Scattering of Photons by Protons*

LLOYD G. HYMAN†
Harvard University, Cambridge, Massachusetts

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A previously proposed model to account for photon scattering from protons in the 50 to 300 Mev range is presented in detail. The scattering amplitude is composed of (1) Klein-Nishina amplitude, (2) resonance magnetic dipole, (3) resonance electric dipole, (4) Low term. Using the measured value of the π^0 lifetime $=(2\pm1)\times10^{-16}$, which enters as a parameter in amplitude (4), the model is compared to experiment. The agreement is poor particularly at the scattering angle $=3\pi/4$.

I. INTRODUCTION

WE proposed previously[1] a relatively simple model to account for photon-proton scattering in the gamma-ray region 50 to 300 Mev. At the time we applied it only to the data for the scattering angle $\theta_s=\pi/2$, with what we believed to be considerable success. In subsequent calculations this model failed to fit the $\theta_s=3\pi/4$ data with reasonable parameters.[2,3] The purpose of this note is to record the details of the model.

II. SCATTERING AMPLITUDES

The model consists of four elementary scattering amplitudes, all computed in the barycentric system:

(1) Klein-Nishina amplitude[4-6]:

$$T_{\mathrm{KN}}=\frac{e^2}{M}\frac{1}{(1+\beta)}\{-\hat{e}\cdot\hat{e}'+\beta(\hat{e}\cdot\hat{k}')(\hat{e}'\cdot\hat{k})$$
$$+\tfrac{1}{2}i\beta\hat{\sigma}\cdot(\hat{e}'\times\hat{e})+\tfrac{1}{2}i\beta\hat{\sigma}\cdot(\hat{e}\times\hat{k})\times(\hat{e}'\times\hat{k}')$$
$$+\tfrac{1}{2}i\beta[\hat{\sigma}\cdot(\hat{k}'\times\hat{e}')(\hat{e}\cdot\hat{k}')-\hat{\sigma}\cdot(\hat{k}\times\hat{e})(\hat{e}'\cdot\hat{k})]\},$$

where $\hat{e}$, $\hat{e}'$ are unit polarization vectors for incoming and outgoing γ rays, and $\hat{k}$, $\hat{k}'$ are unit vectors along direction of incoming and outgoing γ rays. This amplitude is accurate to order β, where β is defined as $E_{\mathrm{lab}}/(M+E_{\mathrm{lab}})$, with E_{lab} the laboratory photon energy and M the nucleon rest mass.

(2) Resonance ($T=\frac{3}{2}$, $J=\frac{3}{2}$) magnetic dipole scattering amplitude[7]:

$$T_{\frac{3}{2},\frac{3}{2}}=a_{33}\{2(\hat{k}'\times\hat{e}')\cdot(\hat{k}\times\hat{e})-i\hat{\sigma}\cdot(\hat{k}'\times\hat{e}')\times(\hat{k}\times\hat{e}),$$

$$a_{33}=\frac{1}{6}\frac{e^2}{M}\left(\frac{\mu_p-\mu_n}{2f}\right)^2\frac{m}{M}\frac{k^2}{\eta^3}\sin\delta_{33}\,e^{i\delta_{33}},$$

where k is the center-of-mass photon momentum in units of pion rest mass, η is pion momentum (measured in units of pion rest mass) in the center-of-mass system at the same total center-of-mass energy $=\omega_\eta=\omega_k$, f^2 is pion-nucleon coupling constant $=0.08$, μ_p and μ_n are proton and neutron moments in units of nucleon Bohr magnetons, and δ_{33} is phase shift for π^+-p scattering through resonance $\frac{3}{2}$, $\frac{3}{2}$ state. δ_{33} is evaluated using a Chew-Low fit wherein $\cot\delta_{33}=\frac{3}{4}[\omega(1-\omega/\omega_r)/f^2\eta^3]$ with $\omega_r=2.11$.[8] The same formulas are used below meson threshold, where η and δ_{33} become imaginary and R is entirely real.

