucleons and pions. Even if the single pion exchange approximation turns out to be poor, we should still observe this resonance for center of mass energies of the electron positron system close to the pion-pion resonant energy. The recent observation of a three-pion resonance[12] at almost the same energy as for the two-pion resonance could alter our results. It is probable, however, that the effect of the three-pion resonance on the electron pair mass distribution would only arise for larger momentum transfer. It would also be worth while to examine this reaction for values of the electron-positron center-of-mass energy below the two-pion resonant energy to look for further structure effects.

[12] B. C. Maglić, L. W. Alvarez, A. H. Rosenfeld, and M. L. Stevenson, Phys. Rev. Letters 7, 178 (1961).

ACKNOWLEDGMENTS

The author would like to thank S. Gasiorowicz for very helpful discussions and N. Hintz, R. L. Cool, and W. K. H. Panofsky for help with the experimental problems involved.

PHYSICAL REVIEW VOLUME 125, NUMBER 5 MARCH 1, 1962

Mass Quantization and Lepton Theory*

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(Received October 9, 1961)

This article discusses some features of mass quantization obtained by the introduction of a continuous inner degree of freedom into a free field. The usual particle interpretation, with discrete mass values, is applicable, as shown in the case of a second-quantized scalar field. A simple class of fermion field equations with unexpected lepton-like properties is also presented and studied in some detail.

1. INTRODUCTION

THE setting up of an eigenvalue problem has for a long time been a tempting method of introducing a mass spectrum into a quantized field theory.[1] Such an approach could be based on the hope that the current Lagrangian models are, by virtue of their interactions, truly consistent only for certain mass ratios; the decoupled fields, of course, are consistent with any mass spectrum.

Alternatively (and this is the approach underlying the present remarks), those models might be modified in such a way that the eigenvalue problem persists even after the coupling constants have been set equal to zero; in this noninteracting case it should be entirely equivalent to the usual attribution of chosen masses to the various decoupled fields. The only interest of formulating this simple assignment of mass values in a less conventional language lies in the possibility that the two languages may become nonequivalent as soon as the "usual" interaction is turned on.

The present discussion is restricted to free fields and is based on the intuitive idea that, if a field ϕ which depends on a continuous parameter λ (in addition to its space-time coordinate x) satisfies the wave equation

$$(\Box+\Lambda)\phi(x,\lambda)=0, \tag{1.1}$$

where Λ is a functional operator acting on λ and having a discrete spectrum, then one should be able to decompose ϕ into free fields whose squared masses are the eigenvalues of Λ. The exact meaning of such a statement will be discussed. In order to illustrate the heuristic value of the approach described here, the example will be presented of a fermion field exhibiting some properties similar to those sought in the theory of leptons.

2. NEUTRAL SCALAR FIELD

The simplest example of mass quantization is provided by the neutral scalar field $\phi(x,\lambda)$ for which a Lagrangian density

$$\mathfrak{L}=-\frac{1}{2}\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x_\mu}+\tfrac{1}{2}m^2\left[\left(\frac{\partial\phi}{\partial\lambda}\right)^2-V(\lambda)\phi^2\right], \tag{2.1}$$

and a Lagrangian

$$L=\int d^3x\int d\lambda\ \mathfrak{L}$$

* Work supported by the National Science Foundation.
[1] See, in particular, E. Minardi, Nuovo cimento 3, 968 (1956); 7, 715, 898 (1958), and the bibliography therein contained.