(3) Amplitude of electric dipole scattering from the tail of the $T=\frac{1}{2}$, $J=\frac{3}{2}$ resonance:

$$T_{\frac{1}{2},\frac{3}{2}}=(e^2/M)a_{13}\{2\hat{e}\cdot\hat{e}'-i\hat{\sigma}\cdot(\hat{e}'\times\hat{e})\}.$$

Given a_{13} at one energy, we extrapolate to other energies as follows: Assuming a one-level resonance form, we have

$$\sigma_{\gamma\gamma}/\sigma_{\gamma\pi}=\sigma_{\pi\gamma}/\sigma_{\pi\pi},\quad \sigma_{\gamma\pi}\sim\sigma_{\pi\gamma}(\eta^2/k^2),$$

by detailed balance; therefore

$$\sigma_{\gamma\gamma}\sim(\sigma_{2\pi\gamma}/\sigma_{\pi\pi})(\eta^2/k^2).$$

Thus, using the notation of partial widths,

$$\frac{\sigma_{2\pi\gamma}}{\sigma_{\pi\pi}}\sim\frac{\Gamma_\pi{}^2\Gamma_\gamma{}^2}{\Gamma_\pi{}^2[(\omega-\omega_r)^2+\frac{1}{4}\Gamma^2]}\frac{1}{\eta^2},$$

or

$$\sigma_{\gamma\gamma}\sim\frac{\Gamma_\gamma{}^2}{k^2[(\omega-\omega_r)^2+\frac{1}{4}\Gamma^2]}.$$

On the tail,

$$\tfrac{1}{4}\Gamma^2\ll(\omega-\omega_r)^2,\quad \Gamma_\gamma\sim(ka)^3/[1+(ka)^2];$$

therefore

$$a_{13}\sim k^2/(\omega-\omega_r)(1+k^2a^2),$$

with $a=$ range of interaction. We take $a=0.88$ meson Compton wavelengths and $\omega_r=4.12$, corresponding to a laboratory energy $=750$ Mev. Because the phase shift is small, a_{13} is real in the region being discussed.

(4) Amplitude due to π^0 interaction with radiation

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† Now at Argonne National Laboratory, Argonne, Illinois.
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[2] G. E. Pugh, R. Gomez, D. H. Frisch, and G. S. Janes, Phys. Rev. **105**, 982 (1957).
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[4] F. E. Low, Phys. Rev. **96**, 1428 (1954).
[5] M. Gell-Mann and M. L. Goldberger, Phys. Rev. **96**, 1433 (1954).
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field[9,10]:

$$T_{\pi^0}=(e^2/M)a_L(\cos\theta)[(\hat{k}-\hat{k}')\cdot(\hat{e}\times\hat{e}')i\hat{\sigma}\cdot(\hat{k}-\hat{k}')],$$

$$a_L(\cos\theta)=\frac{1}{\lambda^{\frac{1}{2}}}\frac{M}{m}\frac{f^2}{\pi}\frac{k}{(1-\cos\theta+1/2k^2)};$$

λ is the π^0 lifetime in units of $\tau_0=(\mu c^2\alpha^2 f^2/\hbar 4\pi^2)^{-1}\simeq 5\times10^{-17}$ sec. The cross section is found by taking the absolute square of the total amplitude, summing over final states, and averaging over initial states. The result is

$$\frac{d\sigma}{d\Omega}=\frac{1}{2}\left(\frac{e^2}{Mc^2}\right)^2\left\{\frac{1}{(1+\beta)^2}(1+\cos^2\theta+2\beta\sin^2\theta\cos\theta)\right.$$
$$+|a_{33}|^2(7+3\cos^2\theta)+|a_{13}|^2(7+3\cos^2\theta)$$
$$+4a_L^2(1-\cos\theta)^3+\mathrm{Re}\,a_{33}\left[20a_{13}\cos\theta-\frac{8\cos\theta}{1+\beta}\right.$$
$$\left.-4a_L(1-\cos\theta)^2+\beta(2(1-\cos\theta)-3\sin^2\theta)\right]$$
$$+a_{13}\left[\frac{-4}{1+\beta}(1+\cos^2\theta)+4a_L(1-\cos\theta)^2\right.$$
$$\left.+\beta(1-2\cos\theta-3\cos^2\theta+4\cos^3\theta)\right]$$
$$\left.-2a_L\beta(1-\cos\theta)^3\right\}.$$

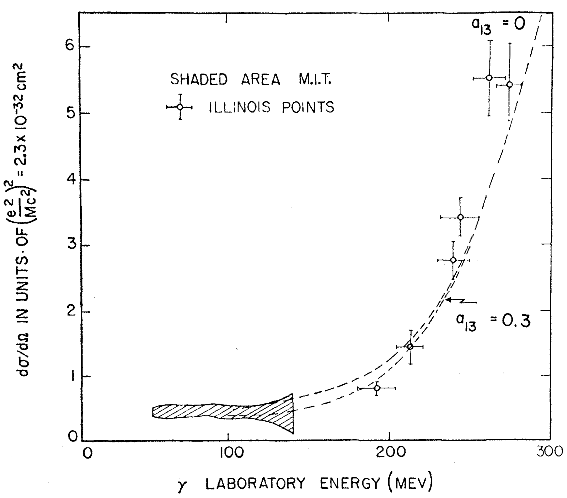

FIG. 1. Differential cross section at $\theta=\pi/2$ vs laboratory energy. The Low term is in with perturbation theory phase. $\tau_{\pi^0}=2\times10^{-16}$ sec.

[9] F. E. Low (private communication).

[10] M. Jacob and J. Mathews, Phys. Rev. **117**, 854 (1960).

III. DISCUSSION

The primary purpose of this model at the time was to see if we could place a lower limit on the π^0 lifetime, which enters as a parameter in amplitude (4). We took the relative sign of the amplitude for process (4) given by perturbation theory[11]; this sign yielded a lower limit for the lifetime, $\tau_{\pi^0}\geq10^{-18}$ sec. Subsequent calculations by Lapidus and Chou Kuang-chao indicate that the term has opposite phase from that taken by us[12]; this would have given $\tau_{\pi^0}\geq10^{-17}$ sec. Since then, the value of $\tau_{\pi^0}=(2\pm1)\times10^{-16}$ sec has been measured.[13,14] Using the lifetime $\tau_{\pi^0}=2\times10^{-16}$ sec and regarding a_{13} at a given energy as a free parameter to be extrapolated to other energies by a one-level resonance technique, we arrive at the set of curves shown in Figs. 1 and 2 for $\theta_s=\pi/2$ and $3\pi/4$, with the perturbation theory sign of π^0 interaction amplitude a_L. If instead we take the Lapidus-Chou sign for a_L, we obtain the curves shown in Figs. 3 and 4. The fit at $\theta_s=3\pi/4$ is poor in all cases; an a_{13} value of approximately 0.2 with the Lapidus-Chou phase for a_L is roughly the best fit at $3\pi/4$ with $\tau_{\pi^0}=2\times10^{-16}$ sec.

The model suffers from the following defects:

(a) According to Jacob and Mathews,[10] when the dispersion integrals are evaluated to include the $T=\frac{1}{2}$, $J=\frac{3}{2}$ resonance, the resonance has little effect below 250 Mev. This implies that a value $a_{13}=0.2$ at 250 Mev

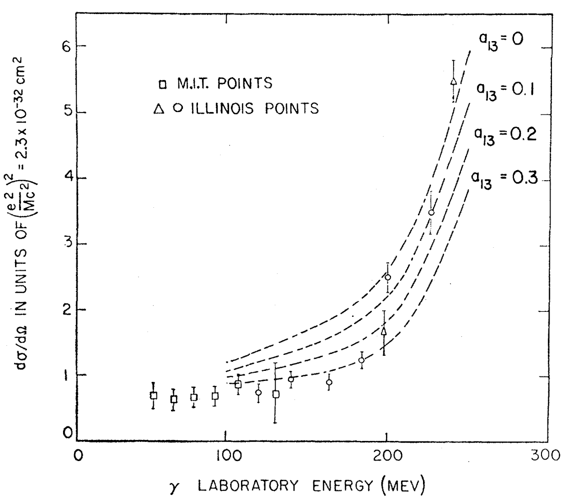

FIG. 2. Differential cross section at $\theta=3\pi/4$ vs laboratory energy. The Low term is in with perturbation theory phase. $\tau_{\pi^0}=2\times10^{-16}$ sec.

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[12] L. I. Lapidus and Chou Kuang-chao, Joint Institute for Nuclear Research Report USSR D-681, 1961 (unpublished).

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[14] R. G. Glasser, N. Seeman, and B. Stiller, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, New York, 1960), p. 30.

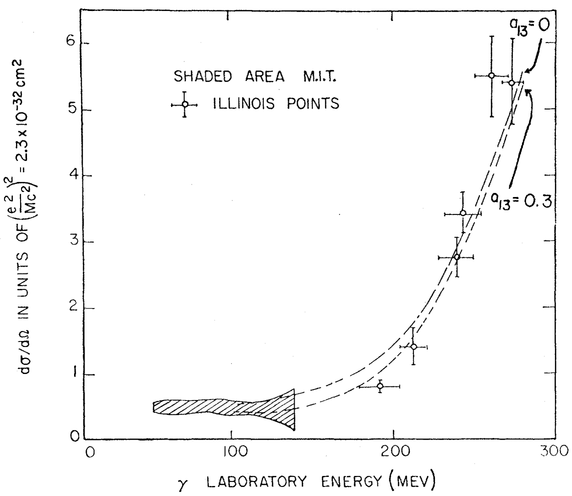

FIG. 3. Differential cross section at $\theta=\pi/2$ vs laboratory energy. The Low term is in with Lapidus-Chou phase. $\tau_{\pi^0}=2\times10^{-16}$ sec.

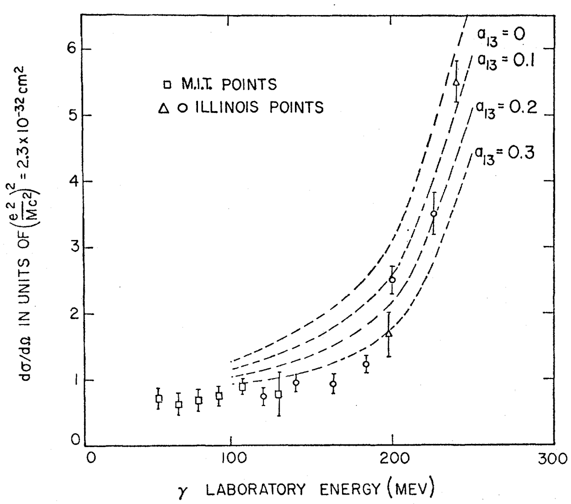

FIG. 4. Differential cross section at $\theta=3\pi/4$ vs laboratory energy. The Low term is in with Lapidus-Chou phase. $\tau_{\pi^0}=2\times10^{-16}$ sec.

is unrealistically large. Berkelman,[15] using a one-level resonance technique and the photoproduction data from the second resonance, estimates a_{13} at 250 Mev of the order 0.1, which in the present model affects the $\theta_s=3\pi/4$ cross sections noticeably. The discrepancy concerning the importance of a_{13} between the dispersion-theoretic approach of Jacob and Mathews and this model may arise because there may be nonresonant contributions to a_{13} which are as important in this energy range as the resonance contributions.

(b) Terms which correspond to electric dipole scattering through the $J=\frac{1}{2}$ state have been left out. Perturbation-theory calculations by Karzas, Watson, and Zachariasen[7] indicate the $J=\frac{1}{2}$ terms to be of the same order as the terms we have included.

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[15] K. Berkelman, Istituto Superiore di Sanita Report ISS-61/13 (unpublished).